Solution

REAL NUMBER

Class 10 - Mathematics

Section A

1.

(b) π

Explanation:

π

2. **(a)** both rational and irrational number

Explanation:

The product of a rational number and an irrational number can be either a rational number or an irrational number.

e.g $\sqrt{5} \times \sqrt{2} = \sqrt{10}$ which is irrational

but $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$ which is a rational number

Thus, the product of two irrational numbers can be either rational or irrational

similarly, the product of rational and irrational numbers can be either rational or irrational

$5 \times \sqrt{2} = 5\sqrt{2}$ which is irrational.

but $0 \times \sqrt{3} = 0$ which is rational.

3.

(b) Irrational

Explanation:

Let rational number + irrational number = rational number

And we know " rational number can be expressed in the form of PQ, where p, q are any integers,

So, we can express our assumption As :

PQ + x = ab (Here x is a irrational number)

x = ab - PQ

So,

x is a rational number, but that contradicts our starting assumption.

Hence rational number + irrational number = irrational number

4.

(c) 2

Explanation:

Since 5 + 3 = 8, the least prime factor of a + b has to be 2, unless a + b is a prime number greater than 2.

If a + b is a prime number greater than 2, then a + b must be an odd number. So, either a or b must be an even number. If a is even, then the least prime factor of a is 2, which is not 3 or 5. So, neither a nor b can be an even number. Hence, a + b cannot be a prime number greater than 2 if the least prime factor of a is 3 or 5.

5. **(a)** an irrational number

Explanation:

Let 2 - $\sqrt{3}$ be rational number

2 - $\sqrt{3} = \frac{p}{q}$ where p and q are composite numbers

$\sqrt{3} = \frac{p}{q} + 2$

$\sqrt{3} = \frac{(p+2q)}{q}$

since p, q are integers, so $\frac{(p+2q)}{q}$ is rational

$\therefore \sqrt{3}$ is an irrational number

it shows our supposition was wrong

hence 2-$\sqrt{3}$ is an irrational number.

6. **(a)** 0.5

Explanation:

$9^{x+2} = 240 + 9^x$

$\Rightarrow 9^x \times 9^2 = 240 + 9^x$

$\Rightarrow 9^x (81 - 1) = 240$

$\Rightarrow 9^x = 3$

$\Rightarrow 9^x = 9^{1/2}$

$\Rightarrow x = \frac{1}{2} = 0.5$

7. **(c)** irrational number

Explanation:

Here, 3 is rational and $2\sqrt{5}$ is irrational.

We know that the sum of a rational and an irrational is an irrational number, therefore, $3 + 2\sqrt{5}$ is irrational.

8. **(b)** a = 2, b = 1

Explanation:

$a - b\sqrt{3} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$

$= \frac{(\sqrt{3})^2+(1)^2-2(1))(\sqrt{3})}{3-1}$

$= \frac{3+1-2\sqrt{3}}{2}$

$= \frac{4-2\sqrt{3}}{2}$

$= \frac{2(2-\sqrt{3})}{2}$

$= 2 - \sqrt{3}$

$\Rightarrow a - b\sqrt{3} = 2 - \sqrt{3}$

$\Rightarrow$ a = 2, b = 1

9. **(b)** 1

Explanation:

1 is neither prime nor composite.

A prime is a natural number greater than 1 that has no positive divisors other than 1 and itself

e.g. 5 is prime because 1 and 5 are its only positive integers factors but 6 is composite because it has divisors 2 and 3 in addition to 1 and 6.

10. **(d)** 17×500

Explanation:

$850 = 2 \times 5 \times 5 \times 17$

$500 = 2 \times 2 \times 5 \times 5 \times 5$

LCM (850, 500) $= 2 \times 2 \times 5 \times 5 \times 5 \times 17 = 17 \times 500$

11. **(b)** m = 5 and n = 3

Explanation:

$\frac{241}{4000} = \frac{241}{2^m \times 5^n}$

$\Rightarrow \frac{241}{2^5 \times 5^3} = \frac{241}{2^m \times 5^n}$

Comparing the denominators of both fractions, we have m = 5 and n = 3

12.

(c) 1679

Explanation:

The dividend is equal to Divisor $\times$ Quotient + Remainder

Number(dividend) = D $\times$ Q + R

$\therefore$ the number (Dividend) = 61 $\times$ 27 + 32

= 1647 + 32

= 1679

13.

(d) 25

Explanation:

H.C.F. of (192, 240, 168) = 2 $\times$ 2$\times$2 $\times$ 3 = 24

Number of rooms for participants in Mathematics, Physics and Biology respectively is $\frac{192}{24}$, $\frac{240}{24}$ and $\frac{168}{24}$

i.e., 8, 10 and 7

$\therefore$Total minimum number of required rooms = 8 + 10 + 7 = 25

14.

(b) p^3q^2

Explanation:

We know that LCM = product of the highest powers of all the prime factors of the numbers pq^2, p^3q^2

LCM = p^3q^2

15. **(a)** 3

Explanation:

$a = 2^2 \times 3^x$

$b = 2^2 \times 3 \times 5$

$c = 2^2 \times 3 \times 7$

LCM (a, b, c) = 3780

3780 = $2^2 \times 3^3 \times 5^1 \times 7^1$

In LCM, we consider highest power

So, x = 3

16.

(c) 2100

Explanation:

As we know HCF $\times$ LCM = Product of the Numbers

Hence HCF $\times$ LCM (30,70) = 30 $\times$ 70 = 2100

17. **(a)** 1200

Explanation:

$HCF \times LCM = N_1 \times N_2$

$HCF \times LCM = 30 \times 40$

= 1200

18.

(c) $2^3 \times 53$

Explanation:

2	424
2	212
2	106
53	53
	1

$424 = 2^3 \times 53$

19.

(b) coprime

Explanation:

We know that the co-prime numbers have no factor in common, or, their HCF is 1.

Thus, p^2 and q^2 have the same factor with exponent 2 each. which again will not have any common factor.

Thus we can conclude that p^2 and q^2 are co-prime numbers.

20. **(a)** $2^6 \times 3^3$

Explanation:

$2^6 \times 3^3$

21.

(c) A is true but R is false.

Explanation:

HCF (a, b) $\times$ LCM (a, b) = a $\times$ b

$\Rightarrow 8 \times$ LCM = 280

$\Rightarrow$ LCM = $\frac{280}{8}$ = 35

A is true but R is false.

22.

(c) A is true but R is false.

Explanation:

A is true but R is false.

23.

(c) A is true but R is false.

Explanation:

Here, reason is not true.

$\sqrt{9} = \pm 3$, which is not an irrational number.

A is true but R is false.

24.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Yes 12 and 17 are coprime numbers and H.C.F. of coprimes is always 1.

25.

(d) A is false but R is true.

Explanation:

A is false but R is true.

26.

(c) A is true but R is false.

Explanation:

We have,

LCM(a, b) × HCF(a, b) = a × b

LCM × 5 = 150

LCM = $\frac{150}{5}$ = 30

LCM = 30

27.

(d) A is false but R is true.

Explanation:

Remainder is less than by divisor not by divident.

28. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

It is a result.

29.

(c) A is true but R is false.

Explanation:

A is true but R is false.

30.

(c) A is true but R is false.

Explanation:

Here reason is not true. $\sqrt{4} = \pm 2$, which is not an irrational number.

Section B

31. The capacity of a container will be equal to HCF of 504 and 735

Let's find out the prime factorization of 504 and 735 by division

2	504
2	252
2	126
3	63
3	21
	7

5	735
3	147
7	49
	7

We get

$504 = \left(2^3 \times 3^2 \times 7\right)$

and $735 = \left(5 \times 3 \times 7^2\right)$.

Therefore; HCF (504, 735) $= (3 \times 7) = 21$.

Therefore, the capacity of a container which can measure the milk of either tank in exact number of times

= 21 liter

32. $(\sqrt{2} + \sqrt{3})^2 = 2 + 3 + 2\sqrt{6} = 5 + 2\sqrt{6}$

Let us assume, to the contrary, that $5 + 2\sqrt{6}$ is rational

$\therefore 5 + 2\sqrt{6} = \frac{a}{b}$; a, b are integers, b $\neq$ 0

$\therefore \sqrt{6} = \frac{a-5b}{2b}$

RHS is a rational number, whereas LHS is an irrational number.

$\therefore$ Our assumption is wrong.

$\Rightarrow 5 + 2\sqrt{6} = (\sqrt{2} + \sqrt{3})^2$ is an irrational number

33. 26 and 91

26= 2 × 13

91 = 7 × 13

HCF = 13

LCM = 2 × 7 × 13 = 182

Product of two numbers 26 and 91 = 26 × 91 = 2366

HCF × LCM = 13 × 182 = 2366

Hence, product of two numbers = HCF × LCM

34. Let us assume that $4+\sqrt{2}$ is rational. Then, there exist positive co-primes a and b such that

$4+\sqrt{2}=\frac{a}{b}$

$\sqrt{2}=\frac{a}{b}-4$

$\sqrt{2}=\frac{a-4b}{b}$

As a - 4b and b are integers.

So, $\frac{a-4b}{b}$ is a rational number .

But $\sqrt{2}$ is not rational number .

Since a rational number cannot be equal to an irrational number . Our assumption that $4+\sqrt{2}$ is a rational number is wrong .

Hence, $4+\sqrt{2}$ is irrational.

35. $4^n = (2^2)^n = 2^{2n}$, the only prime factor of 4^n is 2.

$\therefore$ By fundamental theorem of Arithmetic, there is no other prime factorisation of 4^n.

It does not have 5 as its prime factor, hence 4^n cannot end with digit 0 for any natural number n.

36. Let $\sqrt{3}$ be a rational number.

$\sqrt{3} = \frac{p}{q}$ p, q are coprime, $q \neq 0$

$3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p$ Let $p = 3m$

$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q$

$\therefore$ 3 is common factor of p and q

Contraction to our assumption

Hence $\sqrt{3}$ is irrational No.

37. On applying division lemma on 1288 and 575 we get

$1288 = 575 \times 2 + 138$.....(1)

Since remainder ≠ 0, apply division lemma on divisor 575 and remainder 138

$575 = 138 \times 4 + 23$.....(2)

Since remainder ≠ 0, apply division lemma on divisor 138 and remainder 23

$138 = 23 \times 6 + 0$.

Hence HCF(1288,575) = 23

Now let 1288 = H and 575 = K

Then equation (1) and (2) can be written as:

H = 2k+138

or 138 = H-2K .....(3)

and $K = 138 \times 4 + 23$

or $23 = K - 138 \times 4$

Substituting the value of 138 from (3) we get

$23 = K - 4 \times (H - 2K)$

$= K - 4H + 8K$

$= 9K - 4H$

Now substituting the values of H and K we get

$23 = 9 \times 575 - 4 \times 1288$

38. The prime factorization of 2520 are

$2520 = 2^3 \times 3^2 \times 5 \times 7$----------(1)

It is given that

$2520 = 2^3 \times 3^p \times q \times 7$----------(2)

On comparing equation (1) and (2) we get

p = 2 and q = 5

39. Let us assume that $\sqrt{2}+\sqrt{3}$ is a rational number

Let $\sqrt{2}+\sqrt{3}=\frac{a}{b}$ Where a and b are co-prime positive integers

On squaring both sides, we get

$(\sqrt{2}+\sqrt{3})^2=\frac{a^2}{b^2}$

$2+3+2\sqrt{6}=\frac{a^2}{b^2}$

$5 + 2\sqrt{6} = \frac{a^2}{b^2}$

$2\sqrt{6} = \frac{a^2}{b^2} - 5$

$2\sqrt{6} = \frac{a^2 - 5b^2}{b^2}$

$\sqrt{6} = \frac{a^2 - 5b^2}{2b^2}$

Now $\frac{a^2 - 5b^2}{2b^2}$ is a rational number.

This shows that $\sqrt{6}$ is a rational number.

But this contradicts the fact that $\sqrt{6}$ is an irrational number.

This contradiction has raised because we assume that $\left(\sqrt{2} + \sqrt{3}\right)$ is a rational number.

Hence, our assumption is wrong and $\left(\sqrt{2} + \sqrt{3}\right)$ is an irrational number.

40. Let a and b are numbers and HCF = x

Then LCM = 14x

Now sum of HCF and LCM

x+14x =600

15x = 600

x = 40

Hence HCF=40 and LCM=14×40

Given a=280 and b=?

We know that

a × b = HCF × LCM

So b = $\frac{40 \times 14 \times 40}{280} = 2 \times 40 = 80$

Hence the other number = 80

41. Two positive integers are 687897 and 81445.

By applying Euclid's division lemma

687897 = 81445×8 + 36337

81445 = 36337×2 + 8771

36337 = 8771×4 + 1253

8771 = 1253×7 + 0

∴ HCF = 1253

42. We have: $6 = 2^1 \times 3^1$ and $20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

Now HCF(6, 20) = 2^1 = 2 = Product of the smallest power of each common prime factor

and LCM (6, 20) = $2^2 \times 3^1 \times 5^1$ = 60 = Product of the greatest power of each prime factor

43. $18 = 2 \times 3 \times 3 = 2 \times 3^2$

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$\therefore \quad HCF = 2 \times 3 = 6$

LCM =2×2×2×3×3

= 72

44. Let $3\sqrt{3}$ - 7 be a rational number.

$3\sqrt{3}$ - 7 = x, where x is rational

$\Rightarrow \sqrt{3} = \frac{x+7}{3}$

irrational = rational

This is a contradiction. This contradiction has arisen because of our wrong assumption. Hence $3\sqrt{3}$ - 7 is irrational.

45. Assuming $\frac{2-\sqrt{3}}{5}$ to be a rational number.

$\Rightarrow \frac{2-\sqrt{3}}{5} = \frac{p}{q}$, where p and q are integers $q \neq 0$

$\Rightarrow \sqrt{3} = \frac{2q-5p}{q}$

Here RHS is rational but LHS is irrational.

Therefore our assumption is wrong.

Hence $\frac{2-\sqrt{3}}{5}$ is an irrational number.

Section C

46. The number of participants in each room must be the HCF of 60, 84 and 108.

In order to find the HCF of 60, 84 and 108, we first find the HCF of 60 and 84 by Euclid's division algorithm:

2	60	84	1
	48	60	
	12	24	2
	(HCF)	24	
		0	
		(Remainder)	

Clearly, HCF of 60 and 84 is 12

Now, we find the HCF of 12 and 108

12	108	9
(HCF)	108	
	0	
	(Remainder)	

Clearly, HCF of 12 and 108 is 12. Hence, the HCF of 60,84 and 108 is 12.

Therefore, in each room maximum 12 participants can be seated.

We have,

Total number of participants = 60 + 84 + 108 = 252

$\therefore$ Number of rooms required $= \frac{252}{12} = 21$.

47. We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$is rational.

It means we have some co-prime integers *a* and *b* (*b* ≠ 0)

Such that

$\frac{1}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{b}{a}$ .....(1)

R.H.S of (1) is rational but we know that is$\sqrt{2}$ irrational.

It is not possible which means our supposition is wrong.

Therefore,$\frac{1}{\sqrt{2}}$can not be rational.

Hence, it is irrational.

48. **HCF (highest common factor) :** The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.

We need to find H.C.F of 475 and 495.

By applying Euclid's Division lemma, we have

495 = 475 × 1 + 20.

Since remainder ≠ 0, apply division lemma on 475 and remainder 20

475 = 20 × 23 + 15.

Since remainder ≠ 0, apply division lemma on 20 and remainder 15

20 = 15 × 1 + 5.

Since remainder ≠ 0, apply division lemma on 15 and remainder 5

15 = 5 × 3+ 0.

Therefore, H.C.F. of 475 and 495 = 5

49. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.

That is, we can find coprime numbers a and b (b ≠ 0) such that $5 - \sqrt{3} = \frac{a}{b}$

Therefore, $5 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$

Since a and b are integers, we get $5 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

50. We have to take LCM of 2, 4,3

2 = 2×1

4 = 2×2

3 = 3×1

LCM = 12

Thus they will meet at a gap of 12 days.

7 May - 19 May - 31 May - 12 June - 24 June - 6 July - 18 July - 30 July- 11 August - 23 August - 4 Sept.

Therefore the last day before 4th Sept. will be 23rd of August.

51. Let us find HCF of 48 and 18

$48 = 18 \times 2 + 12$

$18 = 12 \times 1 + 6$

$12 = 6 \times 2 + 0$

Hence HCF (48, 18) = 6

Now, 6 = 18 - 12 $\times$ 1

6 = 18 - (48 - 18 $\times$ 2)

6 = 18 - 48 $\times$ 1 + 18$\times$2

6 = 18 $\times$ 3 - 48 $\times$ 1

6 = 18 $\times$ 3 + 48 $\times$(-1)

i.e., 6 = 18x + 48y ..... (1)

where x= 3, y = -1

$\therefore$ 6 = 18 $\times$ 3 + 48 $\times$ (-1)

= 18 $\times$ 3 + 48 $\times$ (-1) + 18 $\times$ 48 - 18 $\times$ 48

= 18(3 + 48) + 48(-1 - 18)

= 18 $\times$ 51 + 48 $\times$ (-19)

6 = 18x + 48y ..... (2)

where x = 51, y = -19

Hence, x and y are not unique.

52. Using Euclid's algorithm, the HCF(30, 72)

72 = 30 $\times$ 2 + 12

30 = 12 $\times$ 2 + 6.

12 = 6 $\times$ 2 + 0.

HCF(30, 72) = 6

Now it given that

HCF=30x+72y

So 6 = 30x + 72y

or 1 = 5x + 12y

$x = \frac{1-12y}{5}$ ..... (1)

if y = -2 then $x = \frac{1+24}{5} = 5$

if y = -12 then $x = \frac{1+144}{5} = 29$

if y = -22 then $x = \frac{1+264}{5} = 53$

So the value of (x, y) possible are (5, -2), (29, -12) (53, -22)

Hence infinite no. of solutions are possible.

53. Let d be the HCF of 24 and 36.

24 = $2^3 \times 3$

36 = $2^2 \times 3^2$

HCF = $2^2 \times 3 = 12$

$\Rightarrow d = 12$

Now $d = 24a + 36b$

$12 = 24a + 36b$

When $a = -1$ and $b = 1$, we get

12 = 24$\times$(-1) + 36$\times$(1)

= -24 + 36

12 = 12

So, a = -1 , b = 1 satisfy the equation d = 24a + 36b

∴ One possible value of a and b is -1 and 1.

54. Let first we consider, $\sqrt{2}\frac{1}{3}$ be rational. We can write $\sqrt{2}/3$ as

$\frac{1}{3} \times \sqrt{2}$

We know that product of two rational number is always a rational number.

$\frac{\sqrt{2}}{3} \times 3 = \sqrt{2}$

But

$\sqrt{2}$ is irrational.

∴ Here the Contradiction arises by assuming that $\frac{\sqrt{2}}{3}$ is rational. Actually it is irrational.

Hence, $\frac{\sqrt{2}}{3}$ is irrational.

55. Suppose that $(2 + \sqrt{3})$ is rational number.

$\Rightarrow$ there must exist co-prime numbers between a and b $(b \neq 0)$ such that

$(2 + \sqrt{3}) = \frac{a}{b}$

$\Rightarrow \sqrt{3} = \frac{a}{b} - 2 \Rightarrow \sqrt{3} = \frac{a-2b}{b}$

Since a and b are integers, so $\frac{a-2b}{b}$ is rational number

$\Rightarrow \sqrt{3}$ is also rational number .

But, this contradicts the fact that $\sqrt{3}$ is irrational.

∴our assumption is incorrect.

∴ $(2 + \sqrt{3})$ is irrational.

56. For maximum number of students to put into each group

Mr patil sir should have to take H.C.F of 28, 42 and 56

so

maximum number of students Mr. Patil can put into each group is 14.

57. This problem can be solved using H.C.F. because we are cutting or “dividing” the strips of cloth into smaller pieces of 36 and 24 and we are looking for the widest possible strips .

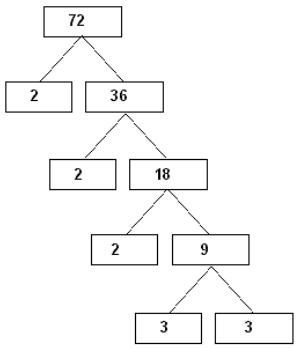
So,

H.C.F. of 36 and 24 is 12

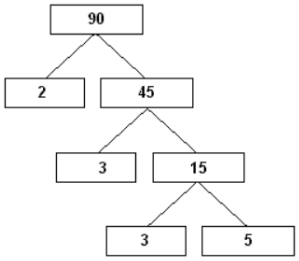
So we can say that

Maya should cut each piece to be 12 inches wide.

58.



So, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$



So, $90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

Therefore,

HCF (72, 90) = $2 \times 3^2 = 18$

LCM (72, 90) = $2^3 \times 3^2 \times 5 = 360$

59. To distribute the fruits equally Renu has to take the H.C.F. of 45 and 20.

H.C.F. of 20 and 45 = 5

i.e. 5 fruits can be placed in 1 pack

$\therefore$ Total no. of packs = $\frac{Total\ available\ fruits}{no.\ of\ fruits\ in\ 1\ packs}$

$= \frac{45+20}{5}$

$= \frac{65}{5}$

= 13

Hence, maximum no. of packets required = 13

60. Let us assume that $2 - \sqrt{3}$ is rational.

Then, there exist positive co-primes a and b such that

$2 - \sqrt{3} = \frac{a}{b}$

$\sqrt{3} = 2 - \frac{a}{b}$

As 2 and $\frac{a}{b}$ are rational number .

So, $\sqrt{3}$ is also rational number .

But $\sqrt{3}$ is not rational number .

Since a rational number cannot be equal to an irrational number. Our assumption that $2 - \sqrt{3}$ is rational wrong .

Hence $2 - \sqrt{3}$ is irrational

Section D

61. i. The Number of room will be minimum if each room accomodates maximum number of participants. Therefore, the number of participants in each room must be the HCF of 60, 84 and 108. The prime factorisations of 60, 84 and 108 are as under.

$60 = 2^2 \times 3 \times 5$, $84 = 2^2 \times 3 \times 7$ and $108 = 2^2 \times 3^3$

$\therefore$ HCF of 60, 84 and 108 is $2^2 \times 3 = 12$

Therefore, in each room 12 participants can be seated.

ii. The Number of room will be minimum if each room accomodates maximum number of participants. Therefore, the number of participants in each room must be the HCF of 60, 84 and 108. The prime factorisations of 60, 84 and 108 are as under.

$60 = 2^2 \times 3 \times 5$, $84 = 2^2 \times 3 \times 7$ and $108 = 2^2 \times 3^3$

$\therefore$ HCF of 60, 84 and 108 is $2^2 \times 3 = 12$

Therefore, in each room 12 participants can be seated.

$\therefore$ Number of rooms required = $\frac{\text{Total number of participants}}{12}$

$= \frac{60+84+108}{12}$

$= \frac{252}{12}$

= 21

iii. Prime factorisation of $60 = 2 \times 2 \times 3 \times 5$

Prime factorisation of $84 = 2 \times 2 \times 3 \times 7$

Hence, LCM of 60, $84 = 2 \times 2 \times 3 \times 5 \times 7 = 420$

And HCF of 60, $84 = 2 \times 2 \times 3 = 12$

Now, LCM $\times$ HCF $= 420 \times 12 = 5040$

Also, $60 \times 84 = 5040$

i.e., HCF $\times$ LCM = Product of the two numbers

OR

Product of numbers = HCF $\times$ LCM

$\Rightarrow 1080 = 30 \times$ LCM

$\therefore$ LCM = $\frac{1080}{30} = 36$

62. i. HCF (96, 240, 336) = 48

ii. Number of stacks = $\frac{336}{48} = 7$

iii. Total number of stacks = $\frac{96}{48} + \frac{240}{48} + \frac{336}{48}$

= 14

OR

Height of each stack of History = $48 \times 1.8 = 86.4$ cm

Height of each stack of Science = 48 × 2.2 = 105.6 cm

Height of each stack of Mathematics = 48 × 2.5 = 120 cm

63. i. The number of students in Section A is 32, and the number of students in Section B is 36.

Step 1: Find the prime factors of each number:

32 = 2 × 2 × 2 × 2 × 2

36 = 2 × 2 × 3 × 3

Step 2: Identify the common and uncommon prime factors. The common ones are 2 × 2.

Step 3: Multiply the common and uncommon prime factors together to get the LCM:

LCM = 2 × 2 × 2 × 2 × 2 × 3 × 3 = 288

So, the minimum number of books needed to be acquired for the class library is 288, so they can be distributed equally among students of Section A or Section B.

ii. Step 1: Find the prime factors of each number:

32 = 2 × 2 × 2 × 2 × 2

36 = 2 × 2 × 3 × 3

Step 2: Identify the common prime factors and their minimum exponent:

The common prime factors are 2 × 2.

Step 3: Calculate the HCF by multiplying the common prime factors:

HCF = 2 × 2 = 4

So, the HCF of 32 and 36 is 4.

iii. Given number (7 × 11 × 13 × 15 + 15)

It can also be written as 15 (7 × 11 × 13 +1).

As it is a product of two composite numbers

hence it is a composite number.

OR

Given:

$p = ab^2$

$q = a^2b$

Take the highest power of each prime factor:

$LCM = a^2 \times b^2$

So, the LCM of p and q is a^2b^2.

64. i. Khushi can invite guests = HCF(36, 60) = 12

So, she can invite at most 12 guests.

ii. Each guest gets bananas = $\frac{60}{12}$ = 5 bananas

Each guest get apples = $\frac{36}{12}$ = 3 apples

iii. If Khushi adds 42 mangoes

She can invite guests = HCF (36, 60, 42)

= 6

OR

Total amount spent = 5 × (60) + 15 × (36) + (42) × (20)

= 300 + 540 + 840

= ₹ 1680

65. $173250 = 2 \times 5^3 \times 3^2 \times 7 \times 11$

i. 3

ii. a. $173250 = 2 \times 5^3 \times 3^2 \times 7 \times 11$

Number of students in the class = 3 + 2 + 1 + 1 = 7

OR

b. $173250 = 2 \times 5^3 \times 3^2 \times 7 \times 11$

Highest prime number used by students = 11

iii. 5

Section E

66. Suppose $\frac{2\sqrt{3}}{5}$ be a rational number

$\therefore \frac{2\sqrt{3}}{5} = \frac{a}{b}$, a and b are co-prime, b$\neq$ 0

$\Rightarrow \sqrt{3} = \frac{5a}{2b}$

$5a \; and \; 2b$ are integers and $\sqrt{3}$ is irrational.

$\frac{5a}{2b}$ is rational.

$\therefore \sqrt{3} \neq \frac{5a}{2b}$

$\therefore$ Our supposition is wrong

$\Rightarrow \frac{2\sqrt{3}}{5}$ is an irrational number.

67. By applying Euclid's division lemma

$592 = 252 \times 2 + 88$

Since remainder ≠ 0, apply division lemma on divisor 252 and remainder 88

$252 = 88 \times 2 + 76$

Since remainder ≠ 0, apply division lemma on divisor 88 and remainder 76

$88 = 76 \times 1 + 12$

Since remainder ≠ 0, apply division lemma on divisor 76 and remainder 12

$76 = 12 \times 6 + 4$

Since remainder ≠ 0, apply division lemma on divisor 12 and remainder 4

$12 = 4 \times 3 + 0.$

Therefore, $H.C.F. = 4.$

HCF of 2 numbers can be expressed as the liner combination the two numbers

$\Rightarrow 4 = 76 - 12 \times 6$

= $76 - 88 - 76 \times 1 \times 6$

= $76 - 88 \times 6 + 76 \times 6$

= $76 \times 7 - 88 \times 6$

= $252 - 88 \times 2 \times 7 - 88 \times 6$

=$252 \times 7 - 88 \times 14 - 88 \times 6$

= $252 \times 7 - 88 \times 20$

= 252 × 7 - 592 - 252 × 2 × 20

= 252 × 7 - 592 × 20 + 252 × 40

= 252 × 47 - 592 × 20

= 252 × 47 + 592 × (-20)

Hence obtained.

68. We have to explain why the numbers 8 × 7× 6 × 5 × 4 + 5 × 4 and 11 × 13 × 15 + 11 are composite numbers.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

8 × 7× 6 × 5 × 4 + 5 × 4 = 6720 + 20

= 20 × (336 + 1)

= 20 × 337

The given expression has 20 and 337 as its factors other than 1 and number itself.

Therefore, it is a composite number.

Now

11 × 13 × 15 + 11 = 11 × (13 × 15 + 1)

= 11 × 196

= 11 ×2 × 2 × 7 × 7

= 2 × 2 × 7 × 7 × 11

The given expression has 2, 7 and 11 as its factors other than 1 and number itself.

Therefore, it is a composite number.

69. Let us first find the HCF of 210 and 55. Applying Euclid's division lemma on 210 and 55, we get

210 = 55 × 3 + 45 ....(i)

$\because \; 55\overline{)210(3}$

$\quad\quad\underline{165}$

$\quad\quad\;\; 45$

Since, the remainder 45 $\neq$ 0. So, we now apply division lemma on the divisor 55 and the remainder 45 to get

55 = 45 × 1 + 10 ....(i)

$\because$ $45\overline{)55}(1$
$\underline{45}$
10

We consider the divisor 45 and the remainder 10 and apply division lemma to get

$45 = 4 \times 10 + 5$ ...(iii)

$\because$ $10\overline{)45}(4$
$\underline{40}$
5

We consider the divisor 10 and the remainder 5 and apply division lemma to get

$10 = 5 \times 2 + 0$ ...(iv)

We observe that the remainder at this stage is zero. So, the last divisor i.e. 5 is the HCF of 210 and 55.

$\therefore$ $5 = 210 \times 5 + 55y$

$\Rightarrow 55y = 5 - 210 \times 5 = 5 - 1050$

$\Rightarrow 55y = -1045$

$\Rightarrow y = \frac{-1045}{55} = -19$.

70. On applying the Euclid's division lemma to find HCF of 152, 272, we get

$152\overline{)272}(1$
$\underline{152}$
120

$120\overline{)152}(1$
$\underline{120}$
32

$272 = 152 \times 1 + 120$

Here the remainder = 0.

Using Euclid's division lemma to find the HCF of 152 and 120, we get

$152 = 120 \times 1 + 32$

Again the remainder = 0.

Using division lemma to find the HCF of 120 and 32, we get

$32\overline{)120}(3$
$\underline{96}$
24

$120 = 32 \times 3 + 24$

Similarly,

$8\overline{)24}(3$
$\underline{24}$
0

$32 = 24 \times 1 + 8$

$24 = 8 \times 3 + 0$

HCF of 272 and 152 is 8.

$272 \times 8 + 152x$ = H.C.F. of the numbers

$\Rightarrow 8 = 272 \times 8 + 152x$

$\Rightarrow 8 - 272 \times 8 = 152x$

$\Rightarrow 8(1 - 272) = 152x$

$\Rightarrow x = \frac{-2168}{152} = \frac{-271}{19}$

71. If possible, let $\sqrt{11}$ be rational and let its simplest form be $\frac{a}{b}$.

Then, a and b are integers having no common factor other than 1, and $b \neq 0$.

Now, $\sqrt{11} = \frac{a}{b}$

$\Rightarrow 11 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow 11b^2 = a^2$ ..... (i0

$\Rightarrow$ 11 divides a^2 So,11 divides by a

Let a = 11c for some positive integer c.

Putting a = 11c in (i), we get

$11b^2 = (11c)^2$

$11b^2 = 121c^2$

$\Rightarrow b^2 = 11c^2$

$\Rightarrow$ 11 divides b^2 [$\because$ 11 divides $11c^2$]

$\Rightarrow$ 11 divides b [$\because$ 11 is prime and 11 divides $b^2 \Rightarrow$ 11 divides b].

Thus, 11 is a common factor of a and b.

But, this contradicts the fact that a and b have no common factor other than 1.

The contradiction arises by assuming that $\sqrt{11}$ is rational.

Hence, $\sqrt{11}$ is irrational.

72. Fundamental theorem of arithmetic:

"Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur."

$2520 = 8 \times 9 \times 5 \times 7 = 2^3 \times 3^2 \times 5 \times 7$

$10530 = 2 \times 81 \times 5 \times 13 = 2 \times 3^4 \times 5 \times 13$

$\therefore$ LCM = $2^3 \times 3^4 \times 5 \times 7 \times 13 = 294840$

73. Maximum number of students that can be seated in one room = HCF of 336, 240, 96

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$\therefore HCF\ of\ 336, 240, 96 = 2 \times 2 \times 2 \times 2 \times 3 = 48$

Now, number of rooms for participants in Mathematics = $\frac{336}{48} = 7$

Number of rooms for participants in Physics = $\frac{240}{48} = 5$

Number of rooms for participants in Biology = $\frac{96}{48} = 2$

$\therefore$ Total no. of rooms = $7 + 5 + 2 = 14$

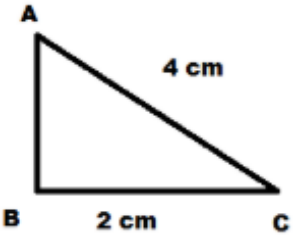
74. Let us assume that $4 + \sqrt{3}$ is a rational number equal to a/b where a and b are two integers

$\Rightarrow 4 + \sqrt{3} = \frac{a}{b}$.

$\Rightarrow \sqrt{3} = \frac{a}{b} - 4$

we know that subtraction of two rational number is always a rational number but we get $\sqrt{3}$ as rational number which contradict the fact as $\sqrt{3}$ is an irrational number.

Hence, $4 + \sqrt{3}$ is a irrational number.

75. 

It is given that BC = 2cm, AC = 4 cm.

We have to prove that the area of $\triangle$ABC is irrational.

In $\triangle$ABC we know that

$AC^2 = AB^2 + BC^2$

$(4)^2 = AB^2 + (2)^2$

$AB^2 = 16 - 4$

$= 12$

$AB = \sqrt{12} = 2\sqrt{3}\ cm$

So area of $\triangle ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}$ cm^2 = irrational (As $\sqrt{3}$ is irrational)

So area of $\triangle$ABC is irrational.