SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

REAL NUMBERS

Class 09 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 183**

Section A

1. The value of $\frac{9^{\frac{1}{3}} \times 27^{\frac{1}{2}}}{3^{\frac{-1}{6}} \times 3^{\frac{1}{3}}}$ is **[1]**

   a) 27 b) 9

   c) 1 d) 3

2. If x = $2+\sqrt{3}$, then $x+\frac{1}{x}$ = **[1]**

   a) 4 b) -5

   c) -4 d) 5

3. The value of 1.9999……. in the form $\frac{p}{q}$, where 'p' and 'q' are integers and q $\neq$ 0, is **[1]**

   a) $\frac{1999}{1000}$ b) $\frac{19}{10}$

   c) 2 d) $\frac{1}{9}$

4. If g = $t^{\frac{2}{3}} + 4t^{\frac{-1}{2}}$, what is the value of g when t = 64? **[1]**

   a) $\frac{31}{2}$ b) $\frac{257}{16}$

   c) $\frac{33}{2}$ d) 16

5. When simplified $(256)^{-\left(4^{\frac{-3}{2}}\right)}$, is **[1]**

   a) 2 b) $\frac{1}{8}$

   c) $\frac{1}{2}$ d) 8

6. An irrational number between $\frac{3}{8}$ and $\frac{5}{8}$ is _____. **[1]**

   a) $\frac{1}{2}\left(\frac{3}{8}+\frac{5}{8}\right)$ b) $\left(\frac{3}{8}\times\frac{5}{8}\right)$

   c) $\sqrt{\frac{3}{8}\times\frac{5}{8}}$ d) $\sqrt{\frac{3}{8}+\frac{5}{8}}$

7. If x = $(7+4\sqrt{3})$ then $\left(x+\frac{1}{x}\right)$ =? **[1]**

   a) 14 b) 48

   c) $8\sqrt{3}$ d) 49

8. The positive square root of 7 + $\sqrt{48}$, is **[1]**

   a) $3+\sqrt{2}$ b) $7+\sqrt{3}$

   c) $2+\sqrt{3}$ d) $7+2\sqrt{3}$

9. An irrational number between $\sqrt{2}$ and $\sqrt{3}$ is **[1]**

a) $(\sqrt{2}+\sqrt{3})$ b) $\sqrt{2}\times\sqrt{3}$

c) $5^{1/4}$ d) $6^{1/4}$

10. π is **[1]**

a) a rational number b) an integer

c) an irrational number d) a whole number

11. The value of $\left(x^{a-b}\right)^{a+b}\times\left(x^{b-c}\right)^{b+c}\times\left(x^{c-a}\right)^{c+a}$ is **[1]**

a) 3 b) 2

c) 1 d) 0

12. The number $0.\overline{3}$ in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0, is **[1]**

a) $\frac{3}{100}$ b) $\frac{3}{10}$

c) $\frac{33}{100}$ d) $\frac{1}{3}$

13. If n is a natural number, then $\sqrt{n}$ is **[1]**

a) always a natural number b) sometimes a natural number and sometimes an irrational number

c) always a rational number d) always an irrational number

14. If $2^x = 4^y = 8^z$ and $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{4z}=4$ then the value of x is **[1]**

a) $\frac{16}{7}$ b) $\frac{3}{4}$

c) $\frac{4}{3}$ d) $\frac{7}{16}$

15. The value of $15\sqrt{15}\div 3\sqrt{5}$ is **[1]**

a) $5\sqrt{3}$ b) $3\sqrt{5}$

c) 3 d) 5

16. On simplification $(3+\sqrt{3})(3-\sqrt{3})$ gives **[1]**

a) 0 b) $-2\sqrt{3}$

c) 16 d) 6

17. The value of $(0.00032)^{\frac{-2}{5}}$ is **[1]**

a) 1 b) 0

c) 5 d) 25

18. If x $=\frac{\sqrt{7}}{5}$and $\frac{5}{x}=p\sqrt{7}$ then the value of p is **[1]**

a) $\frac{15}{7}$ b) $\frac{25}{7}$

c) $\frac{7}{15}$ d) $\frac{7}{25}$

19. The simplest form of $0.5\overline{7}$ is **[1]**

a) $\frac{26}{45}$ b) $\frac{57}{99}$

c) $\frac{57}{100}$ d) $\frac{57}{90}$

20. The decimal form of $\frac{2}{11}$ is **[1]**

a) 0.018

b) 0.18

c) $0.\overline{18}$

d) $0.0\overline{18}$

21. **Assertion (A):** $17^2 \cdot 17^6 = 17^3$ **[1]**

Reason (R): If a > 0 be a real number and p and q be rational numbers. Then $a^p \cdot a^q = a^{p+q}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

22. **Assertion (A):** $\sqrt{2}, \sqrt{3}$, are examples of irrational numbers. **[1]**

Reason (R): An irrational number can be expressed in the form $\frac{p}{q}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

23. **Assertion (A):** Every integer is a rational number. **[1]**

Reason (R): Every integer m can be expressed in the form $\frac{m}{l}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

24. **Assertion (A):** $\sqrt{3}$ is an irrational number. **[1]**

Reason (R): Square root of a positive integer which is not a perfect square is an irrational number.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

25. **Assertion (A):** Rational number lying between two rational numbers a and b is $\frac{a+b}{2}$. **[1]**

Reason (R): There is one rational number lying between any two rational numbers.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

26. **Assertion (A):** $2 + \sqrt{6}$ is an irrational number. **[1]**

Reason (R): Sum of a rational number and an irrational number is always an irrational number.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

27. **Assertion (A):** Three rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$are $\frac{9}{20}, \frac{10}{20}$ and $\frac{11}{20}$ **[1]**

Reason (B): A rational number between two rational numbers p and q is $\frac{1}{2}(p+q)$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

28. **Assertion (A):** e is an irrational number. **[1]**

Reason (R): π is an irrational number.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

29. **Assertion (A):** If $\sqrt{2}$ = 1.414, $\sqrt{3}$ = 1.732, then $\sqrt{5} = \sqrt{2} + \sqrt{3}$. **[1]**

Reason (R): Square root of a positive real number always exists.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

30. **Assertion (A):** $\sqrt{2}$ is an irrational number. **[1]**

Reason (R): A number is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

31. Express the rational number as decimal: $\frac{-2}{15}$ **[2]**

32. Find: $32^{\frac{1}{5}}$ **[2]**

33. Write the following in ascending order of magnitude $\sqrt[6]{6}, \sqrt[3]{7}, \sqrt[4]{8}$. **[2]**

34. Express $0.35\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. **[2]**

35. Express $2.\overline{36} + 0.\overline{23}$ as a fraction in simplest form. **[2]**

36. If $x^{\frac{1}{12}} = 49^{\frac{1}{24}}$. Find the value of x. **[2]**

37. Express $0.\overline{4}$ in the form $\frac{p}{q}$ **[2]**

38. Express the decimal $18.\overline{48}$ in the form $\frac{p}{q}$, where p, q are integers and q $\neq$ 0. **[2]**

39. Simplify the following by rationalizing the denominator : $\frac{1}{\sqrt{6}-\sqrt{5}}$ **[2]**

40. Evaluate by removing the radical sign and negative indices wherever it occurs: $w^2 = 27$ **[2]**

41. Simplify: $\left[9\left(64^{\frac{1}{3}} + 125^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$. **[2]**

42. Write in decimal form and say what kind of decimal expansion: $\frac{329}{400}$ **[2]**

43. Express $0.\overline{37}$ in the form $\frac{p}{q}$ **[2]**

44. Show that 1.272727 = $1.\overline{27}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0. **[2]**

45. Identify a following as rational or irrational number. Give the decimal representation of rational number.$\sqrt{100}$ **[2]**

Section C

46. If $4^{2x-1} - 16^{x-1} = 384$, find the value of x. **[3]**

47. Express in the form of $\frac{p}{q}$:$0.\overline{38} + 1.\overline{27}$ **[3]**

48. Prove that 7.478478... is a rational number. **[3]**

49. Rationalise:$\frac{1}{\sqrt{7}+\sqrt{3}-\sqrt{2}}$. **[3]**

50. State whether the following statements are true or false. Give reasons for your answers. **[3]**

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

51. Find three different irrational numbers between the rational numbers $\frac{5}{9}$ and $\frac{9}{11}$. **[3]**

52. Express $0.\overline{001}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ **[3]**

53. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + y^2$. **[3]**

54. Locate $\sqrt{3}$ on the number line. **[3]**

55. Rationalize the denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$ **[3]**

56. Rationalise the denominator: $\frac{3}{\sqrt{3}+\sqrt{5}-\sqrt{2}}$. **[3]**

57. Find the decimal expansion of $\frac{1}{7}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how? **[3]**

Section D

58. **Read the following text carefully and answer the questions that follow:** **[4]**

In a school 5 out of every 7 children participated in **Save wild life** campaign organised by the school authorities.

i. What is the fraction of students who participated in the campaign? (1)

ii. What is the recurring form of the fraction $\frac{5}{7}$? (1)

iii. How many rational numbers exist between 5 and 7? (2)

OR

Every rational number is a _____ number. (2)

59. **Read the following text carefully and answer the questions that follow:** **[4]**

Democracy has given people a powerful right- that is to VOTE. In India, every citizen over 18 years of age has the right to vote. Instead of enjoying it as a holiday, one must vote if he/she truly wants to contribute to the nation-building process and bring about a change.

A survey was done in a small area in which $\sqrt{9+2x} - \sqrt{2x}$ voters were men and $\frac{5}{\sqrt{9+2x}}$ voters were women.

i. What is the value of x if the number of men is equal to the number of women? (1)

ii. What is the product of the variables $a^p \cdot a^q$? (1)

iii. Simplify $\frac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}}$. (2)

OR

Is it true that if r is rational and s is irrational, then r + s is irrational? (2)

60. **Read the following text carefully and answer the questions that follow:** **[4]**

Real Numbers

N = {1, 2, 3, 4, ...} = Set of all natural numbers

W = {0, 1, 2, 3, 4, ...} = Set of all whole numbers

I = {-2, -1, 0, 1, 2, 3, ...} = Set of all integers

Q = {p/q: p ∈ 1, q ∈ I^+} = Set of all rational numbers

A number which is not rational is irrational number.

The set of all rationals and irrational form set of all real numbers (i.e., R)

Real Numbers are the numbers which include both rational and irrational numbers. Rational numbers are the numbers which can be written in the form of p/q where p and q are integers and q $\neq$ 0. Irrational numbers are those numbers which cannot be expressed as a ratio of two integers.

i. What is the product of two irrational numbers? (1)

ii. How many rational number/numbers lies between two rational numbers? (1)

iii. What is the sum of a rational and irrational number? (2)

OR

Is the number 3.14014001400014... an irrational number? (2)

Section E

61. If a = $3 + 2\sqrt{2}$, then find the value of: **[5]**

    i. $a^2 + \frac{1}{a^2}$

    ii. $a^3 + \frac{1}{a^3}$

62. Write the following in the descending order of magnitude. $\sqrt[3]{2}, \sqrt{3}, \sqrt[6]{5}$. **[5]**

63. If $x = \frac{5-\sqrt{21}}{2}$, prove that $\left(x^3 + \frac{1}{x^3}\right) - 5\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) = 0$. **[5]**

64. It being given that $\sqrt{3} = 1.732$, $\sqrt{5}$ = 2.236, $\sqrt{6} = 2.449$ and $\sqrt{10} = 3.162$, find upto three places of decimal, $\frac{3+\sqrt{5}}{3-\sqrt{5}}$. **[5]**

65. Find the decimal expansions of $\frac{10}{3}$, $\frac{7}{8}$ and $\frac{1}{7}$. **[5]**

66. If $a = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and $b = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$,,show that $3a^2+4ab-3b^2=4+\frac{56}{3}\sqrt{10}$. **[5]**

67. If $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1}$, then find the value of $a^2 + b^2 - 4ab$. **[5]**

68. Find the values of a and b if $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$. **[5]**

69. If $a = \frac{1}{7-4\sqrt{3}}$ and $b = \frac{1}{7+4\sqrt{3}}$, then find the value of: **[5]**

    i. $a^2 + b^2$

    ii. $a^3 + b^3$

70. Simplify: $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$. **[5]**

71. If $a = 3 - 2\sqrt{2}$, find the value of $a^2 - \frac{1}{a^2}$. **[5]**

72. If $p = \frac{3-\sqrt{5}}{3+\sqrt{5}}$ and $q = \frac{3+\sqrt{5}}{3-\sqrt{5}}$, find the value of $p^2 + q^2$. **[5]**

73. If $a = \frac{3+\sqrt{5}}{2}$, then find the value of $a^2 + \frac{1}{a^2}$. **[5]**

74. If $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, find the value of $a^2 + b^2 - 5ab$. **[5]**

75. If x is a positive real number and exponents are rational numbers, simplify **[5]**
    $\left(\frac{x^b}{x^c}\right)^{b+c-a} \cdot \left(\frac{x^c}{x^a}\right)^{c+a-b} \cdot \left(\frac{x^a}{x^b}\right)^{a+b-c}$.