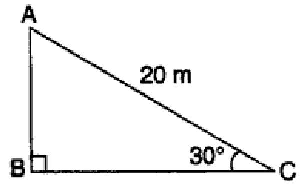
Solution

SOME APPLICATIONS OF TRIGONOMETRY

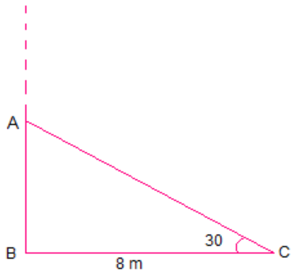
Class 10 - Mathematics

1. In right triangle ABC,



$\sin 30^{\circ} = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{AB}{20} \Rightarrow AB = 10\text{m}$

Hence, the height of the pole is 10 m.

2. 

Let AC be the broken part of the tree.

∴ Total height of the tree = AB + AC

In right △ABC,

cos 30$^{\text{O}}$ = $\frac{BC}{AC}$

⇒ $\frac{\sqrt{3}}{2}$ = 8/AC$\frac{\sqrt{3}}{2} = \frac{8}{AC}$

⇒ AC = $\frac{16}{\sqrt{3}}$

Also,

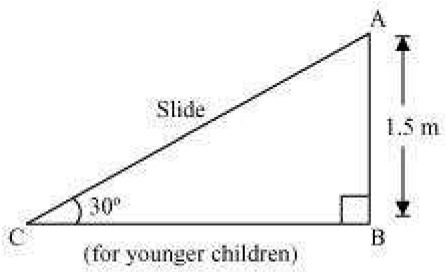
tan 30$^{\text{O}}$ = $\frac{AB}{BC}$

⇒ $\frac{1}{\sqrt{3}} = \frac{AB}{8}$

⇒ AB = $\frac{8}{\sqrt{3}}$

Total height of the tree = AB + AC = $\frac{16}{\sqrt{3}}$ + $\frac{8}{\sqrt{3}}$ = $\frac{24}{\sqrt{3}}$

3. In the first case:



Height of slide= 1.5 m and angle of elevation = 30°

Now,

$\sin \theta = \frac{p}{h}$

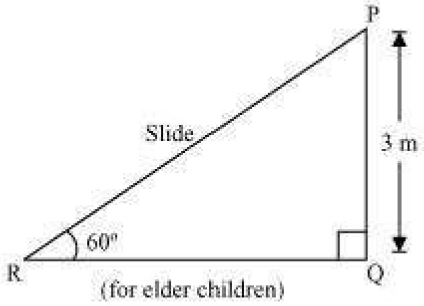
where p = perpendicular, i.e. height of the slide and h = hypotenuse, i.e. length of the slide and θ is the angle of elevation

$\sin 30^{\circ} = \frac{1.5}{h}$

$\frac{1}{2} = \frac{1.5}{h}$

Hence, h = 3 m

In the second case:



Height of slide, = 3 m, angle of elevation = 60°

$\sin\theta = \frac{p}{h}$

$\sin 60^\circ = \frac{3}{h}$

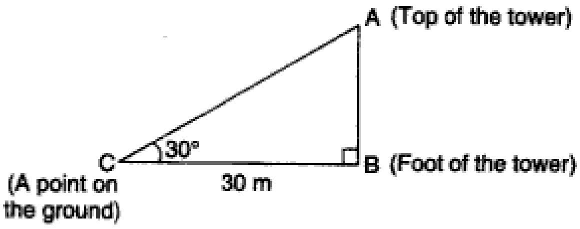
$\frac{\sqrt{3}}{2} = \frac{3}{h}$

Hence, h = $2\sqrt{3}$ m

Therefore, the length of the slide in the first and the second case are 3 m and $2\sqrt{3}$ m respectively.

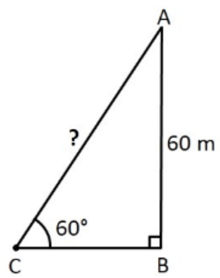
4. In right triangle ABC,

$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} - \frac{AB}{30}$



$AB = \frac{30}{\sqrt{3}} \Rightarrow AB = 10\sqrt{3}\text{m}$

Hence, the height of the tower is $10\sqrt{3}$ m.

5. 

In right triangle ABC,

$\sin 60^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{\text{AC}}$

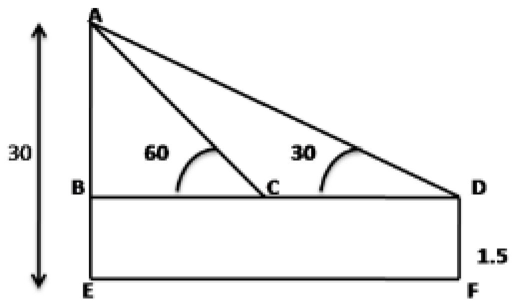
$AC = \frac{120}{\sqrt{3}}$

Multiplying $\sqrt{3}$ in both numerator and denominator,

$\text{AC} = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

AC = $40\sqrt{3}$m

Hence the length of the string is $40\sqrt{3}$m.

6. 

Let AE is the Length of the building.

So AE = 30

Again BE = DF = 1.5

AB = AE - BE = 30 - 1.5 = 28.5

Now in triangle ABC,

tan60 = $\frac{AB}{BC}$

⇒ √3 = $\frac{28.5}{BC}$

⇒ BC = $\frac{28.5}{\sqrt{3}}$

Again in triangle ABD

tan30 = $\frac{AB}{BD}$

$\frac{\sqrt{1}}{\sqrt{3}} = \frac{28.5}{BD}$

⇒ BD = 28.5×√3

⇒ BC + CD = 28.5√3

⇒ 28.5/√3 + CD = 28.5√3

⇒ CD = $\frac{28.5}{\sqrt{3}}$- $\frac{28.5}{\sqrt{3}}$

⇒ CD = $\frac{28.5\times3-28.5}{\sqrt{3}}$

⇒ CD = $\frac{28.5(3-1)}{\sqrt{3}}$

⇒ CD = $\frac{(28.5\times2)}{\sqrt{3}}$

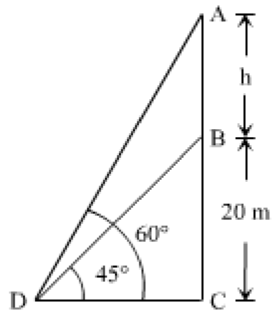
⇒ CD = $\frac{(57)}{\sqrt{3}}$

⇒ CD = $\frac{(57\sqrt{3})}{\sqrt{3}\times\sqrt{3}}$ (Multiply √3 in numerator and denominator)

⇒ CD = $\frac{57\sqrt{3}}{3}$

⇒ CD = 19√3

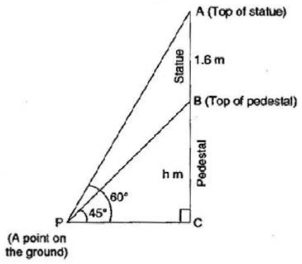
The distance he walked towards the building is 19√3 m

7. 

Let BC be the building, AB be the transmission tower, and D be the point on the ground.

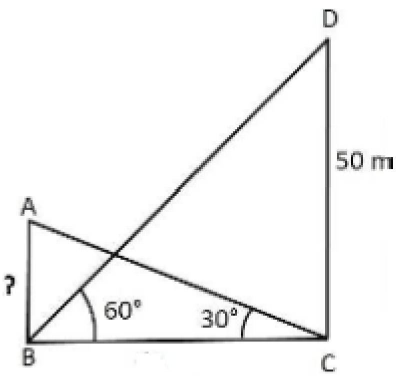
In ΔBCD,

$\frac{BC}{CD} = tan45°$
$\Rightarrow \frac{20}{CD} = 1$
$\Rightarrow CD = 20$
In ΔACD,
$\frac{AC}{CD} = tan60°$
$\Rightarrow \frac{AB+BC}{CD} = \sqrt{3}$
$\Rightarrow \frac{AB+20}{20} = \sqrt{3}$
$\Rightarrow AB + 20 = 20\sqrt{3}$
$\Rightarrow AB = 20(\sqrt{3} - 1)\ m.$

8. 

Let the height of the pedestal be h m.
∴ BC = h m
In right triangle ACP,
$\tan 60^\circ = \frac{AC}{PC}$
$\Rightarrow \sqrt{3} = \frac{AB+BC}{PC}$
$\Rightarrow \sqrt{3} = \frac{1.6+h}{\text{PC}}$..........(i)
In right triangle BCP,
$\tan 45^\circ = \frac{BC}{PC}$
$\Rightarrow 1 = \frac{h}{\text{PC}} \Rightarrow \text{PC} = h$
$\therefore \sqrt{3} = \frac{1.6+h}{h}$[From eq. (i)]
$\Rightarrow \sqrt{3}h = 1.6 + h \Rightarrow h(\sqrt{3} - 1) = 1.6 \Rightarrow \frac{1.6}{\sqrt{3}-1}$
$\Rightarrow \quad \frac{1.6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \Rightarrow h = \frac{1.6(\sqrt{3}+1)}{3-1} \Rightarrow h = \frac{1.6(\sqrt{3}+1)}{2}$
$\Rightarrow h = 0.8(\sqrt{3} + 1)_{\text{m}}$

9. Given,



Let the height of building be AB and height of tower CD
Height of the tower (CD)= 50 m
Angle of elevation of top of building from foot of tower = 30°
Hence, $\angle ACB = 30^o$

Angle of elevation of top of tower from from foot of building = 60°

Hence, $\angle$ DBC = 60°

$\angle$ ABC = 90° & $\angle$ DCB = 90°

In a right angle triangle DBC,

tan B = $\frac{side\ opposite\ to\ angle\ to\ D}{side\ opposite\ to\ angle\ H}$

tan B = $\frac{DC}{BC}$

$tan60^0$ = $\frac{50}{BC}$

$BC = \frac{50}{\sqrt{3}}$

Similarly,

In a right angle triangle ABC,

tan C = $\frac{side\ opposite\ to\ angle\ to\ C}{side\ adjacent\ to\ angle\ C}$

$tan30^0$ = $\frac{AB}{BC}$

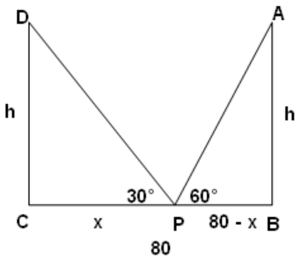
$\frac{1}{\sqrt{3}} = \frac{AB}{\frac{50}{\sqrt{3}}}$

$\frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}} = \mathrm{AB}$

$AB = \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}}$

$AB = \frac{50}{3}m$

10.



Suppose AB and CD are the two poles of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 – x) . Also, $\angle$APB = 60° and $\angle$DPC =30°

In right angled triangle DCP,

tan 30° = $\frac{CD}{CP}$

⇒ $\frac{h}{x} = \frac{1}{\sqrt{3}}$

⇒ $h = \frac{x}{\sqrt{3}}$ ....(1)

In right angled triangle ABP,

Tan 60° = AB/AP$\frac{AB}{AP}$

⇒ $\frac{h}{80-x} = \sqrt{3}$

⇒ $h = \sqrt{3}(80-x)$

⇒ $\frac{x}{\sqrt{3}} = \sqrt{3}(80-x)$

⇒ x = 3(80 – x)

⇒ x = 240 – 3x

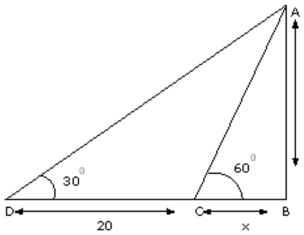
⇒ x + 3x = 240

⇒ 4x = 240

⇒ x = 60

Height of the pole, h = x/√3 = 60/√3 = 20√3.

Thus, the position of the point P is 60 m from C and the height of each pole is 20√3 m.

11. 

Let 'h' (AB) be the height of tower and x be the width of the river

In $\triangle ABC, \frac{h}{x} = \tan 60^\circ$

$\Rightarrow h = \sqrt{3}x$ ....(i)

In $\Delta ABD, \frac{h}{x+20} = \tan 30^\circ$

$\Rightarrow h = \frac{x+20}{\sqrt{3}}$ ....(ii)

Equating (i) and (ii),

$\sqrt{3}x = \frac{x+20}{\sqrt{3}}$

⇒ 3x = x + 20

⇒ 2x = 20

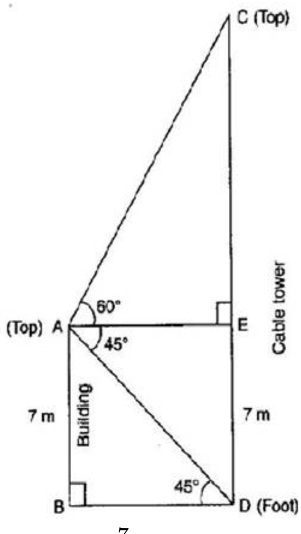
⇒ x = 10 m

Put x = 10 in (i), $h = \sqrt{3}x$

$\Rightarrow h = 10\sqrt{3}\text{m}$

12. In right triangle ABD,

$\tan 45^0 = \frac{AB}{BD}$



$\Rightarrow 1 = \frac{7}{BD}$

⇒ BD = 7 m

⇒ AE = 7 m

In right triangle AEC,

$\tan 60^\circ = \frac{CE}{AE}$

$\Rightarrow \sqrt{3} = \frac{\text{CE}}{7} \Rightarrow \text{CE} = 7\sqrt{3}\text{m}$

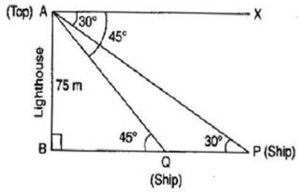
∴ CD = CE + ED

= CE + AB

$= 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)\text{m}$

Hence height of the tower is $7(\sqrt{3} + 1)$ m.

13. In right triangle ABQ,



$\tan 45^{\circ} = \frac{AB}{BQ}$

$\Rightarrow 1 = \frac{75}{\text{BQ}}$

$\Rightarrow$BQ = 75 m .... (i)

In right triangle ABP,

$\tan 30^{\circ} = \frac{AB}{BP}$

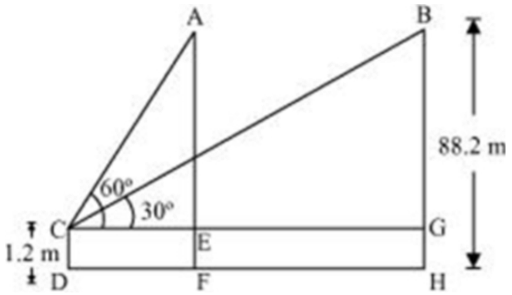
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BQ+QP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{75+QP}$[From eq. (i)]

$\Rightarrow$ 75 + QP = $75\sqrt{3}$

QP = $75(\sqrt{3}-1)$ m

Hence the distance between the two ships is $75(\sqrt{3}-1)$ m.

14. 

Let CD be the girl

Let initial position of balloon A shift to another position B after sometime

In $\triangle$ACE, Tan 60^{O} = $\frac{AC}{CE}$

$\text{Tan}\, 60^{\circ} = \frac{(AF-EF)}{CE}$

$\sqrt{3} = \frac{(88.2-1.2)}{CE}$

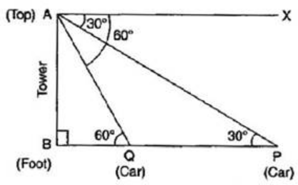
CE = $29\sqrt{3}m$

In $\triangle$BCG, Tan 30^{O} = $\frac{BG}{CG}$

Tan 30^{O} = $\frac{(BH-GH)}{CG}$

CG = $87\sqrt{3}$ m

Distance travelled by the balloon, EG = CG - CE = $(87\sqrt{3}-29\sqrt{3})$ = $58\sqrt{3}\text{m}$

15. 

In right triangle ABP,

$\tan 30^{\circ} = \frac{\text{AB}}{\text{BP}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\text{AB}}{\text{BP}}$

BP = AB$\sqrt{3}$ .... (i)

In right triangle ABQ,

$tan\ 60^0\ = \frac{AB}{BQ}$

$\Rightarrow \sqrt{3} = \frac{AB}{BQ}$

$\Rightarrow_{BQ} = \frac{AB}{\sqrt{3}}$....... (ii)

$\because$ PQ = BP - BQ

$\therefore$ PQ = AB$\sqrt{3} - \frac{AB}{\sqrt{3}} = \frac{3AB - AB}{\sqrt{3}} = \frac{2AB}{\sqrt{3}}$ = 2BQ [From eq. (ii)]

$\Rightarrow$ BQ = $\frac{1}{2}$PQ

$\because$ Time taken by the car to travel a distance PQ = 6 seconds.

$\therefore$ Time taken by the car to travel a distance BQ, i.e. $\frac{1}{2}$PQ = $\frac{1}{2}$ $\times$ 6 = 3 seconds.

Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.