Solution

UNITS AND MEASUREMENT

Class 11 - Physics

Section A

1.
   (c) 2%

   Explanation:

   The error in the measurement of the sides of a rectangle is 1%. The error in the measurement of its area is 2%.

   $A = l \times b$

   $\therefore \frac{\triangle A}{A} = \frac{\triangle l}{l} + \frac{\triangle b}{b} = 1\% + 1\% = 2\%$

2. **(a)** Distance

   Explanation:

   A light year is a unit of distance.

3.
   (d) volt

   Explanation:

   volt

4.
   (c) Force

   Explanation:

   [$L^1M^1T^{-1}$] is the dimensional formula for force.

5.
   (b) Work

   Explanation:

   Dimensions of kinetic energy are the same as that of work.

6. For a given set of measurements of a quantity, the magnitude of the difference between mean value (Most probable value) and each individual value is called absolute error (Δa) in the measurement of that quantity.
7. The maximum possible error in $(A \pm B)$ is $(\Delta A + \Delta B)$.
8. The relative error represented by percentage (i.e., multiplied by 100) is called the percentage error.

   Percentage error $= \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$

Section B

9. **Given:** $X = \frac{a^4 b^3}{c^{1/3} d^{1/2}}$

   Percentage error in a, b, c, d is respectively $2\%, 3\%, 3\%$ and 4%.

   Now, Percentage error in X

   $= \left(4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{1}{3}\frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}\right) \times 100\%$

   $= \left[(4 \times 2) + (3 \times 3) + \left(\frac{1}{3} \times 3\right) + \left(\frac{1}{2} \times 4\right)\right] \times 100\%$

   $= [8 + 9 + 1 + 2] \times 100\% = 20\%$

10. $R = 1.3 \times 10^{-16} \times A^{1/3}\ m$

    For $A = 125$

    $R = 1.3 \times 10^{-16} \times (125)^{1/3}$

    $= 1.3 \times 10^{-16} \times \left(5^3\right)^{1/3}$

    $= 1.3 \times 10^{-16} \times 5$

    $= 6.5 \times 10^{-16}$

$= 0.65 \times 10^{-15}\ m$

$\therefore$ Order of magnitude $= -15$

11. **Given:** $m = 60.0\ g, v = 25.0\ cm/s,\ \ \Delta m = 0.3\ g, \Delta v = 0.1\ cm/s$

To find: Percentage error in E

Formula: Percentage error in E

$= \left(\frac{\Delta m}{m} + 2\frac{\Delta v}{v}\right) \times 100\%$

Calculation: From formula,

Percentage error in $E = \left(\frac{0.3}{60.0} + 2 \times \frac{0.1}{25.0}\right) \times 100\% = 1.3\%$

The percentage error in energy is 1.3%.

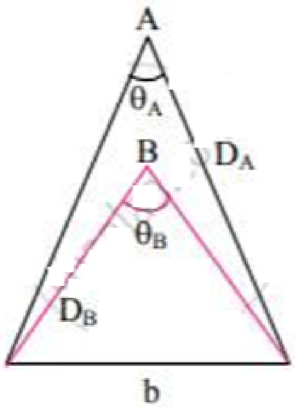
12.

Number	No. of significant figures	Reason
$0.003\ m^2$	1	Rule no. iii
$0.1250\ g\ cm^{-2}$	4	Rule no. iv
$6.4 \times 10^6\ m$	2	Rule no. i
$1.6 \times 10^{-19} C$	2	Rule no. i
$9.1 \times 10^{-31}\ kg$	2	Rule no. i

13. For a given set of measurements of the same quantity, the arithmetic mean of all the absolute errors is called mean absolute error in the measurement of that physical quantity.

$\Delta a_{\text{mean}} = \frac{\Delta a_1 + \Delta a_2 + \ldots\ldots + \Delta a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} \Delta a_i$

14.



i. 'b' is constant for the two stars

$\therefore \theta = \frac{1}{D}$

ii. As star A is farther i.e., DA $> D_B$

$\Rightarrow \theta_A < \theta_B$

Hence, star B will have a larger parallax angle than star A.

Section C

15. **Given:** $a_1 = 6.12\Omega, a_2 = 6.09\Omega, a_3 = 6.22\Omega, a_4 = 6.15\Omega$

To find:

i. Absolute error (Δa_{mean})

ii. Relative error

iii. Percentage error

Formulae: 1. $a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + a_4}{4}$

2. $\Delta a_n = \left|a_{\text{mean}} - \Delta a\right|$

3. $\Delta a_{\text{mean}} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \Delta a_4}{4}$

4. Relative error $= \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$

5. Percentage error $= \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$

Calculation: From formula (i),

$a_{\text{mean}} = \frac{6.12+6.09+6.22+6.15}{4}$
$= \frac{24.58}{4} = 6.145\ cm$

16. **Given:** Angular diameter $(\alpha) = 35.72''$
$= 35.72'' \times 4.847 \times 10^{-6} rad$
$\approx 1.73 \times 10^{-4} rad$
Distance from Earth $(D) = 824.7$ million km
$= 824.7 \times 106\ km$
$= 824.7 \times 109\ m.$
To find: Diameter of Jupiter (d)
Formula: $d = \alpha D$
Calculation: From the formula,
$d = 1.73 \times 10^{-4} \times 824.7 \times 10^{9}$
$= 1.428 \times 10^{8}\ m$
$= 1.428 \times 10^{5}\ km$
The diameter of Jupiter is $1.428 \times 10^{5}\ km$.

17. Volume of ball = Volume enclosed by rope.
$\frac{4}{3}\pi(\text{ radius })^3 =$ Area of cross-section of rope $\times$ length of rope.
$\therefore$ length of rope $I = \frac{\frac{4}{3}\pi r^3}{A}$
Given: $r = 2\ m$ and
Area $= A = 4 \times 4 = 16\ mm^2 = 16 \times 10^{-6}\ m^2$
$\therefore I = \frac{4 \times 3.142 \times 2^3}{3 \times 16 \times 10^{-6}}$
$= \frac{3.142 \times 2}{3} \times 10^6\ m$
$\approx 2 \times 10^6\ m.$
$\therefore$ Total length of rope to the nearest order of magnitude $= 10^6\ m = 10^3\ km$

18. Given: $A \pm \Delta A = 15.7 \pm 0.2\ kg$ and $B \pm \Delta B = 27.3 \pm 0.3\ kg$.
To find: Total mass (Z), and total error (ΔZ)
Formulae:
i. $Z = A + B$
ii. $\pm\Delta Z = \pm\Delta A \pm \Delta B$
Calculation: From formula (i),
$Z = 15.7 + 27.3 = 43\ kg$
From formula (ii),
$\pm\Delta Z = (\pm 0.2) + (\pm 0.3)$
$= \pm(0.2 + 0.3)$
$= \pm 0.5\ kg$
Total mass is $43\ kg$ and total error is $+0.5\ kg$.

19. Given: $v = at + \frac{b}{t+c} + v_0$
As only dimensionally identical quantities can be added together or subtracted from each other, each term on R.H.S. has dimensions of L.H.S. i.e., dimensions of velocity.
$\therefore [\text{ L.H.S. }] = [v] = \left[L^1 T^{-1}\right]$
This means, $[at] = [v] = \left[L^1 T^{-1}\right]$
Given, $t =$ time has dimension $\left[T^1\right]$
$\therefore [a] = \frac{\left[L^1 T^{-1}\right]}{[t]} = \frac{\left[L^1 T^{-1}\right]}{\left[T^1\right]} = \left[L^1 T^{-2}\right] = \left[L^1 M^0 T^{-2}\right]$
Similarly, $[c] = [t] = \left[T^1\right] = \left[L^0 M^0 T^1\right]$
$\therefore \frac{[b]}{\left[T^1\right]} = [v] = \left[LT^{-1}\right]$
$\therefore [b] = \left[L^1 T^{-1}\right] \times \left[T^1\right] = \left[L^1\right] = \left[L^1 M^0 T^0\right]$

20. Volume of sphere $= \frac{4}{3}\pi r^3$
$= \frac{4}{3} \times 3.142 \times \left(\frac{2.14}{2}\right)^3 \ldots \left(\because r = \frac{d}{2}\right)$

$= \frac{4}{3} \times 3.142 \times (1.07)^3$
$= 1.333 \times 3.142 \times (1.07)^3$
$= \{ \text{ antilog } [\log(1.333) + \log(3.142) + 3\log(1.07)]\}$
$= \{ \text{ antilog } [0.1249 + 0.4972 + 3(0.0294)]\}$
$= \{ \text{ antilog } [0.1249 + 0.4972 + 3(0.0294)]\}$
$= \{ \text{ antilog } [0.6221 + 0.0882]\}$
$= \{ \text{ antilog } [0.7103]\}$
$= 5.133\ cm^3$

In multiplication or division, the final result should retain as many significant figures as there are in the original number with the least significant figures.

$\therefore$ Volume in correct significant figures $= 5.13\ cm^3$

21. Given: Distance $(D \pm \Delta D) = (5.2 \pm 0.1)m$, time $(t \pm \Delta t) = (100 \pm 1)s$.

To find: Speed (v), the maximum relative error $\left(\frac{\triangle v}{v}\right)$

Formulae: i. $v = \frac{D}{t}$

ii. $\frac{\Delta v}{v} = \pm\left(\frac{\Delta D}{D} + \frac{\Delta t}{t}\right)$

Calculation: From formula (i),

$v = \frac{5.2}{100} = 0.052\ m/s$

From formula (ii),

$\left(\frac{\Delta v}{v}\right) = \pm\left(\frac{0.1}{5.2} + \frac{1}{100}\right)$

$= \pm\left(\frac{1}{52} + \frac{1}{100}\right) = \pm\frac{19}{650}$

$= \pm 0.029\ m/s$

The speed is $0.052\ m/s$ and its maximum relative error is $0.029\ m/s$.

22. The kinetic energy of a body depends upon mass (m) and velocity (v) of the body.

Let K.E. $\propto m^x v^y$

$\therefore$ K.E. $= km^x vy^y$ ...(1)

where, $k =$ dimensionless constant of proportionality. Taking dimensions on both sides of equation (1),

$[L^2M^1T^{-2}] = [L^0M^1T^0]^x[L^1M^0T^{-1}]^y$

$= [L^0M^xT^0][L^yM^0T^{-y}]$

$= [L^{0+y}M^{x+0}T^{0-y}]$

$[L^2M^1T^{-2}] = [L^yM^xT^{-y}]$ ...(2)

Equating dimensions of L, M, T on both sides of equation (2),

$x = 1$ and $y = 2$

Substituting x, y in equation (1), we have

K.E. $= kmv^2$

23. Quantity $= 1 \times \sqrt{\frac{l}{g}}$ ...(i)

gravitational acceleration, $g = \frac{\text{velocity}}{\text{time}}$

$\therefore g = \frac{\text{distance}}{\text{time} \times \text{time}}$

Substituting in equation (i),

Quantity $= l \times \sqrt{\frac{l \times \text{ time }^2}{\text{distance}}}$

$\therefore$ Dimensional formula of quantity

$= [L] \times \frac{[L^{1/2}][T^{2\times1/2}]}{L^1/2} = [L] \times [T^1] = [L^1\ T^1]$

Section D

24. **Given:** $a_1 = 3.11\ cm, a_2 = 3.13\ cm$,

$a_3 = 3.14cm, a_4 = 3.14cm$

Least count L.C. $= 0.01\ cm$.

To find:

i. Mean length (a_{mean})

ii. Mean absolute error $(\triangle a_{\text{mean}})$

iii. Percentage error

Formulae: 1. $a_{\text{mean }}=\frac{a_1+a_2+a_3+a_4}{4}$

2. $\Delta a_n=\left|a_{\text{mean }}-a_n\right|$

3. $\Delta a_{\text{mean }}=\frac{\Delta a_1+\Delta a_2+\Delta a_3+\Delta a_4}{4}$

4. Percentage error $=\frac{\Delta a_{\text{mean }}}{a_{\text{mean }}} \times 100$

Calculation: From formula (i),

$a_{\text{mean }}=\frac{3.11+3.13+3.14+3.14}{4}=3.13\ cm$

From formula (ii),

$\Delta a_1=|3.13-3.11|=0.02\ cm$

$\Delta a_2=|3.13-3.13|=0$

$\Delta a_3=|3.13-3.14|=0.01\ cm$

$\Delta a_4=|3.13-3.14|=0.01\ cm$

From formula (iii),

$a_{\text{mean }}=\frac{0.02+0+0.01+0.01}{4}=0.01\ cm$

From formula (iii),

$\%$ error $=\frac{0.01}{3.13} \times 100$

$=\frac{1}{3.13}=0.3196 \ldots$ (using reciprocal table)

$=0.32\%$

i. Mean length is $3.13\ cm$.

ii. Mean absolute error is $0.01\ cm$.

iii. Percentage error is 0.32%.

25. **Given:** $I=(4.00 \pm 0.001) cm$,

In order to have same precision, we use,

$(4.000 \pm 0.001), r=(0.0250 \pm 0.001) cm$

In order to have same precision, we use, $(0.025 \pm 0.001) m=(6.25 \pm 0.01) g$

To find: percentage error in density

Formulae:

i. Relative error in volume, $\frac{\Delta V}{V}=\frac{2 \Delta r}{r}+\frac{\Delta l}{l} \ldots .(\because$ Volume of cylinder, $V=\pi r^2 l)$

ii. Relative error $\frac{\Delta \rho}{\rho}=\frac{\Delta m}{m}+\frac{\Delta V}{V} \ldots .\left[\because \text{Density }(\rho)=\frac{\text{mass}(m)}{\text{volume}(v)}\right]$

iii. Percentage error $=$ Relative error $\times 100\%$

Calculation: From formulae (i) and (ii),

$\therefore \frac{\Delta \rho}{\rho}=\frac{\Delta m}{m}+\frac{2 \Delta r}{r}+\frac{\Delta l}{l}$

$=\frac{0.01}{6.25}+\frac{2(0.001)}{0.025}+\frac{0.001}{4.000}$

$=0.0016+0.08+0.00025$

$=0.08185$

$\%$ error in density $=\frac{\Delta \rho}{\rho} \times 100$

$=0.08185 \times 100$

$=8.185\%$

Percentage error in density is 8.185%.

26. Given $=v=k \sqrt{g h}$

Quantity	Formula	Dimension
Velocity (v)	Distance Time	$\left[\frac{L}{T}\right]=\left[L^1\ T^{-1}\right]$
Height (h)	Distance	$[L^1]$
Gravitational acceleration (g)	$\frac{\text{Distance}}{(\text{Time})^2}$	$\left[\frac{L}{T^2}\right]=\left[L^1\ T^{-2}\right]$

k being constant is assumed to be dimensionless.

Dimensions of L.H.S. $=[v]=\left[L^1\ T^{-1}\right]$

Dimension of R.H.S. $= [\sqrt{gh}]$

$= \left[L^1\, T^{-2}\right]^{1/2} \times \left[L^1\right]^{1/2}$

$= \left[L^1\, T^{-2}\right]^{1/2}$

$= \left[L^1\, T^{-1}\right]$

As, [L.H.S.] = [R.H.S.],

$\Rightarrow v = k\sqrt{gh}$ is dimensionally correct equation.

27. **Given:** $|\vec{F}| = B$ e v

Considering only magnitude, given equation is simplified to,

$F = Bev$

$\therefore B = \frac{F}{ev}$

but, $F = ma = m \times \frac{\text{distance}}{\text{time}^2}$

$\therefore [F] = \left[M^1\right] \times \left[\frac{L^1}{T^2}\right]$

$= \left[L^1 M^1 T^{-2}\right]$

Electric charge, e $=$ current $\times$ time

$\therefore [e] = \left[l^1\, T^1\right]$

Velocity $v = \frac{\text{distance}}{\text{time}}$

$\therefore [v] = \left[\frac{L}{T}\right] = \left[L^1\, T^{-1}\right]$

Now, $[B] = \left[\frac{F}{ev}\right]$

$= \frac{\left[L^1 M^1\, T^{-2}\right]}{\left[T^1 I^1\right]\left[L^1\, T^{-1}\right]}$

$\therefore [B] = \left[L^0 M^1\, T^{-2}\right|^{-1}\right]$

28. **Given:** $l = 4.234\; m,\; b = 1.005\; m,\; t = 2.01\; cm = 2.01 \times 10^{-2}\; m = 0.0201\; m$

To find:

i. Area of sheet to correct significant figures (A)
ii. Volume of sheet to correct significant figures (V)

Formulae:

1. $A = 2(lb + bt + tl)$
2. $V = 1 \times b \times t$

Calculation: From formula (i),

$A = 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)$

$= 2\{[anti\log(\log 4.234 + \log 1.005) + \text{antilog}(\log 1.005 + \log 0.0201)\dot{\text{m}} + \text{antilog}(\log 0.0201 + \log 4.234)]\}$

$= 2\{[\text{antilog}(0.6267 + 0.0021) + \text{antilog}(0.0021 + \bar{2}.3010) + \text{antilog}(\bar{2}.3010 + 0.6267)]\}$

$= 2\{[\text{antilog}(0.6288) + \text{antilog}(\bar{2}.3031) + \text{antilog}(\bar{2}.9277)]\}$

$= 2[4.254 + 0.02009 + 0.08467]$

$= 2[4.35876]$

$= 8.71752\; m^2$

In correct significant figure,

$A = 8.71\; m^2$

From formula (ii),

$V = 4.234 \times 1.005 \times 0.0201$

$=$ antilog $[\log(4.234) + \log(1.005) + \log(0.0201)]$

$=$ antilog $[0.6269 + 0.0021 + \bar{2}.3032]$

$=$ antilog $[0.6288 + \bar{2}.3032]$

$=$ antilog $[\bar{2}.9320]$

$= 8.551 \times 10^{-2}$

$= 0.08551 m^3$

In correct significant figure (rounding off),
$V = 0.086 m^3$

i. Area of a sheet to correct significant figures is $8.72 m^2$.
ii. Volume of sheet to correct significant figures is $0.086 m^3$.

29. i. Significant figures in the measured value of a physical quantity is the sum of reliable digits and the first uncertain digit.
OR
The number of digits in a measurement about which we are certain, plus one additional digit, the first one about which we are not certain is known as significant figures or significant digits.
ii. The larger the number of significant figures obtained in a measurement, the greater is the accuracy of the measurement. The reverse is also true.
iii. If one uses the instrument of smaller least count, the number of significant digits increases.

Rules for determining significant figures:

i. All the non-zero digits are significant, for example, if the volume of an object is $178.43\ cm^3$, there are five significant digits which are $1, 7, 8, 4$ and 3 .
ii. All the zeros between two nonzero digits are significant, eg., $m = 165.02\ g$ has 5 significant digits.
iii. If the number is less than 1, the zero/zeroes on the right of the decimal point and to the left of the first nonzero digit are not significant e.g. in $\underline{0.001405}$, the underlined zeroes are not significant. Thus the above number has four significant digits.
iv. The zeroes on the right-hand side of the last nonzero number are significant (but for this, the number must be written with a decimal point), e.g. 1.500 or 0.01500 both have 4 significant figures each.
v. On the contrary, if a measurement yields length L given as $L = 125 m = 12500\ \ cm = 125000\ mm$, it has only three significant digits.

30. Suppose, $Z = \frac{A}{B}$ and measured values of A and B are $(A \pm \Delta A)$ and $(B \pm \Delta B)$ then,
$Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B}$
$\therefore Z\left(1 \pm \frac{\Delta Z}{Z}\right) = \frac{A[1 \pm (\Delta A/A)]}{B[1 \pm (\Delta B/B)]}$
$= \frac{A}{B} \times \frac{1 \pm (\Delta A/A)}{1 \pm (\Delta B/B)}$
As, $\frac{\triangle B}{B} \ll 1$, expanding using Binomial theorem,
$Z\left(1 \pm \frac{\Delta Z}{Z}\right) = Z\left(1 \pm \frac{\Delta A}{A}\right) \times \left(1 \mp \frac{\Delta B}{B}\right) \ldots \left(\because \frac{A}{B} = Z\right)$
$\therefore 1 \pm \frac{\Delta Z}{Z} = \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \times \frac{\Delta B}{B}$
Ignoring term $\frac{\Delta A}{A} \times \frac{\Delta B}{B}$, $\frac{\Delta Z}{Z} = \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B}$
This gives four possible values of $\frac{\Delta Z}{Z}$ as
$\left(+\frac{\Delta A}{A} - \frac{\Delta B}{B}\right), \left(+\frac{\Delta A}{A} + \frac{\Delta B}{B}\right), \left(-\frac{\Delta A}{A} - \frac{\Delta B}{B}\right)$ and $\left(-\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$
$\therefore$ Maximum relative error of $\frac{\Delta Z}{Z} \pm \left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$
Thus, when two quantities are divided, the maximum relative error in the result is the sum of relative errors in each quantity.