Solution

TRIANGLES

Class 09 - Mathematics

Section A

1.

(b) $\angle P$

Explanation:

Since, by corresponding part of congruent $\angle E$ of △EFD is equal to the $\angle P$ of △PQR.

2.

(b) △ABC ≅ △EDF

Explanation:

△ABC ≅ △EDF

3.

(c) △ABD ≅ △FEC

Explanation:

△ABD ≅ △FEC

4. **(a)** CA = RP

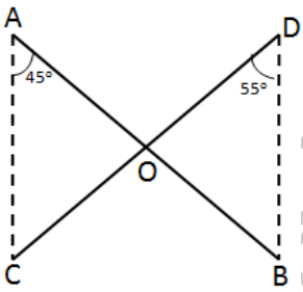
Explanation:

Corresponding sides are equal for two congruent triangles.

5.

(b) 80°

Explanation:



AC||BD

And, AB is transverse to these parallel lines

So ∠CAB = ∠ABD (Alternate angles)

⇒ ∠ABD = 45°

Now In △BOD

∠BOD + ∠ODB + ∠DBA = 180°

∠DBA = ∠ABD = 45°, ∠ODB = 55°

So ∠BOD = 180° - 45° - 55°

= 80°

6.

(c) 53^o

Explanation:

Let

$\angle A - \angle B = 42^\circ$ .....(i) and

$\angle B - \angle C = 21^\circ$ ....(ii)

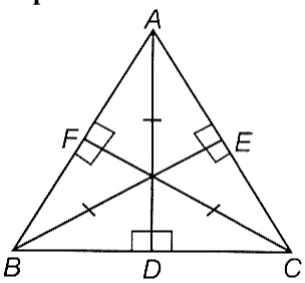
Adding (i) and (ii),we get

$\angle A - \angle C = 63^\circ$ .... (iii)
$\angle B = \angle A - 42^\circ$ .....[using (i)]
$\angle C = \angle A - 63^\circ$ [Using (iii)]
$\therefore \angle A + \angle B + \angle C = 180^\circ$ [Sum of the angles of a triangle]
$\Rightarrow \angle A + \angle A - 42^\circ + \angle A - 63^\circ = 180^\circ$
$\Rightarrow 3\angle A - 105^\circ = 180^\circ$
$\Rightarrow 3\angle A = 285^\circ$
$\therefore \angle B = (95 - 42)^\circ$
$\Rightarrow \angle B = 53^\circ$

7.

(d) 4 : 3 : 2
Explanation:
3A = 4B = 6C
3A = 4B ⇒ A/B = 4/3
This means, A:B=4:3
4B = 6C ⇒ B/C = 6/4 = 3/2
⇒ B:C = 3:2
The value of B in 4:3 is equal to the value of B in 3:2
Hence A : B : C = 4 : 3 : 2

8. **(a)** AC = BC
Explanation:



In right triangles BCE and CBF,
BC = CB [Common]
BE = CF [Given]
∠BEC = ∠CFB [Each 90°]
∴ △BCE = △CBF [By R.H.S. congruency]
⇒ ∠CBE = ∠BCF [By C.P.C.T.]
and ∠ABC = ∠ACB
⇒ AC = AB [Sides opposite to equal angles of a △ are equal]
Similarly, △ABD ≅ △BAE
⇒ ∠ABC = ∠BAC [By C.P.C.T.]
⇒ AC = BC [Sides opposite to equal angles of a △ are equal]

9.

(b) 5 cm
Explanation:
In △ABC and △DEF
AB = DE (given)
∠A = ∠D = 90°
AC = DF (given)
∴ △ABC ≅ △DEF (By SAS congruency)
∴ BC = EF = 5 cm (By C.P.C.T.)

10.

(b) 6 cm

Explanation:

Sides opposite to equal angles are equal.Since,$\angle C = \angle A$, hence , AB=BC = 6 cm

11.

(d) scalene triangle

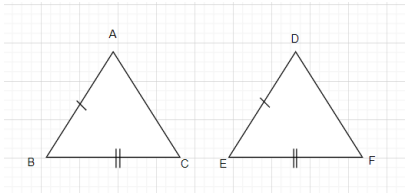
Explanation:

scalene triangle

12.

(c) $\angle B = \angle E$

Explanation:



Given, AB = DE and BC = EF

for, $\angle B = \angle E$

$\Delta ABC \cong \Delta DEF$ [SAS]

13.

(d) △AEB ≅ △ADC

Explanation:

We have, AE = AD and CE = BD

⇒ AE+ CE = AD + BD

⇒ AC = AB ...(i)

Now, in △AEB and △ADC, we have

AE = AD [Given]

∠EAB = ∠DAC [Common]

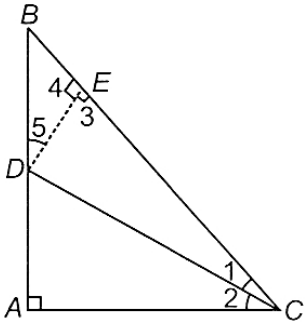
AB = AC [From (i)]

∴ △AEB ≅ △ADC [By SAS Congruency]

14. **(a)** BC

Explanation:

Given a right angled △ABC, where AB = AC and CD is the bisector of ∠C.



Let us draw DE ⊥ BC.

In right angled △ABC, AB = AC and ∠A = 90°

Now, in △DAC and △DEC,

∠A = ∠3 [Each 90°]

∠2 - ∠1 [∵ CD is the bisector of ∠C]

DC = DC [Common]

∴ △DAC ≅ △DEC [By AAS congruency]

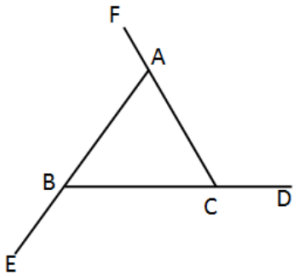
⇒ DA = DE ...(1)
and AC = EC [By C.P.C.T.] ...(2)
Also, in △ABC, AB = AC ⇒ ∠C = ∠B ...(3) [Angles opposite to equal sides are equal]
Again, in △ABC, ∠A + ∠B + ∠C = 180° [by angle sum property of triangle]
⇒ 90° + ∠B + ∠B = 180° [From (3)]
⇒ 2∠B = 90°⇒ ∠B = 45°
In △BED,∠5 = 180° - (∠B + ∠4)
∴ ∠5 = 180°- (45° + 90°) = 180° - 135° = 45°
∠B = ∠5 => DE = BE ...(4) [Sides opposite to equal angles are equal]
From (1) and (4),
DA = DE = BE ...(5)
∵ BC = EC + BE = AC + AD [from (2) and (5)]
Thus, AD + AC = BC

15.

(b) DF = 5 cm, ∠E = 60°
Explanation:
Given that: In △ABC, AB = 5 cm, ∠B = 40° and ∠A = 80°
Using angles sum property of triangle, we have
∠A + ∠B + ∠C = 180°
⇒ 80° + 40° + ∠C = 180
⇒ 120° + ∠C = 180° [∵ ∠B = 40° and ∠A = 80°]
⇒ ∠C = 180° – 120°
⇒ ∠C = 60°
It is given that △ABC ≅ △FDE, so we have
AB = FD, BC = DE and AC = FE & ∠A = ∠F, ∠B = ∠D and ∠C = ∠E
⇒ AB = FD = 5cm and ∠C = ∠E = 60°.

16.

(d) 360°
Explanation:



In △ABC
∠A + ∠B + ∠C = 180°
Now, ∠FAB = 180°- ∠A ...(i)
∠DCA = 180° - ∠C ...(ii)
∠EBC = 180° - ∠B ...(iii)
Adding equations (i), (ii) and (iii)
∠FAB + ∠DCA + ∠EBC = 180° - ∠A + 180° - ∠C + 180° - ∠B
= 540° - (∠A + ∠B + ∠C)
= 540° - 180°
⇒ Sum of All exterior angles = 360°

17.

(b) 45°
Explanation:

The measures of angles of a triangle are in ratio 3: 4: 5.

Let the angles be 3x, 4x and 5x.

In any triangle, sum of all angles = 180°

$\Rightarrow$ 3x + 4x + 5x = 180°

$\Rightarrow$ 12x = 180°

$\Rightarrow$ x = 15°

So, smallest angle = $3 \times 15^\circ$ = 45°

18.

(b) Isosceles

Explanation:

In $\triangle$ADB and $\triangle$ADC

$\angle$BAD = $\angle$CAD ($\because$ AD bisects $\angle$A)

$\angle$ADB = $\angle$ADC ($\because$ Each 90°)

AD = AD (common)

$\triangle$ADS $\cong$ $\triangle$ADC (By ASA congruency)

$\therefore$ AS = AC (by C.P.C.T.)

$\therefore$ $\triangle$ABC is an isosceles triangle.

19. **(a)** 40 m

Explanation:

Cost of turfing a triangular field at the rate of Rs.45 per 100 = Rs.900

$\frac{\text{Area} \times 45}{100} = 900$

$\Rightarrow$ Area = 2000 sq. m

According to question,

2 $\times$ Base = 5 $\times$ Height

$\Rightarrow$ Base = $\frac{\text{Height} \times 5}{2}$

Area of triangle = 2000 sq. m

$\Rightarrow \frac{1}{2} \times$ Base $\times$ Height = 2000

$\Rightarrow \frac{1}{2} \times \frac{\text{Height} \times 5}{2} \times$ Height = 2000

$\Rightarrow$ $(\text{Height})^2$ = 1600

$\Rightarrow$ Height = 40 m

20.

(c) SSA

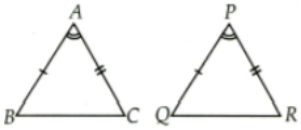
Explanation:

The criteria for congruence of triangles are SSS(Side-Side-Side), SAS (Side-Angle-Side), ASA (Angle-Side- Angle) and RHS (Right angle- Hypotenuse-Side)

21.

(c) A is true but R is false.

Explanation:



In ΔABC and ΔPQR

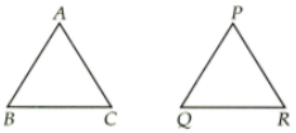
AB = PQ

AC = PR

$\angle$BAC = $\angle$QPR

ΔABC $\cong$ ΔPQR (By SAS Rule)

22. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:



$\angle A = \angle P, \angle C = \angle R$ and AC = PR
$\triangle ABC \cong \triangle PQR$ [ASA congruence rule]

23.
(c) A is true but R is false.
Explanation:
A is true but R is false.

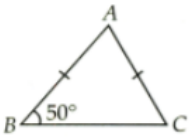
24. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
In ΔACB and ΔADB
AC = AD [Given]
∠CAB = ∠DAB [AB bisects ∠A]
AB = AB [Common]
ΔACB ≅ ΔADB [By SAS congruence rule]

25.
(c) A is true but R is false.
Explanation:
A is true but R is false.

26.
(d) A is false but R is true.
Explanation:
A is false but R is true.

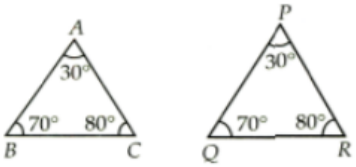
27.
(d) A is false but R is true.
Explanation:
A is false but R is true.

28. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:



Since AB = AC,
then ∠B = ∠C

29. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:



In ΔABC and ΔPQR
$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$
But sides are different

There is no rule of congruence as AAA.

$\triangle ABC \ncong \triangle PQR$

Section B

30. In △ABD and △ACD

∠BAD = ∠CAD ..... [Given]

∠ADB = ∠ADC ..... [Each 90^o]

AD = AD ..... [Common]

$\therefore \triangle ABC \cong \triangle ACD$ ..... [ASA axiom]

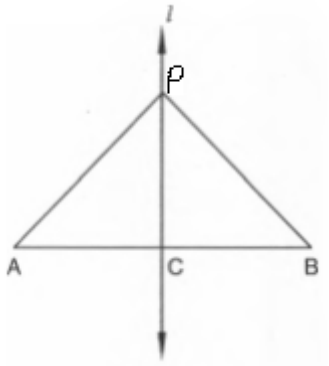
△ AB = AC ..... [c.p.c.t.]

Hence, ABC is an isosceles triangle.

31. Let C be the mid-point of AB.

Clearly, line l passes through C is perpendicular to AB.

In △PCA and △PCB, we have



AC = BC [$\because$ C is the mid-point of AB]

PC = PC [common side]

$\angle PCA = \angle PCB$ [Each equal to 90^o as l $\perp$ AB]

So, by SAS congruence rule, we obtain

$\triangle PCA \cong \triangle PCB$

$\Rightarrow$ PA = PB

32. m $\parallel$ n and AB intersects them

$\therefore$ ∠MAC = ∠MBD ...[Alternate angles] ...(1)

In DMAC and DMBD,

MA = MB

∠MAC = ∠MBD ...[From (1)]

∠AMC = ∠BMD ...[Vertically opposite angles]

$\therefore$ DMAC $\cong$ DMBD ...[ASA axiom]

$\therefore$ MC = MD ...[c.p.c.t.]

$\therefore$ M is the mid-point of CD.

33. In $\triangle ABC$,we have

AB = AC

$\Rightarrow \quad \angle C = \angle B \ldots(1)$

[$\because$Angles opposite to equal sides of a triangle are equal]

BC = AC

$\Rightarrow \quad \angle A = \angle B \ldots(2)$

[$\because$Angles opposite to equal sides of a triangle are equal]

Now, $\angle A + \angle B + \angle C = 180^\circ$ [$\because$Angle sum property of a triangle]

$\Rightarrow \quad \angle A + \angle A + \angle A = 180^\circ$ [From (1) and (2)]

$\Rightarrow \quad 3\angle A = 180^\circ$

$\Rightarrow \quad \angle A = \frac{180^\circ}{3} = 60^\circ$

$\therefore \quad \angle A = \angle B = \angle C = 60^\circ$

34. Consider the triangles AEB and CDB

∠EBA = ∠DBC ... (Common angle) ... (i)

Further, we have:

∠BEA = 180 - y

∠BDC = 180 - x

Since x = y, we have:

180 - x = 180 - y

∠BEA = ∠BDC ... (ii)

AB = CB ... (Given) ... (iii)

From (i), (ii) and (iii), we have:

△BDC≅△BEA ... (AAS criterion)

∴ AE = CD (CPCT)

Hence, proved.

35. Given : AC = AE, AB = AD and ∠BAD = ∠EAC.

To prove ; BC = DE

Proof : In △ABC and △ADE

AC = AE, AB = AD and ∠BAD = ∠EAC ...[Given]

∴ ∠BAD + ∠DAC = ∠DAC + ∠EAC ...[Adding ∠DAC to both sides]

∴ ∠BAC = ∠DAE ...(1)

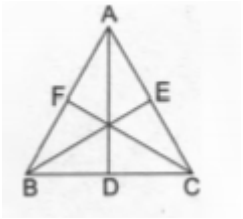
AC = AE ...[Given]

∠BAC = ∠DAE ...[From (1)]

AB = AD ...[Given]

∴ DABC ≅ DADE ...[By SAS congruence]

∴ BC = DE ...[c.p.c.t.]

36. 

In ADC and BEA, we have

AC = BA given

DC = EA $\left[\because BC = AC \Rightarrow \frac{1}{2}BC = \frac{1}{2}AC\right]$

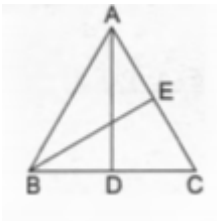
and $\angle ACD = \angle BAE$

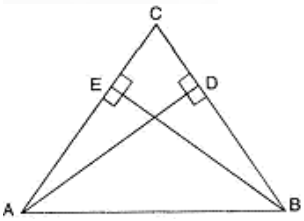
$\therefore \triangle ADC \cong \Delta BEA$

AD = BE

similarly, BE = CF

Hence, AD = BE = CF



37. 

In right triangles ADB and BEA.

∠ADB = ∠BEC . . .[Each 90^o]

∠ABD = ∠BAE As AC = BC ∴ ∠ABD = ∠BAE . . . [Angles opposite to equal sides of triangle]

AB = AB . . . .[Common]

∴ △ADB = △BEA . . .[By AAS property]

∴ AE = BD . . . [c.p.c.t.]

38. In △PQT,

PQ = PT ..... [Given]

△∠PTQ = ∠PQT ..... [Angles opposite to equal sides] .....(1)

In △PST and △PRQ

PT = PQ . . . [Given]

∠TPS = ∠QPR . . .[Given]

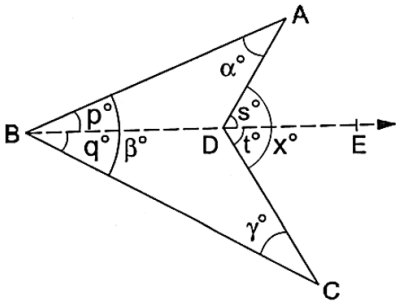
∠PTQ = ∠PQT . . . [From (1)]

∠PTS = ∠PQR

$\therefore \triangle \mathrm{PST} \cong \triangle \mathrm{PRQ}$ ..... [By ASA property]

∴ PS = PR ..... [c.p.c.t.]

∴ △PRS is isosceles.

39. 

Join B and D and produce BD to E.

Let $\angle ABD = p^\circ$, $\angle CBD = q^\circ$ and let

$\angle ADE = s^\circ$ and $\angle CDE = t^\circ$

Then, p + q = β and s + t = x.

Now, side BD of has been produced to E.

$\therefore s = p + \alpha$ ....(i)

Again, side BD of ACBD has been produced to E.

$\therefore t = q + \gamma$ ...(ii)

Adding the corresponding sides of (i) and (ii), we get

s + f = (p + q) + α + γ

$\Rightarrow x = \beta + \alpha + \gamma$ [$\because$ s + t = x and p + q= β]

Hence the required , $x = \alpha + \beta + \gamma$.

40. Produce CB to a point D such that BC = BD and join AD.

In △ABC and △ABD, we have

BC = BD (By construction)

AB = AB (Same side)

∠ABC = ∠ABD (Each of 90°)

Therefore, △ ABC ≅ △ABD (SAS)

So, ∠CAB = ∠DAB .....(1)

and AC = AD.....(2)

Thus, ∠CAD = ∠CAB + ∠BAD = x + x = 2x [From (1)] ....(3)

and ∠ACD = ∠ADB = 2x [From (2), AC = AD] ...(4)

That is, △ ACD is an equilateral triangle. [From (3) and (4)]

or AC = CD, i.e., AC = 2 BC (Since BC = BD)

41. In the given figure, EF II BD and CE is the transversal.

$\therefore \angle \mathrm{CAD} = \angle \mathrm{AEF}$ (Pair of corresponding angles)

$\Rightarrow \angle CAD = 55^\circ$

In $\triangle ABC$

$\angle \mathrm{CAD} = \angle \mathrm{ABC} + \angle \mathrm{ACB}$ (Exterior angle of a triangle is equal to the sum of the two interior opposite angles)

$\Rightarrow 55^\circ = \angle ABC + 25^\circ$

$\Rightarrow \angle ABC = 55^\circ - 25^\circ = 30^\circ$

Thus, the measure of $\angle ABC$ is 30^o

42. In △ABE and △ACF,

∠A = ∠ A [Common]

∠AEB = ∠AFC = [90°]

AB = AC [Given]

∴ △ABE ≅ △ACF [By ASA congruency]

∴ BE = CF [By C.P.C.T.]

So Altitudes are equal.

43. Side AC of triangle ABC is produced to E

$\therefore \angle EAB = \angle B + \angle C$

$\Rightarrow 110^\circ = x + \angle C$ .....(i)

Also, $\angle ACD + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 60^\circ$

$\Rightarrow \angle C = 60^\circ$

Substituting the value of angle C in (i), we get x = 50

44. Given : ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively.

To Prove : BF = CE.

Proof : AB = AC ..... [As ABC is an isosceles triangle]

∴ ∠ABC = ∠ACB ..... [∠s opposite to equal side of a △] ..... (1)

In △BEC and △CFB

∠BEC = ∠CFB ..... [Each = 90^o]

BC = CB ..... [Common]

∠EBC = ∠FCB . . . [From (1)]

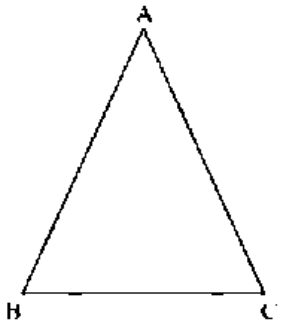
$\therefore \triangle BEC \cong \triangle CFB$ ..... [By AAS property]

∴ CE = BF ..... [c.p.c.t.]

Section C

45. In the given problem, angles of $\triangle ABC$ are in the ratio 2: 1: 3

We need to find the measure of the smallest angle.



Let ∠A = 2x

∠B = x

∠C = 3x

According to the angle sum property of the triangle, in $\triangle ABC$, we get

2x + 1x + 3x = 180°

6x = 180°

x = $\frac{180^\circ}{6}$ = 30°

thus,

∠A = $2(30^\circ) = 60^o$

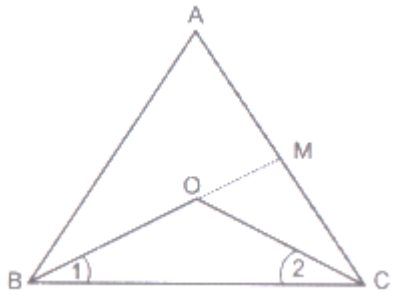
∠B = $1(30^\circ) = 30^o$

∠C = $3(30^\circ) = 90^\circ$

since, the measure of $\angle B$ is the smallest of all the three angles.

Therefore, the measure of the smallest angle is 30°

46. Bisector of the angles B and C of an isosceles triangle ABC with AB = AC intersect each other at O. BO is produced to a point M.Figure is drawn below.



In ΔABC, we have

AC = AB. [Given]

$\therefore \angle ABC = \angle ACB$ [$\because$ Angles opposite to equal sides of a triangle are equal]

Dividing both sides by 2, we get:-

$\Rightarrow \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

$\Rightarrow \angle OBC = \angle OCB$ [$\because$ BO and CO are bisectors of $\angle B$ and $\angle C$]

$\Rightarrow \angle 1 = \angle 2$ (from figure).....(1)

In ΔOBC, Exterior angle $\angle MOC = \angle 1 + \angle 2$

[$\because$ Exterior angle of a triangle is equal to the sum of interior opposite angles]

$\Rightarrow \angle MOC = 2\angle 1$ [Since $\angle 1 = \angle 2$] ..... (2)

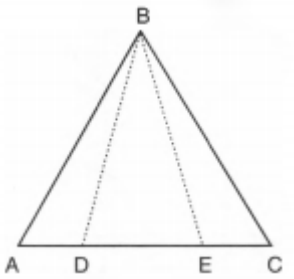
Also ,from figure, $\angle 1 = \frac{1}{2}\angle ABC$.....(3)$\because$ Angles opposite to equal sides of a triangle are equal]

From (2) & (3):-

$\angle MOC = 2 \times \frac{1}{2}\angle ABC$ $\because$ Angles opposite to equal sides of a triangle are equal]

$\Rightarrow \angle MOC = \frac{1}{2}\angle ABC$.

Hence Proved.

47. 

Given: In Fig. AB = BC and AD = EC.

i. In △ABC, we have

BA = BC (given)

$\Rightarrow$ ∠BCA = ∠BAC.....(i) [$\because$ Angles opp. to equal sides are equal]

AD = EC (given)

$\Rightarrow$ AD + DE = DE + EC [Adding DE on both sides]

$\Rightarrow$ AE = CD ...(ii)

Thus, In △ABE and △CBD, we have

AB = BC [Given]

∠BAE = ∠BCD [From (i)]

and, AE = CD [From (ii)]

△ABE ≅ △CBD [By SAS criterion of congruence]

Hence Proved.

ii. As △ABE ≅ △CBD

$\Rightarrow$ BE = BD [CPCT]

Hence Proved.

48. We have,

$\angle DCA = \angle ECB$

$\Rightarrow \angle DCA + \angle ECD = \angle ECB + \angle ECD$

$\Rightarrow \angle ECA = \angle DCB$ ...(i)

Now, in △DBC and △EAC, we have

$\angle DCB = \angle ECA$ [From (i)]

BC = AC [Given]

and, $\angle DBC = \angle EAC$ [Given]

So, ASA (Angle-Side-Angle) criterion of congruence, we obtain

$\Delta DBC \cong \Delta EAC$

$\Rightarrow$ DC = EC and BD = AE

49. Let $\angle$ADC = Q

$\Rightarrow \angle$CAD = Q [$\because$ CA = CD]

Exterior$\Rightarrow \angle$CAD = Q [$\because$CA = CD]

= 2Q

$\Rightarrow \angle$BAC = 2Q [$\because$ BA = BC]

$\angle$BAD = $\angle$BAC + $\angle$CAD

Hence = 2Q + Q

= 3Q = 3$\angle$ADC = 3$\angle$ADB

50. In DPDB and DPEA,

$\angle$PDB = $\angle$PEA . . . . [Each 90^o]

$\angle$BPD = $\angle$APE . . . [Vertically opposite angles]

AE = BD . . . . [Given]

$\therefore$ DPDB $\cong$ DPEA . . . .[By AAS property]

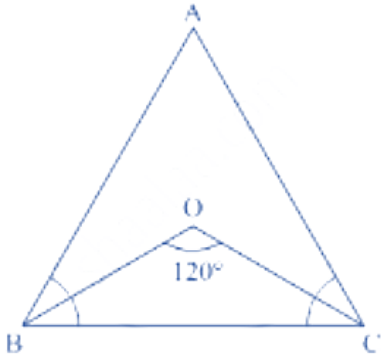
$\therefore$ PA = PB . . . [c.p.c.t.] . . . . (1)

PD = PE . . . [c.p.c.t.] . . . . (2)

PA + PD = PB + PE

$\Rightarrow$ AD = BE . . . [By adding (1) and (2)]

51. We need to find the measure of $\angle$A



So here, using the corollary, if the bisectors of $\angle$ABC and $\angle$ACB of a $\triangle$ABC meet at a point O, then $\angle BOC = 90^\circ + \frac{1}{2}\angle A$

Thus, in $\triangle$ABC

$\angle BOC = 90^\circ + \frac{1}{2}\angle A$

$120^\circ = 90^\circ + \frac{1}{2}\angle A$

$120^\circ - 90^\circ + \frac{1}{2}\angle A$

$\angle$A = 2(30^o).

$\angle$A = 60^o

52. 

Given : $\triangle$ABC is an isosceles triangle in which AB = AC.

Side BA is produced to D such that AD = AB.

To Prove : $\angle$BCD is a right angle.

Proof : As ABC is an isosceles triangle

$\angle$ABC = $\angle$ACB ..... (1)

AC = AD ..... [As given : AB = AC and AD = AB]

In $\triangle$ACD,

∠CDA = ∠ACD ..... [∠s opposite to equal side of a △]

∠CBD = ∠ACD . . . (2)

∠ABC + ∠CDB = ∠ACB + ∠ACD ..... [Adding corresponding sides from (1) and (2)]

∠ABC + ∠CDB = ∠BCD . . . (3)

In △BCD

∠BCD + ∠DBC + ∠CDB = 180^o . . .[Sum of three angles of a triangle]

∴ ∠BCD + ∠ABC + ∠CDB = 180^o

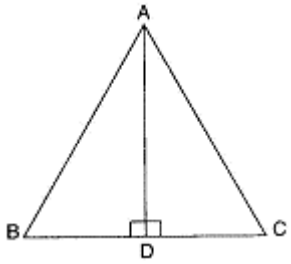
∠BCD + ∠BCD = 180^o . . .[From (3)]

∴ $2∠BCD = 180^o$

∴ $∠BCD = 90^o$

∴ ∠BCD is a right angle proved.

53. In right triangles △ABD and △ACD,



AB = AC . . . [Given]

side AD = side AD . . . [Common]

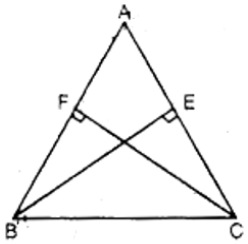
∴ △ABC ≅△ACD . . . [R.H.S. Axiom]

∴ BD = DC . . . [c.p.c.t.]

AD is a median of △ABC

54. In △BEC and △CFB,

∠ BEC = ∠CFB [Each 90°]



BC = BC [Common]

BE = CF [Given]

∴ △BEC ≅ △CFB [RHS congruency]

⇒ EC = FB[By C.P.C.T.] ...(i)

Now In △AEB and △AFC

∠ AEB = ∠AFC [Each 90°]

∠ A = ∠A [Common]

BE = CF[Given]

∴ △AEB △AFC[ASA congruency]

⇒AE = AF[By C.P.C.T.] ...(ii)

Adding eq. (i) and (ii), we get,

EC + AE = FB + AF⇒ AB = AC

⇒ ABC is an isosceles triangle.

55. i. in ΔIPC and ΔIQC

$\angle IPC = \angle IQC = 90^\circ$

$\angle ICP = \angle ICQ$

CI = CI {same side}

$\therefore \Delta IPC \cong \Delta IQC$ [ASA congruancy]

$\therefore \quad IP = IQ$

similarly, $IQ = IR$

Hence, IP = IQ = IR.

ii. In $\triangle IQA$ and ΔIRA

IQ = IR

$\angle IQA = \angle IRA$

$hyp.\, IA = hyp.\, IA$ [same side]

$\therefore \Delta IQA \cong \Delta IRA$ [by SAS congruancy]

$\therefore \angle IAQ = \angle IAR$

Hence, IA bisects $\angle A$

56. SOLUTION: Since AD bisects the exterior A,

∠EAD = ∠DAC ...(1)

∠2 = ∠3 [OPPOSITE ANGLE OF EQUAL SIDE] ...(2)

∠EAC = ∠EAD + ∠DAC ...(3)

∠EAC = ∠2 + ∠3 [EXTERIOR ANGLE THEOREM FOR A TRIANGLE] ...(4)

From equation (3) & (4)

∠EAD + ∠DAC = ∠2 + ∠3

From equation (1) & (2)

∠DAC + ∠DAC = ∠3 + ∠3

2∠DAC = 2∠3

∠DAC = ∠3

alternate angle are equal so - AD || BC

57. Side BC of △ABC has been produced to D.

$\therefore \angle ACD = \angle BAC + \angle ABC$

$\Rightarrow \frac{1}{2}\angle ACD = \frac{1}{2}\angle BAC + \frac{1}{2}\angle ABC$

$\Rightarrow \angle ECD = \frac{1}{2}\angle BAC + \frac{1}{2}\angle ABC$ ...(i)

Again, side BC of AEBC has been produced to D

$\therefore \angle ECD = \angle CBE + \angle BEC$

$\Rightarrow \angle ECD = \frac{1}{2}\angle ABC + \angle BEC$

From (i) and (ii), we get

$\frac{1}{2}\angle ABC + \angle BEC = \frac{1}{2}\angle BAC + \frac{1}{2}\angle ABC$ [each equal to ∠ECD]

$\therefore \angle BEC = \frac{1}{2}\angle BAC$

58. In DOAB and DODC

OA = OD ...[Given]

OB = OC ...[Given]

∠AOB = ∠DOC ...[Vertically opposite angles]

∴ DOAB ≅ DODC ...[By SAS property]

∴ AB = DC ...[c.p.c.t.]

But DC = 540 cm ...[Given]

∴ AB = 540 cm.

59. In △ABC,

AB = AC[Given]

∴ ∠ ABC = ∠ACB [angles opposite to equal side are equals]

Similarly in, △DBC, DB = DC [Given] ...(1)

∴ ∠DBC = ∠DCB ...(2)

Adding (1) and (2)

∠ABC + ∠DBC = ∠ACB + ∠DCB

or ∠ABD = ∠ACD

Section D

60. i. In △APD and △BQC

AD = BC (given)

AP = CQ (opposite sides of rectangle)

$\angle APD = \angle BQC = 90^o$

By RHS criteria $\triangle APD \cong \triangle CQB$

ii. $\triangle APD \cong \triangle CQB$

Corresponding part of congruent triangle

side PD = side BQ

iii. In $\triangle ABC$ and $\triangle CDA$

AB = CD (given)

BC = AD (given)

AC = AC (common)

By SSS criteria $\triangle ABC \cong \triangle CDA$

OR

In $\triangle APD$

$\angle APD + \angle PAD + \angle ADP = 180^o$

$\Rightarrow 90^o + (180^o - 110^o) + \angle ADP = 180^o$ (angle sum property of $\triangle$)

$\Rightarrow \angle ADP = m = 180^o - 90^o - 70^o = 20^o$

$\angle ADP = m = 20^o$

Section E

61. In DAPC and DBPD,

$\angle ACP = \angle BDP$ ...[Given]

$\angle APC = \angle BPD$ ...[Vertically opposite angles]

$\therefore \angle A = \angle B$ ...[sum of three angles of triangle]

In DACD and D BDC,

CD = CD ...[Common]

$\angle A = \angle B$ ...[As proved above]

$\angle BCD = \angle ADC$ and $\angle ACB = \angle BDA$ ...[Given]

$\angle BCD + \angle ACB = \angle ADC + \angle BDA$ ...[By addition]

$\Rightarrow \angle ACD = \angle BDC$

$\therefore$ DACD $\cong$ DBDC ...[By AAS property]

$\therefore$ AD = BC ...[c.p.c.t.]

62. Given: ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$

To prove:

$\Delta ABD \cong \Delta BAC$

Proof:

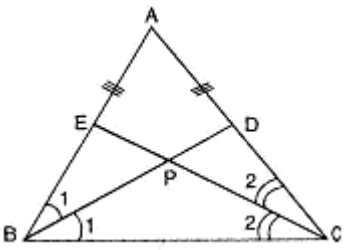
In ΔABD and ΔBAC,

AD = BC ...[Given]

$\angle DAB = \angle CBA$ ...[Given]

AB = BA ...[Common]

$\therefore \Delta ABD \cong \Delta BAC$ proved ...[By SAS Property] ...(i)

63. 

As AB = AC

$\therefore \angle ACB = \angle ABC$ . . . . [Angles opposite to equal sides of triangle]

$\therefore 2\angle 2 = 2\angle 1$

As CE and BD are the bisectors of $\angle C$ and $\angle B$ respectively.

$\angle 2 = \angle 1$

BP = PC . . . [Side opposite to equal angles of triangle] . . .(1)

In $\triangle BPE$ and $\triangle CPD$,

BP = CP . . .[Proved as above]

∠EBP = ∠DCP . . . [Proved as above]

∠BPE = ∠CPD . . .[Vertically opposite angles]

△BPE ≅△CPD. . .[By ASA property]

PE = PD . . .[c.p.c.t.]

PD = PE . . . . (2)

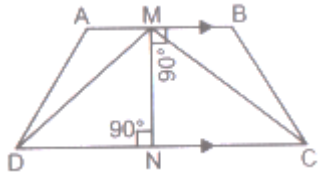
BP + PD = PC + PD . . . .[Adding (1) and (2)]

∴ BD = CE

64. Join MD and CM.

We have,$\angle DNM = \angle NMB$ [Alt. $\angle s$]

$\therefore AB \| CD$



Now, in $\triangle DMN$ and $\triangle CNM$

CN = DN

[∵ N is the mid-point of DC]

$\angle DNM = \angle CNM$ [Each = 90^o]

NM = NM [Common side]

$\therefore \triangle DMN \approx \triangle CNM$ [By SAS congruence rule]

$\therefore DM = CM$ and $\angle NMC = \angle NMD$ …(1) [CPCT]

Now, $\angle AMN = \angle BMN$ [Each = 90^o]

And$\angle NMD = \angle NMC$ [Proved above]

$\therefore \angle AMN - \angle NMD = \angle BMN - \angle NMC$ [On subtraction]

$\Rightarrow \angle AMD = \angle BMC$ ...(2)

AM = BM [Given]

DM = CM [From (1)]

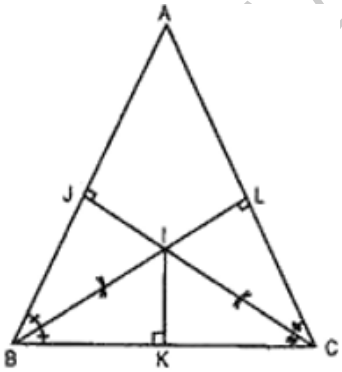
$\angle AMD = \angle BMC$ [From (2)]

$\angle AMD \cong \triangle BMC$ [By SAS congruence rule]

$\therefore AD = BC$ [CPCT]

65. The point which is equidistant from all the sides of a triangle is known as its incentre and it is also the point of intersection of the angular bisectors of angles of the triangle. Hence we will proceed with finding the incentre of the given triangle.

Let ABC be a triangle.



Draw bisectors of ∠ B and ∠ C.

Let these angular bisectors intersect each other at point I.

Draw IK ⊥ BC

Also, draw IJ ⊥ AB and IL ⊥ AC.

Join BI & CI

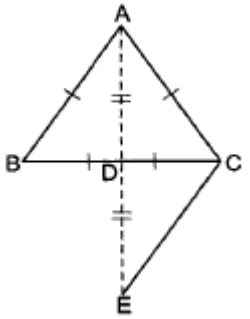
In Δ BIK and Δ BIJ,

∠ IKB = ∠ IJB = 90° [By construction]

∠ IBK = ∠ IBJ

[∵ BI is the bisector of ∠ B (By construction)]
BI = BI [Common side]
∴ $\Delta BIK \cong \Delta BIJ$ [ASA congruency criterion of triangle]
∴ IK = IJ [C.P.C.T.] …….(i)
Now, in ΔCIK and ΔCIL,
$\angle IKC = \angle ILC$ (each 90°)
$\angle ICK = \angle ICL$ [since, CI is angular bisector of $\angle C$]
CI = CI [common side]
Hence, $\Delta CIK \cong \Delta CIL$ [ASA congruency criterion of triangle]
∴ IK = IL [C.P.C.T.] …….(ii)
From eq (i) and (ii),
IK = IJ = IL
Hence, I is the point of intersection of angular bisectors of any two angles of Δ ABC and is also equidistant from the sides of the triangle.

66. AM is the median of △ABC.
∴ BM = MC = $\frac{1}{2}$ BC ...(i)
PN is the median of △PQR.
∴ QN = NR = $\frac{1}{2}$ QR ...(ii)
Now BC = QR [Given] ⇒ $\frac{1}{2}BC = \frac{1}{2}QR$
∴ BM = QN ...(iii)
   i. Now in △ABM and △PQN,
      AB = PQ[Given]
      AM = PN[Given]
      BM = QN[From eq.(iii)]
      ∴ △ABM ≅ △PQN [By SSS congruency]
      ⇒ ∠B = ∠Q [By C.P.C.T.] ...(iv)
   ii. In △ABC and △PQR,
      AB = PQ [Given]
      ∠B = ∠Q [Prove above]
      ∴ PR = QR [Given]
      ABC ≅ PQR [By SAS congruency]

67. 

Given: In Δ ABC, AD is the angular bisector of ∠A which meets BC in D such that BD = DC.....(a)
To prove: ΔABC is an isosceles.
Construction: Produce AD to E such that AD = DE and then join CE.
Proof: In Δ ABD and Δ ECD, we have
BD = CD [from (a)]
AD = ED (By construction)
and ∠ADB = ∠EDC (Vertically opposite angles)
Therefore, Δ ABD ≅ Δ ECD (SAS congruency criterion of triangle)
So, AB = EC (CPCT) ...(i)
and ∠BAD = ∠CED (CPCT) ...(ii)
Also, ∠BAD = ∠CAD (Given, BD is angular bisector of $\angle A$).....(iii)
Therefore, from (ii) and (iii)
∠CAD = ∠CED

So, AC = EC (Sides opposite to equal angles are equal) ...(iv)

From (i) and (iv), we get

AB = AC.

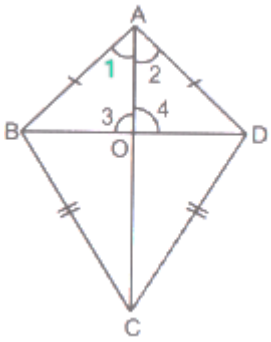
Therefore, Δ ABC is isosceles.

Hence, Proved.

68. Given: ABCD is a quadrilateral . AB = AD & CB = CD

To prove: AC is the perpendicular bisector of BD.

Proof:



Let diagonals AC & BD intersect at O.

Let, $\angle BAC = \angle 1$, $\angle DAC = \angle 2$, $\angle AOB = \angle 3$ and $\angle AOD = \angle 4$

In △ABC & △ADC, we have :-

AB = AD [Given]

BC = CD [Given]

AC = AC [Common side]

So, By SSS criterion of congruency of triangles , we have

$\Delta ABC \cong \Delta ADC$

$\therefore \angle 1 = \angle 2$ [CPCT]

Now, in ΔAOB and ΔAOD , we have :-

AB = AD [Given]

$\angle 1 = \angle 2$ [Proved above]

AO = AO [Common side]

So, By SAS criterion of congruency of triangles , we have :-

$\Delta AOB \cong \Delta AOD$

$\therefore \quad BO = DO$ [CPCT]

And $\angle 3 = \angle 4$ [CPCT]

But, $\angle 3 + \angle 4 = 180^o$ [Linear pair axiom]

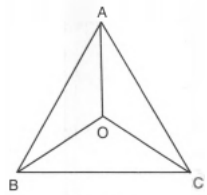
$\Rightarrow \angle 3 + \angle 3 = 180^\circ$ [$\because \angle 3 = \angle 4$]

$\Rightarrow 2\angle 3 = 180^\circ$

$\Rightarrow \angle 3 = \frac{180^\circ}{2} = 90^\circ$

$\therefore$ AC is perpendicular bisector of BD. [$\because \angle 3 = 90^\circ$ and BO = DO]

Hence, proved.

69. 

In △ABC, we have

AB = AC

$\Rightarrow \quad \angle B = \angle C$ [$\because$ Angles opposite to equal sides are equal]

$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\Rightarrow \angle OBC = \angle OCB$ $\begin{bmatrix} \because OB \text{ and } OC \text{ are bisectors of } \angle B \text{ and } \angle C \text{ respectively} \\ \therefore \angle OBC = \frac{1}{2}\angle B \ \& \ \angle OCB = \frac{1}{2}\angle C \end{bmatrix}$ ..... (i)

$\Rightarrow$ OB = OC [$\because$ Sides opposite to equal angles are equal] ..... (ii)

Now, in △ABO and △ACO, we have

AB = AC [Given]

$\angle OBC = \angle OCB$ [From (i)]

and, OB = OC [From (ii)]

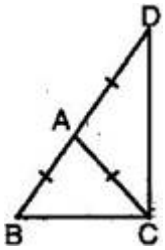
So, by SAS criterion of congruence, we obtain

$\triangle ABO \cong \triangle ACO$

$\Rightarrow \angle BAO = \angle CAO$ [c.p.c.t]

$\Rightarrow$ AO is the bisector of ∠BAC.

70.



From figure & according to the question, in ΔABC,

AB = AC .....(1)

$\Rightarrow$ ∠ACB = ∠ABC ...(2) [Angles opposite to equal sides are equal]

Again given, AD = AB

But AB = AC [from (1)]

$\therefore$ AD = AB = AC

$\Rightarrow$ AD = AC..... (3)

Now in Δ ADC,

AD = AC [from (3)]

$\Rightarrow \angle ADC = \angle ACD$ …(4) [Angles opposite to equal sides are equal]

In Δ BCD,

$\angle ABC + \angle BCD + \angle CDA = 180^\circ$ [Angle sum property]

$\Rightarrow \angle ACB + \angle BCD + \angle CDA = 180^\circ$ [Because ∠ACB = ∠ABC, from (2)]

$\Rightarrow \angle ACB + \angle ACB + \angle ACD + \angle CDA = 180^\circ$ [Because $\angle BCD = \angle ACB + \angle ACD$]

$\Rightarrow 2\angle ACB + \angle ACD + \angle CDA = 180^\circ$

$\Rightarrow 2\angle ACB + \angle ACD + \angle ACD = 180^\circ$ [Because ∠ADC = ∠ACD, see (4)]]

$\Rightarrow 2\angle ACB + 2\angle ACD = 180^\circ$

$\Rightarrow 2(\angle ACB + \angle ACD) = 180^\circ$ [Taking out 2 common]

$\Rightarrow 2\angle BCD = 180^\circ$ [Because, $\angle ACD + \angle ACB = \angle BCD$]

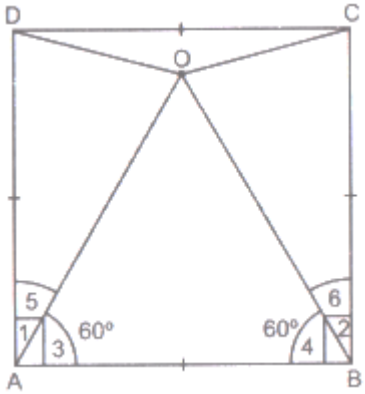
$\Rightarrow \angle BCD = 90^\circ$

Hence ∠BCD is a right angle.

71. Given: ABCD is a square & O is an interior point. Δ OAB is an equilateral triangle.

Hence, OA = OB = AB.....(1)

So, $\angle OAB = \angle OBA \Rightarrow \angle 3 = \angle 4$ .....(2)



TO prove: ΔOCD is an isosceles triangle.

Proof: In square ABCD,

$\angle 1 = \angle 2$ …....(3)

[$\because$ Each equal to 90^o]

Now, in ΔOAB, we have

$\angle 3 = \angle 4$ .....(4) [from (2)]

Subtracting (2) from (1), we get

$\angle 1 - \angle 3 = \angle 2 - \angle 4$

$\Rightarrow \angle 5 = \angle 6$. [see figure].....(5)

Now, in ΔDAO and ΔCBO

AD = BC [sides of square]

$\angle 5 = \angle 6$ [from (5)]

OA = OB [from (1)]

So, By SAS criterion of congruency of triangles , we have

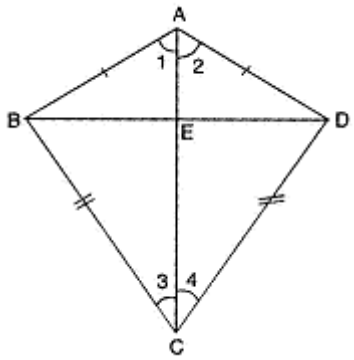
$\Delta DAO \cong \Delta CBO$

$\therefore OD = OC$

$\Rightarrow \Delta OCD$ is an isosceles triangle.

Hence, proved.

72. △ABC and △ADC,



i. AB = AD, BC = DC . . . [Given]

AC = AC . . . [Common]

△ABC ≅ △ADC . . .[SSS axiom

∠1 = ∠2 . . . .[c.p.c.t.]

AC bisects ∠A and ∠3 = ∠4 . . .[c.p.c.t.]

AC bisects ∠C.

Hence, AC bisects each of the angles A and C.

ii. In △ABE and △ADE,

AB = AD . . . [Given]

∠1 = ∠2 . . .[As proved above]

AE = AE . . . .[Common]

∴ △ABE ≅ △ADE . . . [SAS axiom]

∴ BE = ED . . . [c.p.c.t.]

iii. △ABC ≅ △ADC . . . [As proved above]

∴ ∠ABC = ∠ADC . . . [c.p.c.t.]

73. Side BC of triangle ABC is produced to D.

$\therefore \angle ABC = \angle A + \angle C$

$\Rightarrow 106^\circ = \angle A + \angle C \quad \ldots (i)$

Also, side BC of triangle ABC is produced to E.

$\angle ACE = \angle A + \angle B$

$\Rightarrow 118^\circ = \angle A + \angle B \quad \ldots (ii)$

Adding (i) and (ii), we get:

$\angle A + \angle A + \angle B + \angle C = (106 + 118)^\circ$

$\Rightarrow (\angle A + \angle B + \angle C) + \angle A = 224^\circ \quad [\because \angle A + \angle B + \angle C = 180^\circ]$

$\Rightarrow 180^\circ + \angle A = 224^\circ$

$\Rightarrow \angle A = 44^\circ$

and

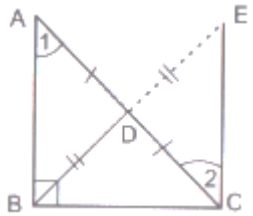
$\angle C = 106^\circ - \angle A \quad [$ Using $(i)]$

$\Rightarrow \angle C = (106 - 44)^\circ$

$\Rightarrow \angle C = 62^\circ$

74. ABC is a right triangle, right-angled at B and D is the mid-point of AC.

We have to prove that $BD = \frac{1}{2}AC$



Now, produce BD to E such that BD = DE. Join EC.

In $\triangle ADB$ and $\triangle CDE$ we have

AD = CD [$\because$D is the mid-point of AC]

$\angle ADB = \angle CDE$ [Vertically opposite $\angle s$]

BD = DE [By construction]

$\triangle ADB \cong \triangle CDE$ [By SAS criterion of congruence]

$\therefore$ AB = EC [CPCT]

And $\angle 1 = \angle 2$ [CPCT]

But, $\angle 1$ and $\angle 2$ are alternate angles.

$\therefore$ EC || BA

Now, EC is parallel to BA and BC is the transversal

$\therefore \angle ABC + \angle BCE = 180^\circ$

$\Rightarrow 90^\circ + \angle BCE = 180^\circ$

$\Rightarrow \angle BCE = 180^\circ - 90^\circ = 90^\circ$

In $\triangle ABC$ and $\triangle EBC$ we have

BC = BC [Common side]

AB = EC [Proved above]

$\angle CBA = \angle BCE$ [$\because$Each = 90^o]

$\triangle ABC \cong \triangle EBC$ [By SAS criterion of congruence]

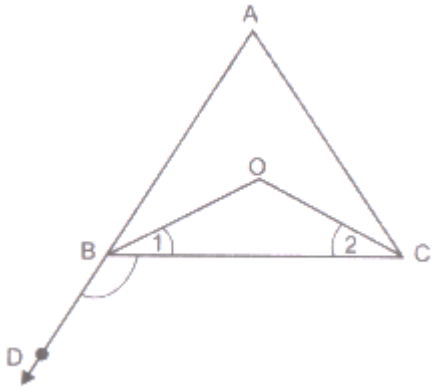
$\therefore$ AC = EB [CPCT]

$\Rightarrow \frac{1}{2}AC = \frac{1}{2}EB \Rightarrow \frac{1}{2}AC = BD$

Hence, $BD = \frac{1}{2}AC$

75. In ΔABC , OB & OC are angular bisectors of $\angle$ABC & $\angle$ACB respectively. Side AB is extended to point D so that $\angle DBC$ is external angle of triangle adjacent to $\angle ABC$.

To prove :- $\angle DBC = \angle BOC$



Proof :-

Given, AB = AC

$\therefore \angle B = \angle C$

[$\because$ Angles opposite to equal sides of a triangle are equal]

$\therefore \frac{1}{2}\angle B = \frac{1}{2}\angle C$ ...(1)

In ΔOBC, we have

$\angle 1 = \frac{1}{2}\angle B$
And $\angle 2 = \frac{1}{2}\angle C$
$\therefore \angle 1 = \angle 2$ [By (1)]
$\angle DBC + \angle 1 + \angle OBA = 180^\circ$ [$\because$ ABD is a straight line, so using linear pair axiom]
$\Rightarrow \angle DBC + 2\angle 1 = 180^\circ$ [Since, $\angle 1 = \angle OBA$,as OB is angular bisector of angle ABC]…(2)
In ΔOBC,
$\angle 1 + \angle 2 + \angle BOC = 180^\circ$ [Angle sum property of triangle]
$\Rightarrow 2\angle 1 + \angle BOC = 180^\circ$ [$\because \angle 1 = \angle 2$] …...(3)
From (2) and (3), we get
$\angle DBC + 2\angle 1 = 2\angle 1 + \angle BOC$
$\Rightarrow \angle DBC = \angle BOC$. Hence, Proved.