Solution

SURFACE AREAS AND VOLUMES

Class 09 - Mathematics

Section A

1.

(c) 2 : 3

Explanation:

SA of Sphere $= 4\pi r^2$

SA of two hemisphere $= 2 \times 3\pi r^2$

$= 6\pi r^2$

$\frac{SA \text{ of sphere}}{\text{SA of Hemisphere}} = \frac{4\pi r^2}{6\pi r^2} = \frac{4}{6} = \frac{2}{3}$

2.

(b) $36\pi \text{cm}^3$

Explanation:

Largest sphere that can be cut out of a cube will have its diameter as the side of the cube.

Radius of the largest sphere cut out of a cube of side 6 cm = 6/2 = 3 cm

Volume of a sphere = $(4/3)\pi r^3$

$\Rightarrow$ Volume of the largest sphere that is cut off from a cube of side 6 cm = $\frac{4}{3} \times \pi \times 3^3 = 36\pi \text{cm}^3$

3.

(b) 30.48 cm^3

Explanation:

Gap between the two = volume of cube - volume of sphere

= $\text{edge}^3 - \frac{4}{3}\pi r^3$

= $4^3 - \frac{4}{3} \times \frac{22}{7} \times 2^3$ (sphere touches cube, so diameter of sphere would be 4)

= 64 - 33.52

= 30.48 cm^3

4. **(a)** 25 : 16

Explanation:

Let r_1 and r_2 be the radius of the two spheres respectively. Therefore, the ratio of their surface areas,

$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{125}{64}$

$\Rightarrow \frac{r_1^3}{r_2^3} = \frac{125}{64}$

$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{5}{4}\right)^3$

$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$

Now, Ratio of their surface area,

$\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

$\therefore SA_1 : SA_2 = 25 : 16$

5.

(b) 1cm

Explanation:

Let the radius of sphere be r cm

and the radius of the cylinder is R cm.

Then according to question,

The volume of sphere = Volume of cylinder

$\Rightarrow \frac{4}{3}\pi r^3 = \pi R^2 h$

$\Rightarrow \frac{4}{3}r^3 = R^2 h$

$\Rightarrow$ h = $\frac{4r^3}{R^2}$

$\Rightarrow$ h = $\frac{4\times3\times3\times3}{3\times6\times6}$ = 1 cm

6.

(d) 25.344 kg wt.

Explanation:

Volume of conical vessel = $\frac{1}{3} \times \pi \times r^2 \times h$

= $\frac{1}{3} \times \frac{22}{7} \times 24 \times 24 \times 42$

= 25344 cm^3

= 25.344 dm^3 ($1dm^3 = 1000\ cm^3$)

= 25.344 kg wt. (1 kg- wt = $1dm^3$)

7. **(a)** 28 cm

Explanation:

Let h be the height of the cone.

Diameter of the cone = 42 cm

Radius of the cone = 21 cm

Then, volume of the cone = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times h$

= $22 \times 21 \times h$

Therefore

$22 \times 21 \times h = 12936$

$\Rightarrow h = \left(\frac{12936}{22\times21}\right)$

$\Rightarrow$ h = 28 cm

Hence, the height of the cone is 28 cm

8.

(b) 4 : 3

Explanation:

Let l_1 and l_2 be the slant heights of two cones respectively.

Given: $\frac{l_1}{l_2} = \frac{4}{3}$

Now, required ratio = $\frac{\pi r l_1}{\pi r l_2} = \frac{l_1}{l_2} = \frac{4}{3}$

$\Rightarrow CSA_1:CSA_2 = 4:3$

9. **(a)** 2 : 3

Explanation:

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

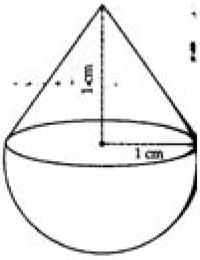
If a cylinder circumscribes a sphere of radius r , then its base radius is ‘r’ and height is diameter = 2r

Ratio between the volume of a sphere and volume of a circumscribing right circular cylinder

= $\frac{\frac{4}{3}\pi r^3}{\pi\times r^2\times 2r}$ = 2 : 3

10. **(a)** $\pi\ cm^3$

Explanation:



Radii of cone = r = 1 cm
Radius of hemisphere = r = 1 cm (h) = 1cm
Height of cone $(h) = 1$ h =1 cm
Volume of solid = Volume of cone + Volume of a hemisphere
= $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ = $\frac{1}{3}\pi r^2 (h + 2r)$
= $\frac{1}{3} \times \pi \times (1)^2 (1 + 2 \times 1)$
= $\frac{1}{3} \times \pi \times 3$ = π cm^3

11. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

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13. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

14.
(d) A is false but R is true.
Explanation:
A is false but R is true.

15.
(d) A is false but R is true.
Explanation:
Volume of metallic sphere = $\frac{4}{3}\pi(4)^3$ cm^3
Density of metal = 10 g/cm^3
Mass of shot put = Density $\times$ Volume
= 10 $\times$ $\frac{4}{3}\pi(4)^3$
= 2681.90g
= 2.68 kg $\neq$ 2 kg

16. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

17. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Volume of sphere, $\mathrm{V} = \frac{4}{3}\pi \mathrm{r}^3 \Rightarrow \mathrm{r} = \left(\frac{3\mathrm{~V}}{4\pi}\right)^{\frac{1}{3}}$
Surface area of sphere, $S = 4\pi r^2 = 4\pi\left(\frac{3\mathrm{~V}}{4\pi}\right)^{\frac{2}{3}}$
$\mathrm{S}^3 = 64\pi^3\left(\frac{3\mathrm{~V}}{4\pi}\right)^2 = \frac{64\pi^2 \times 9\mathrm{~V}^2}{16\pi^2} = 36\mathrm{~V}^2$
S^3 = 36V^2

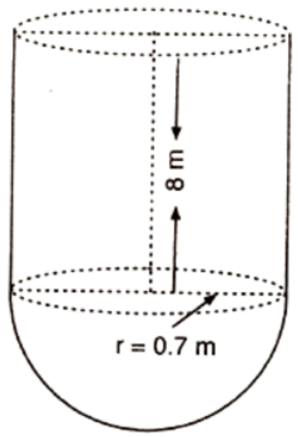
18. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

19.
(d) A is false but R is true.
Explanation:
A is false but R is true.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

Section B

21. Let the radius of the sphere be r cm.
Then, surface area = $4\pi r^2\ cm^2$
and, volume $= \frac{4}{3}\pi r^3\ cm^3$
According to the question, $4\pi r^2 = \frac{4}{3}\pi r^3$
⇒ r = 3
⇒ 2r = 6
∴ The diameter of the sphere is 6 cm.

22. 

We know that,
Curved surface area of a cylinder = 2πrh,
Curved surface area of hemisphere = $2\pi r^2$
Given, external diameter of the cylinder be 1.4, m and its length be 8 m....
Thus r = 0.7m and h = 8m....
Total curved surface area = 2πrh + $2\pi r^2$
⇒ Total curved surface area = 2 × (22/7) × 0.7 × 8 + 2 × (22/7) × 0.7^2 = 38.28 m^2
Cost of painting it on the outside at the rate of Rs. 10 per m^2 = 38.28 × 10 = Rs. 382.80.

23. Let the radius of the sphere be r cm.
Then, its volume = $\left(\frac{4}{3}\pi r^3\right)$cm^3
Therefore, $\frac{4}{3}\pi r^3$ = 4851
⇒ $\frac{4}{3} \times \frac{22}{7} \times r^3$ = 4851
⇒ r^3 = $\left(4851 \times \frac{3}{4} \times \frac{7}{22}\right)$ = $\left(\frac{441\times21}{8}\right)$ = $\left(\frac{21}{2}\right)^3$
⇒ r = $\frac{21}{2}$ = 10.5
Thus, the radius of the sphere is 10.5 cm.
Surface area of the sphere = ($47\pi r^2$) sq units
= $\left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right)$ cm^2
= 1386 cm^2

24. Radius of the base of conical tent (r) = 5 cm

Also, curved surface area of cone = πrl

$\Rightarrow 165 = \frac{22}{7} \times 5 \times l$

$\Rightarrow l = \frac{165 \times 7}{22 \times 5}$

$\Rightarrow l = \frac{21}{2} m = 10.5m$

Also, $h^2 = l^2 - r^2$

$\Rightarrow h = \sqrt{(10.5)^2 - 5^2} = \sqrt{15.5 \times 5.5} \approx 9.23$

Volume of conical tent = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 9.23 m^3 = 241.74\ m^3$.

25. r = 7 cm, l = 25 cm.

$r^2 + h^2 = l^2$

$\Rightarrow (7)^2 + h^2 = (25)^2$

$\Rightarrow h^2 = (25)^2 - (7)^2$

$\Rightarrow h^2 = 625 - 49$

$\Rightarrow h^2 = 576$

$\Rightarrow h = \sqrt{576}$

$\Rightarrow$ h = 24 cm

$\therefore$ Capacity $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24$

= 1232 cm^3 = 1.232 l

26. For conical vessel

Diameter = 120 cm

$\therefore$Radius (r) = $\frac{120}{2}$ cm = 60 cm = 6 dm

Depth (h) = 105 cm = 10.5 dm

$\therefore$Volume of water contained in the vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (6)^2 \times (10.5)\ dm^3 = 396\ dm^3$

$\therefore$ Weight of water contained in the vessel = 396 × 1.5 kg = 594 kg.

27. Given that,

Height of conical pardal (A) = 100 m

Base radius (r) = 240 m

Therefore Slant height (l) $= \sqrt{r^2 + h^2}$

$= \sqrt{(240)^2 + (100)^2} = \sqrt{57600 + 10000}$

$= \sqrt{67600}$ = 260 m

Now area of curved surface $= \pi rl$

$= \pi \times 240 \times 260 \mathrm{m}^2 = 62400\pi \mathrm{m}^2$

Width of canvas cloth = 100 π m

Therefore Length of cloth $= \frac{\text{Area}}{\text{Width}} = \frac{62400\pi}{100\pi}$

= 624 m

28. Given that,

Radius of the sphere = 8 cm

Volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 8^3 = 2145.52\ cm^3$

Radius of one lead ball = 1 cm

Volume of one lead ball = $\frac{4}{3} \times \frac{22}{7} \times 1^3 = 4.19\ cm^3$

Therefore, Number of lead balls = $\frac{volume\ of\ the\ sphere}{volume\ of\ one\ lead\ ball} = \frac{2145.52}{4.19} = 512.05 \approx 512$

29. Given that,

r = Radius of the hemisphere = 3.5 cm

Therefore Volume of the hemisphere = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 \mathrm{cm}^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \mathrm{cm}^3$

= $\frac{11\times49}{3\times2}$cm^3 = 89.83 cm^3

Now ,

Total surface area of the hemisphere

= $3\pi r^2$ = $3\times\frac{22}{7}\times3.54\times3.5$cm^2 = $3\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$cm^2 = $\frac{231}{2}$cm^2 = 115.5 cm^2

30. Slant height (l) = 25 m

Base diameter (d) = 14 m

∴ Base radius (r) = $\frac{14}{2}$m = 7 m

∴ Curved surface area of the tomb = πrl

= $\frac{22}{7}$× 7 × 25 = 550 m^2

∴ Cost of white-washing the curved surface of the tomb at the rate of Rs. 210 per 100 m^2

= $\frac{210}{100}$ × 550 = Rs. 1155.

31. Let the radius of base of hemisphere and cone, each be r cm. Let the height of the cone be h cm.

Volume of the cone = $\frac{1}{3}\pi r^2$ hcm^3

Volume of the hemisphere = $\frac{2}{3}\pi$r^3 cm^3

According to the question, $\frac{1}{3}\pi r^2h=\frac{2}{3}\pi r^3$

⇒ h = 2r

⇒ Height of the cone = 2r cm.

Height of the hemisphere = r cm

∴ Ratio of their heights = 2r : r = 2 : 1

32. Inner radius of the bowl, r = $\frac{10.5}{2}$ = 5.25 cm

Therefore, Inner curved surface area of the bowl = 2πr^2 = $2\times\frac{22}{7}\times(5.25)^2$ = 173.25 cm^2

Rate of tin-plating = 32 per 100 cm^2

Therefore, Cost of tin-plating the bowl on the inside

= Inner curved surface area of the bowl × Rate of tin-plating

= $173.25\times\frac{32}{100}$

= 55.44

Therefore, the cost of tin-plating the bowl on the inside is 55.44.

33. Inside surface area hemisphere = $2\pi r^2$

∴ Cost of white-washing at the rate of ₹ 2 per m^2 = ₹ ($2\pi r^2$ x 2) = ₹ $4\pi r^2$

It is given that the cost of white-washing is ₹ 498.96.

∴ $4\pi r^2$ = 498.96

⇒ $4\times\frac{22}{7}\times r^2$ = 498.96

⇒ $r^2=\frac{498.96\times7}{22\times4}$ ⇒ r^2 = 39.69 ⇒ r = 6.3

∴ Inside surface area of the dome = $2\pi r^2=2\times\frac{22}{7}$ x (6.3)^2 cm^2 = 249.48 cm^2

Volume of the dome = $\frac{2}{3}\pi r^3=\frac{2}{3}\times\frac{22}{7}\times(6.3)^3$ = 523.908 cm^3

34. Internal diameter of hemispherical tank = 14 m

∴ Internal radius of hemispherical tank = 14 m ÷ 2 = 7 m

Volume of hemispherical tank $=\frac{2}{3}\pi r^3=\frac{2}{3}\times\frac{22}{7}\times(7)^3$

$=\frac{44\times49}{3}=718.66m^3$.

The tank contains 50 Kilolitres of water = 50,000 litres $=\frac{50,000}{1,000}m^3=50m^3$

Volume of water pumped into the tank = 718.66 m^3 – 50 m^3 = 668.66 m^3.

35. Diameter of semicircular sheet is 28 cm. It is bent to from an open conical cup. The radius of sheet becomes the slant height of the cup. The circumference of the sheet becomes the circumference of the base of the cone.

∴ $l=$ Slant height of conical cup = 14 cm.

Let r cm be the radius and h cm be the height of the conical cup circumference of conical cup of the semicircular sheet

∴ $2\pi r=\pi\times14\Rightarrow r=7cm$

Now, $l^2=r^2+h^2\Rightarrow h=\sqrt{l^2-r^2}$

$=\sqrt{(14)^2-(7)^2}=\sqrt{196-49}=\sqrt{147}=12.12cm$

$\therefore$ Capacity of the cup

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12.12$

= 622.16 cm^3

36. Diameter of base of conical cap = 10 cm

$\therefore$ Radius of conical cap (r) = 5 cm

Slant height of cone (l) $= \sqrt{r^2 + h^2} = \sqrt{(5)^2 + (12)^2}$

$= \sqrt{25 + 144} = \sqrt{169}$ = 13 cm

Curved surface area of a cap = $\pi rl = 3.14 \times 5 \times 13 = 204.1\ cm^2$

Curved surface area of 15 caps = 15 $\times$ 204.1 = 3061.5cm^2

Area of a sheet of paper used for making caps $= 25 \times 40 = 1000\ cm^2$

82% of sheet is used after cutting = 82% of 1000 cm^2

$= \frac{82}{100} \times$ 1000 = 820 cm^2

Number of sheet $= \frac{3061.5}{820}$ = 3.73

Hence 4 sheets area needed.

37. Let the base radii of two right circular cones be 3x and 5x respectively.

Let their common height be h.

Then, volume of the first cone (v_1) = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(3x)^2 h$

and, volume of the second cone (v_2) = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(5x)^2 h$

$\therefore$ Ratio of their volumes = $\frac{v_1}{v_2} = \frac{\frac{1}{3}\pi(3x)^2 h}{\frac{1}{3}\pi(5x)^2 h}$

$= \frac{9}{25}$ = 9 : 25

38. Let the radius of the sphere be r cm.

Then, its surface area = ($4\pi r^2$) cm^2

Therefore, $4\pi r^2$ = 346.5 $\Rightarrow 4 \times \frac{22}{7} \times r^2 = 346.5$

$\Rightarrow r^2 = \left(346.5 \times \frac{7}{88}\right) = \left(\frac{3465 \times 7}{10 \times 88}\right) = \frac{441}{16}$

$\Rightarrow$ r = $\frac{21}{4}$ = 5.25

Thus, the radius of the sphere is 5.25 cm.

Therefore, Volume of the sphere = $\left(\frac{4}{3}\pi r^3\right)$ cm^2

= $\left(\frac{4}{3} \times \frac{22}{7} \times 5.25 \times 5.25 \times 5.25\right)$ cm^3

= $\left(\frac{4}{3} \times \frac{22}{7} \times \frac{525}{100} \times \frac{525}{100} \times \frac{525}{100}\right)$ cm^3

= $\frac{4851}{8}$ cm^3 = 606.375 cm^3

Hence, the volume of the sphere is 606.375 cm^3.

39. Case I : r = 7 cm

Surface area = $4\pi r^2$

= 4 × $\frac{22}{7}$ × $(7)^2$ = 616 cm^2

Case II : r = 14 cm

Surface area = $4\pi r^2$

= 4 × $\frac{22}{7}$ × $(14)^2$ = 2464 cm^2

$\therefore$ Ratio of surface area of the balloon = 616 : 2464

=1 : 4

40. i. h = 10 m, r = 24 m

$l = \sqrt{r^2 + h^2} = \sqrt{(24)^2 + (10)^2}$

$= \sqrt{576 + 100} = \sqrt{676}$

= 26 m

$\therefore$ the slant height of the tent is 26 m.

ii. Curved surface area of the tent = πrl

$= \frac{22}{7} \times 24 \times 26\ m^2$

$\therefore$ Cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹ 70.

$= \frac{22}{7} \times 24 \times 26 \times 70$ = ₹ 137280

$\therefore$ The cost of the canvas is ₹ 137280.

Section C

41. Heap of rice is in shape of cone, so

r = $\frac{9}{2}$ m = 4.5 m

h = 3.5 m

$\therefore$ V = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 3.5$

$\Rightarrow$ V = $\frac{22\times9\times9\times35}{3\times7\times2\times2\times10} = \frac{33\times9}{4} = \frac{297}{4}$

$\Rightarrow$ V = 74.25 m^3

Hence, volume of rice = 74.25 m^3.

For canvas:

Area of canvas = Curved surface area of cone

= πrl

Here, $l^2 = r^2 + h^2 = (4.5)^2 + (3.5)^2$ = 20.25 + 12.25

$\Rightarrow l^2$ = 32.50

$\Rightarrow$ l = $\sqrt{32.5}$ = 5.7 m

$\therefore$ Area of canvas = $\frac{22}{7}$ × 4.5 × 5.7 = 80.614

$\Rightarrow$ Area of canvas = 80.61 m^2

42. We have Diameter of each sphere, d = 12 cm

Radius of each sphere, r = 6 cm

Therefore, Volume of each sphere = $\frac{4}{3}\pi r^3$ = $\frac{4}{3}\pi(6)^3$ cm^3

Again Diameter of base of the cylinder, D = 8 cm

Radius of base of cylinder, R = 4 cm

Height of the cylinder, h = 90 cm

Therefore Volume of the cylinder = $\pi R^2 h$ = $\pi(4)^2 \times 90$ cm^3

Number of spheres = $\frac{volume\ of\ the\ cylinder}{volume\ of\ the\ sphere}$

= $\frac{\pi R^2 h}{\frac{4}{3}\pi r^3}$ = $\frac{4^2\times90\times3}{4\times6^3}$ = $\frac{12\times90}{216}$ = 5

43. Let the radius of the cone $(r) = 3x\ cm$

Height of the cone $(h) = 4x\ cm$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow$ 301.44 = $\frac{1}{3} \times 3.14 \times (3x)^2 \cdot 4x$

$\Rightarrow x^3 = \frac{301.44}{3.14\times12} = 8$

$\Rightarrow x^3 = 2^3$

$\Rightarrow x = 2cm$

Radius of the cone $= 3x = 3 \times 2 = 6cm$

Height of the cone $= 4x = 4 \times 2 = 8cm$

Slant height of the cone (l) =$\sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2}$ =$\sqrt{100} = 10\ cm$

44. Let, the radius of the base of cone be r cm

Height of the cone $= 15cm$

Volume of the cone $= 1570cm^3$

$\Rightarrow \frac{1}{3}\pi r^2 h = 1570$

$\Rightarrow \frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$

$\Rightarrow r^2 = \frac{1570\times3}{3.14\times15}$=100

$\Rightarrow$r= $\sqrt{100}$= 10 cm

Thus, the diameter of the base of cone$= 2r = 2 \times 10cm = 20cm$

45. A hemispherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties it at the rate of 7 litres per second.We have to find the time it will take to empty the tank completely.

Suppose the pipe takes x seconds to empty the tank. Then,

Volume of the water that flows out of the tank in x seconds

= Volume of the hemispherical tank.

⇒ Volume of the water that flows out of the tank x in seconds = Volume of the hemispherical shell of radius 175 cm

⇒ $7000x = \frac{2}{3} \times \frac{22}{7} \times 175 \times 175 \times 175$

⇒ $x = \frac{2}{3} \times \frac{22}{7} \times \frac{175\times175\times175}{7000} = 1604.16$ seconds

⇒ $x = \frac{1604.16}{60}$ minutes = 26.73 minutes

46. For heap of wheat

Diameter = 10.5 m

∴ Radius (r) = $\frac{10.5}{2}$ cm = 5.25 m

Height (h) = 3 m

∴ Volume = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5.25)^2 \times 3$

= 86.625 m^3

Slant height, $l = \sqrt{r^2 + h^2}$

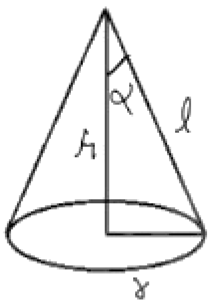
$= \sqrt{(5.25)^2 + (3)^2} = \sqrt{27.5625 + 9}$

$= \sqrt{36.5625}$ = 6.05 m

∴ Curved surface area =πrl

= $\frac{22}{7}$ × 5.25 × 6.05 = 99.825 m^2

∴ The area of the canvas required is 99.825 m^2

47. 

⇒ sin α = $\frac{r}{l}$

⇒ l = $\frac{r}{sin\alpha}$

⇒ l = r cosecα .....(1)

⇒ tan α = $\frac{r}{h}$

⇒ h = $\frac{r}{tan\alpha}$

⇒ h = r cot α .....(2)

height of cone = $r\ cot\alpha$

Area of base (A) = α

Volume of right circular cone = $\frac{1}{3}\pi r^2 h$

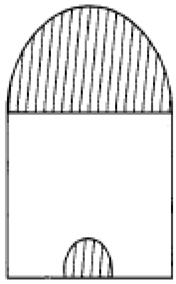
= $\frac{1}{3}\pi r^2 \times$ r cotα [from eq(2)]

= $\frac{1}{3}\pi r^3$ cot α

Therefore , Surface area of cone = πrl =πr(r cosecα)

= πr^2 cosec α.

Hence proved

48. 

Let radii of the bases of two domes be r and R.

$\therefore 2\pi r = 17.6$

$\Rightarrow 2\times \frac{22}{7} \times r = 17.6$

$\Rightarrow$r = $\frac{17.6\times7}{2\times22}$ = 2.8 cm and 2πR $= 70.4$

$\Rightarrow 2\times\frac{22}{7}\times$R $= 70.4$

$\Rightarrow$R = $\frac{70.4\times7}{2\times22} = 11.2\ cm$

Now, area of two hemispherical domes $= 2\pi r^2 + 2\pi R^2$

=2 $\times\frac{22}{7}\times2.8\times2.8\ \times2\ \times\frac{22}{7}\times11.2\ \times11.2$

$= 49.28 + 788.48\ cm^2$

$= 837.76\ cm^2$

Cost of painting at the rate of Rs.10 per cm^2 $= 837.76 \times 10 = Rs.8377.6$

49. Given, base diameter = 40 cm

$\therefore$ Base radius (r) = $\frac{40}{2}\ cm = 20\ cm = \frac{20}{100}\ m = 0.2\ m$

Height (h) = 1 m

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{(0.2)^2 + (1)^2} = \sqrt{0.04 + 1}$

$= \sqrt{1.04} = 1.02\ m$

$\therefore$ Curved surface area = πrl

= 3.14 × 0.2 × 1.02

= 0.64056 m^2

$\therefore$ Curved surface area of 50 cones = 0.64056 × 50 m^2

= 32.028 m^2

$\therefore$ Cost of painting all these cones = 32.028 × 12

= 384.336 = Rs. 384.34

50. Diameter of Conical heap = 24 m

Radius of conical heap, r =$\frac{diameter}{2}$ =$\frac{24}{2}$m = 12 m

Height of conical heap, h = 3.5 m

Volume of rice = $\frac{1}{3} \times \pi \times r^2 \times h$ = $\frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$ m^3 = 528 m^3

Area of canvas cloth required = πrl

Now, l = $\sqrt{12^2 + (3.5)^2}$ = 12.5 m

$\therefore$ Area of canvas required = $\frac{22}{7} \times 12 \times 12.5$ = 471.4 m^2

51. 

We are given that ,

Outer radius of the vessel, R = 14 cm.

Inner radius of the vessel, r = 10 cm.

Area of the outer surface = (2πR^2) sq units

= (2$\pi \times 14 \times 14$) cm^2 = (392π) cm^2.

Area of the inner surface = (2πr^2) sq units

= (2$\pi \times 10 \times 10$) cm^2 = (200π) cm^2.

Area of the ring at the top = π(R^2 - r^2) sq units

$= \pi[(14)^2 - (10)^2]$ cm^2

$= \pi(14 + 10)(14 - 10)$ cm^2 = (96π) cm^2.

Total area to be painted = $(392\pi + 200\pi + 96\pi)$ cm^2 = (688π) cm^2

Cost of painting = ₹$\left(688\pi \times \frac{35}{100}\right)$ = ₹$\left(688 \times \frac{22}{7} \times \frac{35}{100}\right)$

= ₹ $\frac{3784}{5}$ = ₹756.80

52. Radius (r) of heap = $\left(\frac{9}{2}\right)$m = 4.5m

Height (h) of heap = 3.5m

Volume of heap = $\frac{1}{3}\pi r^2 h$

= $[\frac{1}{3} \times 3.14 \times (4.5)^2 \times 3.5]$ m^3 = 74.18 m^3

Slant height (l) = $\sqrt{r^2 + h^2} = \sqrt{(4.5)^2 + (3.5)^2}$ = 5.70 m

Area of canvas required = CSA of cone

= πrl = $(3.14 \times 4.5 \times 5.7)$ m^2 = 80.54 m^2

53. Diameter of spherical capsule = 3.5 mm

$\therefore$ Radius of spherical capsule (r)=$\frac{3.5}{2}$=$\frac{35}{20}$=$\frac{7}{4}$mm

Medicine needed to fill the capsule = Volume of sphere

$=\frac{4}{3}\pi r^3$

$=\frac{4}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}$

$=\frac{11\times7\times7}{3\times2\times4}$

=539/24

$= 22.46\, mm^3$ (Approx.)

54. Here we are given that,

Radius of a cone, r = 35 cm

Height of a cone, h = 12 cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 12\right)$ cm^3

= 15400 cm^3

Slant height of a cone, $l = \sqrt{r^2 + h^2} = \sqrt{35^2 + 12^2} = \sqrt{1369}$ = 37 cm

Curved surface area of a cone $= \pi rl$

$= \left(\frac{22}{7} \times 35 \times 37\right)$ cm^2

= 4070 cm^2

Total surface area of a cone $= \pi r(l + r)$

$= \left[\frac{22}{7} \times 35(37 + 35)\right]$ cm^2

$= [22 \times 5 \times 72]$cm^2

= 7920 cm^2

55. Here we are given that,

Radius of a conical tent, r = 24 m

Height of a conical tent, h = 10 m

Therefore Slant height of a conical tent,

$l = \sqrt{r^2 + h^2} = \sqrt{24^2 + 10^2}$ = $\sqrt{576 + 100} = \sqrt{676}$ =26 cm

Curved surface area of a conical tent = πrl

$= \left(\frac{22}{7} \times 24 \times 26\right)$ m^2

$= \frac{13728}{7}$m^2

cost of 1 m^2 canvas = Rs. 70

$\Rightarrow$ cost of $\frac{13728}{7}$m^2 canvas =Rs.$\left(70 \times \frac{13728}{7}\right)$= Rs. 137280

56. CSA of cone $= \pi rl = \pi r\sqrt{r^2 + h^2}$

=$(3.14 \times r \times \sqrt{r^2 + 64})$m^2

$\therefore 3.14 \times r \times \sqrt{r^2 + 64} = 188.4$

$\Rightarrow r\sqrt{r^2 + 64} = \frac{188.4}{3.14}$ =60

$\Rightarrow r^2\left(r^2+64\right)$ = 3600 $\left[\begin{array}{l}\because x^2+64x-3600=0,\ \text{where } r^2=x \\ \Rightarrow (x+100)(x-36)=0\end{array}\right]$

$\Rightarrow (r^2+100)(r^2-36) = 0$

$\Rightarrow r^2 = 36$ [$\therefore r^2 = -100$ gives imaginary value of r]

$\Rightarrow r = 6$.

So, the radius of the base = 6 m.

Volume = $=\left(\frac{1}{3}\pi r^2 h\right)$ cubic units

$=\left(\frac{1}{3}\times 3.14\times 6\times 6\times 8\right)$ m^3 = 301.44 m^3

Hence, the volume of the cone is 301.44 m^3

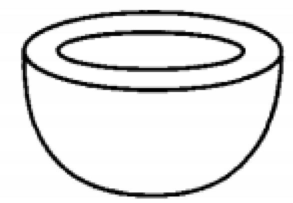
57. We know that the surface area S and total surfaces area S_1 of a hemisphere of radius r are given by

S = $2\pi r^2$ and, S_1 = $3\pi r^2$ respectively.

Here, r = 21 cm

$\therefore$ S = $2\times\frac{22}{7}\times 21\times 21$ cm^2 and, S_1 = $3\times\frac{22}{7}\times 21\times 21$ cm^2

$\Rightarrow$ S = 2772 cm^2 and, S_1 = 4158 cm^2

58. 

Internal radius = $\frac{24}{2}$ cm

External radius = $\frac{25}{2}$ cm

Area to be painted

= Outer C.S.A. + inner C.S.A + area of the ring

$= 2\pi \mathrm{R}^2 + 2\pi r^2 + \pi\left[\mathrm{R}^2 - r^2\right]$

$=2\times\frac{22}{7}\times\frac{25}{2}\times\frac{25}{2}+2\times\frac{22}{7}\times\frac{24}{2}\times\frac{24}{2}+\frac{22}{7}\left[\left(\frac{25}{2}\right)^2-\left(\frac{24}{2}\right)^2\right]$

$=2\times\frac{22}{7}\times\frac{1}{4}$ [625 + 576] + $\frac{22}{7}\left[\frac{25}{2}-\frac{24}{2}\right]\left[\frac{25}{2}+\frac{24}{2}\right]$

$=\frac{22}{7}\left[\frac{1201}{2}+\frac{1}{2}\times\frac{49}{2}\right]$

$=\frac{22}{7}\times\frac{1}{4}[2402+49]$

$=\frac{22}{7}\times\frac{1}{4}\times 2451\mathrm{cm}^2$

Cost of painting

$= 0.05\times\frac{22}{7}\times\frac{1}{4}\times 2451$= Rs 96.28.

59. Inner diameter of bowl = 10.5 cm

$\therefore$ Inner radius of bowl $(r)=\frac{10.5}{2}$ = 5.25 cm

Now, Inner surface area of bowl = $2\pi r^2$

$=2\times\frac{22}{7}\times 5.25\times 5.25$

$=2\times\frac{22}{7}\times\frac{21}{4}\times\frac{21}{4}$

$=\frac{693}{4}cm^2$

$\because$ Cost of tin-plating per100 cm^2 = ₹ 16

$\therefore$ Cost of tin-plating per1 $cm^2=\frac{16}{100}$

$\therefore$ Cost of tin-plating per $\frac{693}{4}cm^2=\frac{16}{100}\times\frac{693}{4}$ = ₹ 27.72

60. The diameter of the largest sphere which can be carved out of a cube is 7 cm

$\therefore$ Radius of the sphere = r = $\frac{7}{2}$ cm

Hence, Volume of the sphere = $\frac{4}{3}\pi r^3$

$\Rightarrow$ Volume of the sphere = $\frac{4}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^3$ cm^3

= $\frac{4}{3}\times\frac{22}{7}\times\frac{343}{8}$ cm^3 = 179.66 cm^3

Section D

61. Since the grains of corn are found on the curved surface of the corn cob.

So, Total number of grains on the corn cob = Curved surface area of the corn cob $\times$ Number of grains of corn on 1 cm^2

Now, we will first find the curved surface area of the corn-cob.

We have, r = 2.1 and h = 20

Let l be the slant height of the conical corn cob. Then,

$l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (20)^2} = \sqrt{4.41 + 400} = \sqrt{404.41}$ = 20.11

$\therefore$ Curved surface area of the corn cub = $\pi r l$

= $\frac{22}{7} \times$ 2.1 $\times$ 20.11 cm^2

= 132.726 cm^2 = 132.73 cm^2

Hence, Total number of grains on the corn cob = 132.73 $\times$ 4 = 530.92

So, there would be approximately 531 grains of corn on the cob.

62. Diameter of cone = 40 cm

$\Rightarrow$ Radius of cone (r) = $\frac{40}{2}$

= 20 cm

= $\frac{20}{100}$ m

= 0.2 m

Height of cone (h) = 1 m

Slant height of cone $(l) = \sqrt{r^2 + h^2}$

$= \sqrt{(0.2)^2 + (1)^2}$

= $\sqrt{1.04}$ m

Curved surface area of cone = $\pi r l$

= $3.14 \times 0.2 \times \sqrt{1.04}$

= 0.64056 m^2

$\because$ Cost of painting 1m^2 of a cone = Rs.12

$\therefore$ Cost of painting 0.64056m^2 of a cone = 12 $\times$ 0.64056 = Rs. 7.68672

$\therefore$ Cost of painting of 50 such cones = 50 $\times$ 7.68672 = Rs. 384.34 (approx.)

63. i. Diameter of cone = 40 cm

$\Rightarrow$ Radius of cone (r) = $\frac{40}{2}$

= 20 cm

= $\frac{20}{100}$ m

= 0.2 m

Height of cone (h) = 1 m

Slant height of cone (l) = $\sqrt{r^2 + h^2}$

$= \sqrt{(0.2)^2 + (1)^2}$

= $\sqrt{1.04}$ m

Curved surface area of cone = $\pi r l$

= $3.14 \times 0.2 \times \sqrt{1.04}$

= 0.64056 m^2

ii. Radius of base of cone = 20 cm = 0.2 m

Height of cone = 1 m

Volume of each cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 0.2 \times 0.2 \times 1$

= 0.042 m^3

iii. $\therefore$ Cost of painting 1m^2 of a cone = ₹12

$\therefore$ Cost of painting 0.64056 m^2 of a cone = 12 $\times$ 0.64056 = ₹ 7.68672

$\therefore$ Cost of painting of 50 such cones = 50 $\times$ 7.68672 = ₹ 384.336

OR

Cost of 1 m^2 cardboard = ₹ 100

Curved surface area of 50 cones = 0.640 $\times$ 50 = 32 m^2

Cost of card board of these 50 cones = 50 $\times$ 32 = ₹ 1600

64. i. First we will find the curved surface area of the corn cob.

We have, r = 2.1 and h = 20

Let l be the slant height of the conical corn cob. Then,

l = $\sqrt{r^2+h^2}=\sqrt{(2.1)^2+(20)^2}$ = $\sqrt{4.41+400}=\sqrt{404.41}$ = 20.11 cm

$\therefore$ Curved surface area of the corn cub = πrl

= $\frac{22}{7}\times$ 2.1 $\times$ 20.11 cm^2

= 132.726 cm^2 = 132.73 cm^2

ii. The volume of the corn cub

$=\frac{1}{3}\pi r^2 h=\frac{1}{3}\times\frac{22}{7}\times 2.1\times 2.1\times 20$

= 92.4 cm^3

iii. Now

Total number of grains on the corn cob = Curved surface area of the corn cob $\times$ Number of grains of corn on 1 cm^2

Hence, Total number of grains on the corn cob = 132.73 $\times$ 4 = 530.92

So, there would be approximately 531 grains of corn on the cob.

OR

Volume of a corn cub = 92.4 cm^3

Volume of the cartoon = 20 $\times$ 25 $\times$ 20 = 10,000 cm^3

Thus no. of cubs which can be stored in the cartoon

$\frac{10000}{92.4}\approx 108$ cubs

65. i. Height of the tent h = 10 m

Radius r = 7 cm

Thus Latent height l = $\sqrt{r^2+h^2}$ = $\sqrt{7^2+10^2}=\sqrt{149}$ = 12.20 m

Curved surface of tent = $\pi rl=\frac{22}{7}\times 7\times 12.2$ = 268.4 m^2

Thus the length of the cloth used in the tent = 268.4 m^2

The remaining cloth = 300 - 268.4 = 31.6 m^2

Hence the cloth used for the floor = 31.6 m^2

ii. Height of the tent h = 10 m

Radius r = 7 cm

Thus the volume of the tent = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 10$

= 513.3 m^3

iii. Radius of the floor = 7 m

Area of the floor = $\pi r^2=\frac{22}{7}\times 7\times 7$

= 154 m^2

OR

Radius of the floor r = 7 m

Latent height of the tent l = 12.2 m

Thus total surface area of the tent = $\pi r(r+l)$

= $\frac{22}{7}\times 7(7+12.2)$

= 22 $\times$ 19.2

= 422.4 m^2

Section E

66. Since only the rounded surface of the dome is to be painted, we would need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done. Now, circumference of the dome = 17.6 m. Therefore, 17.6 = $2\pi r$

2 $\times\frac{22}{7}r$ = 17.6 m

So, the radius of the dome = 17.6 $\times\frac{7}{2\times 22}$ m = 2.8 m

The curved surface area of the dome = $2\pi r^2$

= 2 $\times$ $\frac{22}{7}$ $\times$ 2.8 $\times$ 2.8 m^2

= 49.28 m^2

Now, the cost of painting 100 cm^2 is Rs. 5.

So, the cost of painting 1 m^2 = Rs. 500

Therefore, the cost of painting the whole dome

= Rs. 500 $\times$ 49.28

= Rs. 24640

67. Since the grains of corn are found on the curved surface of the corn cob.

So, Total number of grains on the corn cob = Curved surface area of the corn cob $\times$ Number of grains of corn on 1 cm^2

Now, we will first find the curved surface area of the corn-cob.

We have, r = 2.1 and h = 20

Let l be the slant height of the conical corn cob. Then,

$l=\sqrt{r^2+h^2}=\sqrt{(2.1)^2+(20)^2}=\sqrt{4.41+400}=\sqrt{404.41}$ = 20.11

$\therefore$ Curved surface area of the corn cub = πrl

= $\frac{22}{7}\times$ 2.1 $\times$ 20.11 cm^2

= 132.726 cm^2 = 132.73 cm^2

Hence, Total number of grains on the corn cob = 132.73 $\times$ 4 = 530.92

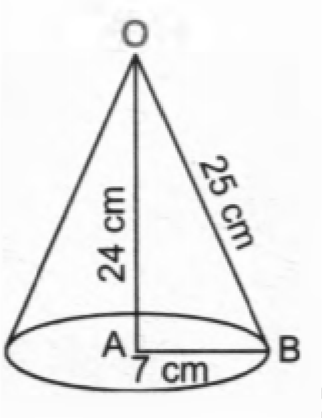
So, there would be approximately 531 grains of corn on the cob.

68. Here we are given that, h = 24 cm and r = 7 cm.

Let the slant height be l cm. Then

$l=\sqrt{h^2+r^2}=\sqrt{(24)^2+(7)^2}$ = $\sqrt{625}=25$

Therefore slant height = l cm = 25 cm.



Volume of the cone = $\frac{1}{3}\pi r^2 h$

$=\left(\frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 24\right)$ cm^3

= 1232 cm^3

Curved surface area of the cone = πrl

$=\left(\frac{22}{7}\times 7\times 25\right)$ cm^2 = 550 cm^2.

Total surface area of the cone = $\pi r(l+r)$

$=\left[\frac{2}{7}\times 7\times 125+7\right]$ cm^2

= 704 cm^2

69. Let r be the radius of hemisphere $\therefore$ r = $\frac{21}{2}$ cm

Volume of remaining block = $a^3-\frac{2}{3}\pi r^3$

= $(21)^3$ - $\frac{2}{3}\pi\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}$

= 9261 $\left[1-\frac{\pi}{12}\right]$ cm^3

= 6853 cm^3 (Approx.)

70. Radius (r) = 2.8 cm

Apparent capacity of glass = $\frac{22}{7}\times 2.8\times 2.8\times 10$

= 246.4 cm^3

Volume of hemispherical part = $\frac{2}{3}\times\frac{22}{7}\times 2.8\times 2.8\times 2.8$

= 45.9 cm^3

∴ Actual capacity of glass = 246.4 - 45.9

= 200.5 cm^3 or 200.5 ml

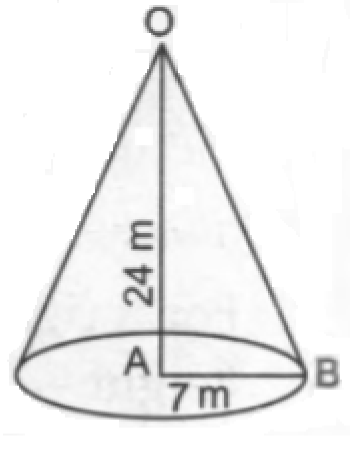
71. i. We have, Volume = 9856 cm^3 and r = 14 cm.
Let the height of the cone be h,cm
Then, volume of cone = $\frac{1}{3}\pi r^2 h$
$\Rightarrow$9856 = $\frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times$h
$\Rightarrow h = \left(\frac{9856\times3\times7}{22\times14\times14}\right)$ = 48
height of the cone is 48 cm.

ii. We have, Volume = 9856 cm^3 and r = 14 cm.
Let the slant height of the cone be l, cm. Then,
$l^2 = r^2 + h^2$
$\Rightarrow l^2 = (14)^2 + (48)^2 = 196 + 2304 = 2500$
$\Rightarrow l = \sqrt{2500} = 50$
∴ slant height of the cone is 50 cm

iii. We have, Volume = 9856 cm^3 and r = 14 cm.
Curved surface area of the cone
$= \pi r l$
$= \left(\frac{22}{7} \times 14 \times 50\right) \mathrm{cm}^2$ = 2200 cm^2
∴ curved surface area of the cone is 2200 cm^2.

72. Diameter of cone = 40 cm
$\Rightarrow$ Radius of cone (r) = $\frac{40}{2}$
= 20 cm
= $\frac{20}{100}$ m
= 0.2 m
Height of cone (h) = 1 m
Slant height of cone $(l) = \sqrt{r^2 + h^2}$
$=\sqrt{(0.2)^2 + (1)^2}$
= $\sqrt{1.04}$ m
Curved surface area of cone = $\pi r l$
= $3.14 \times 0.2 \times \sqrt{1.04}$
= 0.64056 m^2
∵ Cost of painting 1m^2 of a cone = Rs.12
∴ Cost of painting 0.64056m^2 of a cone = 12 $\times$ 0.64056 = Rs. 7.68672
∴ Cost of painting of 50 such cones = 50 $\times$ 7.68672 = Rs. 384.34 (approx.)

73. Radius of the tent, r = 7m and its height, h = 24 m.
∴ Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(7)^2 + (24)^2}$m
$= \sqrt{625}$m = 25 m



Area of the curved surface = (πrl) sq units

$= \left(\frac{22}{7} \times 7 \times 25\right) \mathrm{m}^2$

= 550 m^2

Thus, the area of the cloth = 550 m^2

Length of the cloth required = $\left(\frac{\text{area}}{\text{width}}\right) = \left(\frac{550}{5}\right) \mathrm{m}$ = 110m

Hence, length of the required cloth is 110 m.

74. Height of the conical tent (h) = 8 m and Radius of the conical tent (r) = 6 m

Slant height of the tent $(l) = \sqrt{r^2 + h^2}$

$=\sqrt{(6)^2 + (8)^2}$

$=\sqrt{36 + 64}$

$=\sqrt{100}$

= 10 m

Area of tarpaulin = Curved surface area of tent = $\pi rl = 3.14 \times 6 \times 10 = 188.4\, m^2$

Width of tarpaulin = 3 m

Let Length of tarpaulin = L

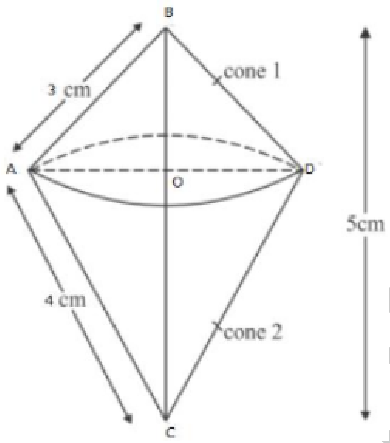
$\therefore$ Area of tarpaulin = $Length \times Breadth \;=\; L \times 3$ = 3L

Now According to question, 3L = 188.4

$\Rightarrow$ L = 188.4/3 = 62.8 m

The extra length of the material required for stitching margins and cutting is 20 cm = 0.2 m.

So the total length of tarpaulin bought is (62.8 + 0.2) m = 63 m

75. 

AB = 3 cm, AC = 4 cm

In $\triangle$BAC, by pythagoras theorem

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = 3^2 + 4^2$

$\Rightarrow BC^2 = 25$

$\Rightarrow$BC = $\sqrt{25}$ = 5 cm

In $\triangle$AOB and $\triangle$CAB

$\angle$ABO = $\angle$ABC [common]

$\angle$AOB = $\angle$BAC [each 90^o]

Then, $\triangle$AOB - $\triangle$CAB [by AA similarity]

$\therefore \frac{AO}{CA} = \frac{OB}{AB} = \frac{AB}{CB}$ [c.p.s.t]

$\Rightarrow \frac{AO}{4} = \frac{OB}{3} = \frac{3}{5}$

Then, AO = $\frac{4\times3}{5}$ and OB = $\frac{3\times3}{5}$

$\Rightarrow$ AO = $\frac{12}{5}$ cm and OB = $\frac{9}{5}$ cm

$\therefore$OC = 5 - $\frac{9}{5}$ = $\frac{16}{5}$ cm

$\therefore$ Volume of double cone thus generated = volume of first cone + volume of second cone

$= \frac{1}{3}\pi(AO)^2 \times BO + \frac{1}{3}\pi(AO)^2 \times OC$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{12}{5}\right)^2 \times \frac{9}{5} + \frac{1}{3} \times \frac{22}{7} \times \left(\frac{12}{5}\right)^2 \times \frac{16}{5}$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{12}{5} \times \frac{12}{5}\left[\frac{9}{5} + \frac{16}{5}\right]$
$= \frac{1}{3} \times \frac{22}{7} \times \frac{12}{5} \times \frac{12}{5} \times 5$
$= \frac{1056}{35} = 30\frac{6}{35} \mathrm{cm}^3.$