Solution

QUADRILATERALS

Class 09 - Mathematics

Section A

1.

(b) 80°, 100°

Explanation:

Let the adjacent angles of a parallelogram be 4x and 5x and sum of adjacent angles of parallelogram is 180°.

$\therefore$ 4x + 5x = 180°

$\Rightarrow$ 9x = 180° $\Rightarrow$ x = 20°

$\therefore$ Angles are 80° and 100°.

2.

(c) 3.6 cm

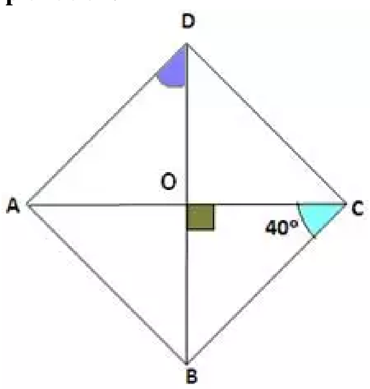
Explanation:

E and F are midpoints of sides AB and AC. By midpoint theorem, EF is parallel to BC and EF is $\frac{1}{2}$ of BC.

So, EF = $\frac{1}{2}$ of (7.2) = 3.6 cm

3. **(a)** 50^{o}

Explanation:



Given ABCD is a rhombus. Diagonals bisect each other perpendicularly.

Hence $\angle$BOC = 90°

Given $\angle$OCB = 40°

AD||BC and BD is the transversal

$\therefore$ $\angle$ADB = $\angle$DBC (Alternate angles)

Hence in right angled $\triangle$BOC,

$\angle$BOC + $\angle$OCB + $\angle$OBC = 180°

$\Rightarrow$ 90° + 40° + $\angle$OBC = 180°

$\Rightarrow$130° + $\angle$OBC = 180°

$\Rightarrow$ $\angle$OBC = 180° - 130° = 50°

But $\angle$OBC = $\angle$DBC

Therefore, $\angle$ADB = 50°

4.

(c) 9.5 cm, 9.5 cm, 5.5 cm, 5.5 cm

Explanation:

Perimeter of ABCD = AB + BC + CD + DA = 30

In a parallelogram, opposite sides are equal.

AB = CD = 9.5 and BC = DA = x

So, 9.5 + x + 9.5 + x = 30

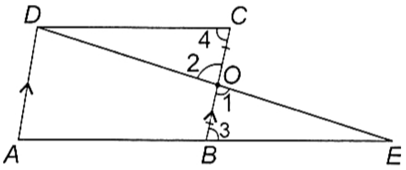
2x = 30 - 19

x = 5.5
AB = 9.5 = CD and BC = DA = 5.5

5.

(d) 135^o, 135^o
Explanation:
AB is parallel to DC.
angle A + angle D = 180^o (co-interior angle)
angle D = 180^o - 45^o = 135^o
Similarly by following same argument, angle C = 135^o

6.

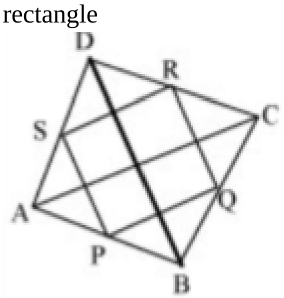
(d) AB = BE
Explanation:


In the figure, $\triangle$BCD is a parallelogram, where AB is produced to E such that OC = OB
In $\triangle$OBE and $\triangle$OCD,
$\angle 1 = \angle 2$ (Vertically opposite angles)
$\angle 3 = \angle 4$ (Alternate interior angles)
OB = OC (given)
$\therefore \triangle OBE \cong \triangle OCD$ (By ASA congruency)
$\Rightarrow$ BE = CD (By CPCT)
Also, AB = CD (y ABCD is parallelogram)
$\therefore$ AB = BE

7.

(b) 40°
Explanation:
$\angle BOC + \angle COD = 180^o$ (linear pair)
$\angle COD = 180^o - 90^o = 90^o$
In triangle DOC, $\angle DOC + \angle DCO + \angle ODC = 180^o$ (angle sum property)
$90^o + \angle DCO + 50 = 90^o$
$\angle DCO = 180^o - 140^o = 40$
$\angle DCO = \angle OAB = 40$ (alternate angles)

8.

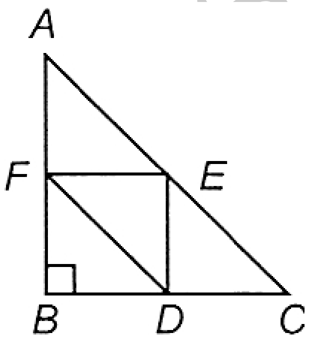
(b) rectangle
Explanation:
rectangle


Let ABCD be a rhombus and P,Q,R and S be the mid-points of sides AB, BC, CD and DA respectively.

In $\triangle ABD$ and $\triangle BDC$ we have
SP ∥ BD and SP = $\frac{1}{2}$BD ..... (1) [By mid-point theorem]
RQ ∥ BD and RQ = $\frac{1}{2}$BD ..... (2) [By mid-point theorem]
From (1) and (2) we get,
SP ∥ RQ
PQRS is a parallelogram
As diagonals of a rhombus bisect each other at right angles.
∴ AC⊥BD
Since, SP ∥ BD, PQ ∥ AC and AC⊥BD
∴ SP⊥PQ
∴ ∠QPS = 90^o
∴ PQRS is a rectangle.

9.
(b) 190°
Explanation:
∠ADC + ∠DCB = 180° (Sum of adjacent angles of a parallelogram is 180°)
⇒ 85° + x = 180° ⇒ x = 95°
Now, DC || AE and CB is a transversal.
∴ y - x - 95° (Alternate interior angles)
∴ x + y = 95° + 95° = 190°

10.
(d) 70^o
Explanation:
∠OAD = 90^o - (∠OAB)
= 90^o - 35^o = 55^o.
Now, ∠ODA = ∠OAD = 55^o [∵ OA = OD since diagonals of a rectangles are equal and bisect each other].
∠AOD = 180^o - (∠OAD + ∠ODA)
= 180^o - (55^o + 55^o) = 70^o.

11.
(b) Right angled
Explanation:


Let ABC be right angled triangle and ∠ABC = 90°.
Let D, E, F are mid-points of sides BC,
AC and AB respectively.
∴ EF || BD and BF || DE (By mid-point theorem)
⇒ BDEF is a parallelogram.
∴ ∠FED = ∠FBD = 90° (∵ Opposite angles of a parallelogram are equal)
∴ DEF is right angled triangle.

12. **(a)** Diagonals of ABCD are equal

Explanation:

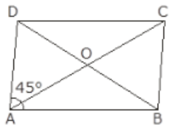
The diagonals of a square bisect its angles. Opposite sides of a square are both parallel and equal in length. All four angles of a square are equal.

13.

(d) 135^o

Explanation:

Given,



ABCD is a quadrilateral

$\angle A = 45^o$,

$\because$ diagonals of quadrilateral bisects each other hence ABCD is a parallelogram,

$\Rightarrow \angle A + \angle B = 180^o$

$\Rightarrow 45^o + \angle B = 180^o$

$\Rightarrow \angle B = 180^o - 45^o = 135^o$

14.

(b) Parallelogram

Explanation:

In quadrilateral AXCY,

AX || CY ($\because$ AB || CD) ...(i)

AX = $\frac{1}{2}$ AB and CY = $\frac{1}{2}$ CD ($\because$ X and Y are midpoint of AB and CD)

Also, AB = CD (Opposite sides of parallelogram)

So, AX = CY ...(ii)

$\Rightarrow$ AXCY is a parallelogram (from (i) and (ii))

Similarly, quadrilateral DXBY is a parallelogram.

In quadrilateral SXRY, SX || YR ($\because$ SX is a part of DX and YR is a part of YB)

Similarly, SY || XR

So, SXRY is a parallelogram.

15.

(d) 10 cm

Explanation:

Let us assume a rhombus ABCD where,

AB = BC = CD = DA

Now, in triangle OBC by using Pythagoras theorem we get:

$BC^2 = OB^2 + OC^2$

$BC^2 = 6^2 + 8^2$

$BC^2 = 36 + 64$

$BC^2 = 100$

$BC = \sqrt{100}$

BC = 10 cm

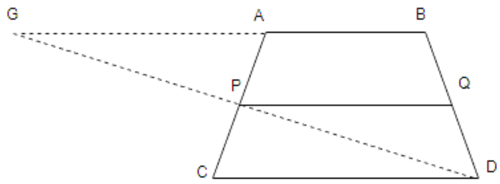
$\therefore$ AB = BC = CD = DA = 10 cm

16.

(b) $\frac{1}{2}$(AB + CD)

Explanation:

Join PD and Produce it to meet BA at G.



In $\triangle$PCD and $\triangle$APG,

$\angle$DPC = $\angle$GPA,

$\angle$PDC = $\angle$AGP

$\therefore \triangle PCD \cong \triangle APG$

CD = AG and PD = PG

In $\triangle$BGD,

P is the mid - point of GD

Q is the mid -point of BD

Therefore, By mid - point theorem, PQ || AB

and PQ = $\frac{1}{2}$(GB)

but GB = GA + AB = CD + AB

$\therefore PQ = \left(\frac{1}{2}\right)(AB + CD)$

17. **(a)** Parallelogram

Explanation:

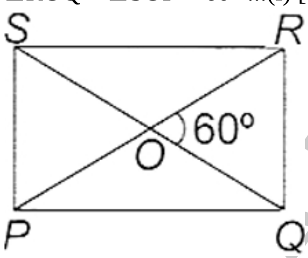
Two diagonals of quadrilateral form four triangles. Out of these four triangles two triangles of opposite to each other are congruent by SAS. By using CPCT property we can prove that both pair of opposite sides in a quadrilateral are parallel. A quadrilateral with both pair of opposite sides parallel is called parallelogram.

18.

(b) 60°

Explanation:

$\angle$ROQ = $\angle$SOP = 60° ...(i) [Vertically opposite angles]



$\therefore$ PR = SQ $\Rightarrow$ PO= SO (Diagonals of a rectangle are equal and bisect each other)

$\Rightarrow$ $\angle$OPS = $\angle$OSP ...(ii) [$\because$ In a triangle, angles opposite to equal sides are equal]

In $\triangle$POS, by angle sum property

$\angle$OSP + $\angle$OPS + $\angle$SOP = 180°

$\Rightarrow$ 2$\angle$OSP = 180° - 60° [Using (i) & (ii)]

$\Rightarrow$ $\angle$OSP = 60°

19.

(d) Diagonals of PQRS are at right angles.

Explanation:

Diagonals of PQRS are at right angles form all the internal angles as right angles. [according to angle property of rectangle, i.e, all the angles of a rectangle are right angle(90^0)]

20.

(b) 768 m^2

Explanation:

According to the question,
Area of given quadrilateral = $\frac{1}{2} \times$ Product of diagonals
= $\frac{1}{2} \times 48 \times 32$
= 768 sq. m

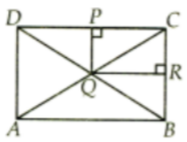
21. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Since, opposite angles of a parallelogram are equal.
Therefore, 3x - 2 = 50 - x
x = 13
One angle is 37^{o}

22. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).
Explanation:
It is given that, ABCD is a quadrilateral in which P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Then, PQRS is a parallelogram.
Also, the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
Hence, both assertion and reason are true and reason is correct explanation of the assertion

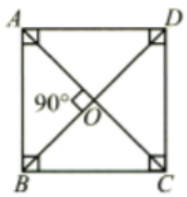
23. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

24. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

25.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
In ΔADC, Q is the midpoint of AC such that PQ || AD.
P is the mid-point of DC
DP = PC [Using converse of midpoint theorem]



26. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Since, diagonals of a square bisect each other at right angles.
$\angle$AOB = 90^{o}



27.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
In ΔABC, E and F are midpoint of the sides AC and AB respectively.
FE || BC [By mid-point theorem]
Now, in ΔABP, F is mid-point of AB and FQ || BP. Q is mid-point of AP

AQ = QP

28.

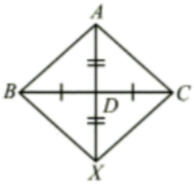
(c) A is true but R is false.

Explanation:

In quadrilateral ABXC, we have

AD = DX [Given]

BD = DC [Given]

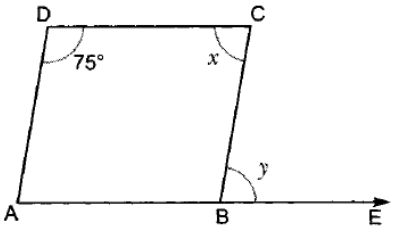


So, diagonals AX and BC bisect each other but not at right angles.

Therefore, ABXC is a parallelogram.

Section B

29.



Since ABCD is a parallelogram, we have

$\angle D + \angle C = 180^o$ ....(Sum of co-interior angles)

or $75^o + x = 180^o$

or $x = 180^o - 75^o$

or $x = 105^o$

Again, we have $x = y = 105^o$ (Alternate angles)

Hence, $x + y = 105^o + 105^o = 210^o$

30. From the given figure; In $\triangle BDC$, Q is the mid-point of BD.

Again, QR $\parallel$ DC ....(As ABCD is a rectangle and PQRB is a rectangle)

$\Rightarrow$ R is the mid-point of BC ....(By converse of mid-point theorem)

Since Q and R are the mid-points of BD and BC, we can write

QR = $\frac{1}{2}$DC

5 = $\frac{1}{2}$DC

So, DC = 10 cm

Also, DC = AB ....(Opposite sides of rectangle)

Therefore, DC = AB =10 cm.

31. Let ABCD be a $||$gm in which $\angle A = 115^o$.

Since AD $||$ BC and AB cuts them, so

$\angle A + \angle B = 180^o$

$\Rightarrow 115° + \angle B = 180^o$

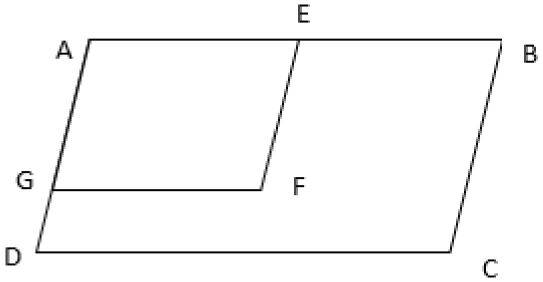
$\Rightarrow \angle B = (180^o - 115^o) = 65^o$.

Since the opposite angles of ||gm are equal, we have

$\angle C = \angle A = 115^o$ and $\angle D = \angle B = 65^o$.

Hence, $\angle B = 65^o$, $\angle C = 115^o$ and $\angle D = 65^o$

32. Given: $\angle C = 55°$



Opposite angles of a parallelogram are equal

$\because$ ABCD is a parallelogram

$\therefore \angle A = \angle C = 55°$

Also,

$\because$ AEFG is a parallelogram

$\therefore \angle F = \angle A = 55°$

Hence, $\angle F = 55°$

33. PQRS is a rhombus.

So the diagonals bisect each other at right angle.

$\therefore y = 90^o$

$\angle SRT = \angle QSR + y$ (external angle is equal to sum of opposite internal angles) I

$\Rightarrow 152 = \angle QSR + 90^o$

$\Rightarrow \angle QSR = 62^o$

Since SR || PQ

So, $\angle QRS = x = 62^o$

In $\triangle SPR$, SP = SR

$\Rightarrow \angle SRP = \angle SPR = z$

$\angle SRP = 180^o - \angle SRT = 180^o - 150^o = 28^o$

Hence the required, $z = 28^o$

34. Given: ABCD is a parallelogram with diagonal AC = diagonal BD

To prove: ABCD is a rectangle

Proof: In triangles ABC and ACD,

AB = AB [Common]

AC = BD [Given]

AD = BC [opp. Sides of a $\parallel$ gm]

$\therefore \Delta$ ABC $\cong \Delta$ ACD [By SSS congruency]

$\Rightarrow \angle$ DAB = $\angle$ CBA [By C.P.C.T.] ……….(i)

But $\angle$ DAB + $\angle$ CBA = 180° ……….(ii)

[$\because$ AD $\parallel$BC and AB cuts them, the sum of the interior angles of the same side of transversal is 180°]

From eq. (i) and (ii),

$\angle$ DAB = $\angle$ CBA = 90°

Hence ABCD is a rectangle.

35. Let ABCD be the parallelogram.

We know that opposite angles of parallelogram are equal.

$\therefore \angle A = \angle C$ and $\angle B = \angle D$

By given conditions,

Let $\angle A = x^o$ and $\angle B = \frac{4x^\circ}{5}$

Also, adjacent angles of parallelogram are supplementary,

$\therefore x^{o} + \frac{4x^\circ}{5} = 180^{o}$

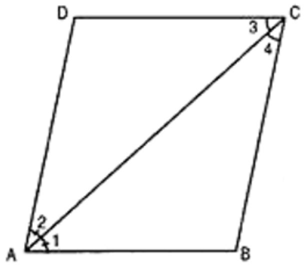
$\frac{9x^\circ}{5} = 180°$

$\therefore x = 100^{o}$

Hence, $\angle A = 100^{o}$ and $\angle B = \frac{4 \times 100^\circ}{5} = 80^{o}$

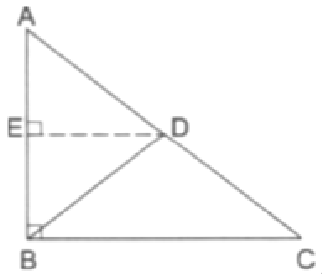
Hence, $\angle A = \angle C = 100^{o}$; $\angle B = \angle D = 80°$

36. Diagonal AC bisects $\angle A$ of the parallelogram ABCD.



i. Since AB $\parallel$ DC and AC intersects them.
   $\therefore \angle 1 = \angle 3$ [Alternate angles] ...(i)
   Similarly $\angle 2 = \angle 4$ ...(ii)
   But $\angle 1 = \angle 2$ [Given] ...(iii)
   Thus AC bisects $\angle C$.
ii. $\angle 2 = \angle 3 = \angle 4 = \angle 1$
   $\Rightarrow$ AD = CD [Sides opposite to equal angles]
   $\therefore$ AB = CD = AD = BC
   Hence ABCD is a rhombus.

37. Through D, draw DE ‖ BC, meeting AB at E.



Now, $\angle AED = \angle ABC = 90^{o}$ [corres. angles]

$\therefore \angle BED = \angle AED = 90^{o}$ [$\because \angle AED + \angle BED = 180^{o}$].

Now, in $\triangle$ ABC, it is given that D is the midpoint of AC and DE || BC (by construction).

$\therefore$ E must be the midpoint of AB ... (by converse of midpoint theorem).

$\therefore$ AE = BE.

Now, in $\triangle$AED and BED, we have

AE = BE ... (proved)

ED = ED ... (common),

$\angle AED = \angle BED$ (each equal to 90^{o}).

$\therefore \triangle AED \cong \triangle BED$

$\therefore$ DA = DB.

But, DA = DC ... [$\because$ D is the midpoint of AC].

Hence, DA = DB = DC.

38. In $\triangle$ABC, AD is median.

$\therefore$ BD = DC

We know that the line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side.

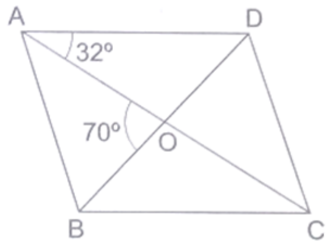
So, in $\triangle$ABC, D is the mid point of BC and DE || BA.

Hence, DE bisects AC.

$\therefore$ AE = EC

This means that E is the midpoint of AC.

$\therefore$ BE is median of $\triangle$ABC.

39. 

$\angle$AOB + $\angle$AOD = 180^o ... (linear pair)

$\Rightarrow$ 70° + $\angle$AOD = 180^o

In $\triangle$AOD, we have

$\angle$DAO + $\angle$AOD + $\angle$ADO = 180^o ... (sum of angles of a $\triangle$)

$\Rightarrow$ 32° +110° + $\angle$ADO = 180^o

$\Rightarrow$ $\angle$ADO = (180^o - 32^o -110^o) = 38^o.

Now, $\angle$OBC = $\angle$ADO = 38^o .... (alt. interior angles)

$\therefore$ $\angle$DBC = $\angle$OBC = 38° .

Hence , the $\angle DBC = 38^\circ$

40. We have $\angle$Q = 56^o

If diagonals bisects each other, then PQRS is parallelogram.

then we have,

$\angle$Q + $\angle$R = 180^o (interior angles on same side of transversal)

56^o + $\angle$R = 180^o

$\angle$R = 180^o - 56^o = 124^o

$\angle$R = 124^o

41. $\angle$A + $\angle$B + $\angle$C + $\angle$D = 360^o

$\angle$C + $\angle$D = 360^o - ($\angle$A + $\angle$B) ...(i)

In $\triangle$AOB

$\angle$AOB + $\frac{1}{2}\angle$A + $\frac{1}{2}\angle$B = 180^o

On multiplying by 2 on both sides

2$\angle$AOB + $\angle$A + $\angle$B = 360^o

$\angle$A + $\angle$B = 360^o - 2$\angle$AOB ...(ii)

On substituting value of 2 in ...(i)

$\angle$C + $\angle$D = 2$\angle$AOB

So k = 2

42. Given: In ABCD, in which BM $\perp$ AC and DN $\perp$ AC and BM = DN.

To prove: AC bisects BD ie. DO = BO

Proof:

Now, in $\triangle$OND and $\triangle$OMB, we have,

$\angle$OND = $\angle$OMB … 90^o each

$\angle$DON = $\angle$ BOM … Vertically opposite angles

Also, DN = BM … Given Hence, by AAS congruence rule,

$\triangle$OND $\cong$ $\triangle$OMB

$\therefore$ OD = OB … (by CPCT)

Hence, AC bisects BD.

43. From the given figure we have:

$\angle$C = $\angle$A ....(Opposite angles of a parallelogram)

$\Rightarrow$ $\angle$C = 60° (because $\angle$A = 60°)

Now, in $\triangle$BDC

$\angle$C + $\angle$CDB + $\angle$DBC = 180°

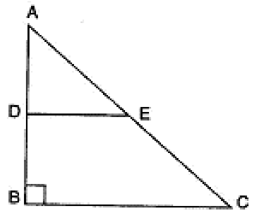
$\Rightarrow 60° + \angle CDB + 80° = 180°$

$\Rightarrow \angle CDB = 180° - 140°$

$\Rightarrow \angle CDB = 40°$

Section C

44. i. In right angled triangle ABC,



$BC^2 = AC^2 - AB^2$ (Using Pythagoras theorem)

$= (15)^2 - (9)^2 = 225 - 81$

$= 144$

$\Rightarrow BC = \sqrt{144} = 12$ cm.

ii. As D and E are the mid-points of AB and AC respectively.

$\therefore$ DE || BC and DE $= \frac{1}{2}$ BC $= \frac{1}{2}(12) = 6$ cm.

AD = BD $= \frac{1}{2}$AB $= \frac{1}{2}$ (9) $= \frac{9}{2}$cm.

As DE || BC and AB intersects them

$\therefore \angle ADE = \angle ABC = 90^o$ . . . .[Corresponding angles]

$\Rightarrow \triangle$ ADE is a right-angled triangle.

$\therefore$ Area of $\triangle$ ADE $= \frac{(AD)(DE)}{2} = \frac{9}{2} \cdot \frac{6}{2} = \frac{27}{2} = 13.5\text{cm}^2$

45. Given: ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal.

To Prove :

i. SR || AC and SR $= \frac{1}{2}$AC

ii. PQ = SR

iii. PQRS is a parallelogram

Proof :

i. In $\triangle$DAC,

As S is the mid-point of DA and R is the mid-point of DC

$\therefore$ SR || AC and SR $= \frac{1}{2}$AC . . . [Mid point theorem]

ii. In $\triangle$BAC,

As P is the mid-point of AB and Q is the mid-point of BC

$\therefore$ PQ || AC and PQ $= \frac{1}{2}$AC . . . [Mid point theorem]

But from (i) SR $= \frac{1}{2}$AC

$\therefore$ PQ = SR

iii. PQ || AC . . .[From (i)]

SR || AC . . .[From (i)]

$\therefore$ PQ || SR . . .[Two lines parallel to the same line are parallel to each other]

Similarly, PQ = SR . . .[From (ii)]

$\therefore$ PQRS is a parallelogram . . . [A quadrilateral is a parallelogram if a pair of opposite sides is parallel and is of equal length]

46. Given: A quad. ABCD in which AB = BC = CD = DA and O is a point within it such that OB = OD.

To prove: $\angle AOB + \angle COB = 180°$

Proof: In $\triangle$OAB and $\triangle$OAD,

we have AB=AD (given)

OA = OA (common)

OB = OD (given)

$\triangle OAB \cong \triangle OAD$

$\angle AOB = \angle AOD$. ... (i) (c.p.c.t.)

Similarly, $\triangle OBC \cong \triangle ODC$

$\angle COB = \angle COD$ ...(ii)

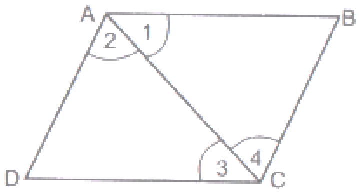
Now, ∠AOB + ∠COB + ∠COD + ∠AOD = 360° [angles at a point]

⇒ 2(∠AOB + ∠COB) = 360°

⇒ ∠AOB + ∠COB = 180°

Hence, A, O and C are in the same straight line.

47. ABCD is a parallelogram and diagonal AC bisect $\angle A$. We have to show that ABCD is a rhombus.



$\angle 1=\angle 2$ …(1) [∵ AC bisect $\angle A$]

$\angle 2=\angle 4$ …(2) [Alt. interior angles]

From (1) and (2), we get

$\angle 1=\angle 4$

Now, in ΔABC, we have

$\angle 1=\angle 4$ [Proved above]

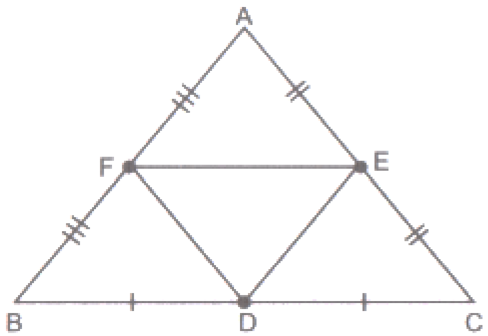
∴ BC = AB [∵ Side. Opp. To equal $\angle s$ are equal]

Also, AB = DC and AD = BC [∵ Opposite sides of a parallelogram are equal]

So, ABCD is a parallelogram in which its sides AB = BC = CD = AD.

Hence, ABCD is a rhombus.

48. Given: ABC is an equilateral triangle. D, E and F are the mid-points of the sides BC, CA and AB, respectively of ΔABC.



To prove: Δ DEF is an equilateral triangle.

Proof: EF joints mid-points of sides of AB and AC respectively.

∴ $EF=\frac{1}{2}BC$ ...(1) [Mid-point theorem]

Similarly, D E = $\frac{1}{2}$ A B ....(2) [Mid-point theorem]

$DF=\frac{1}{2}AC$ ....(3) [Mid-point theorem]

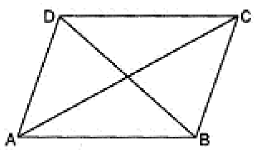
But, AB = BC = CA …(4) [Sides of an equilateral Δ ABC]

From (1), (2), (3) and (4), we have

DE = EF = FD

∴ ΔDEF is an equilateral triangle.

49. Given: The diagonals of a parallelogram are equal.



To prove: Parallelogram is a rectangle.

Proof : In △ACB and △BDA,

AC = BD . . . [Given]

AB = BA . . . [Common]

BC = AD . . . [Opposite sides of parallelogram]

∴ △ACB ≅△BDA . . .[By SSS property]

∴ ∠ABC = ∠BAD . . . [c.p.c.t.] . . . .(1)

As AD || BC . . . [Opposite sides of parallelogram] transversal AB intersects them.

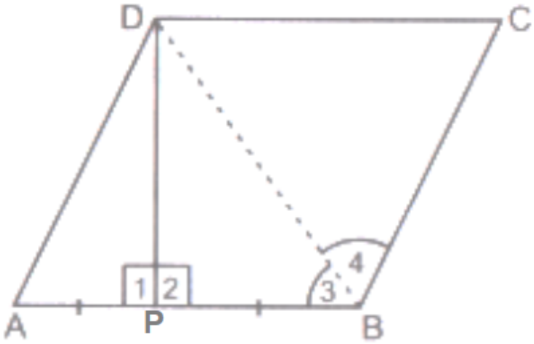
$\therefore$ ∠BAD + ∠ABC = 180^{o} . . . [Sum of interior angle on the same side of a transversal] . . . .(2)

∠BAD = ∠ABC = 90^{o} . . . [From (1) and (2)]

$\therefore$ ∠A = 90^{o}

$\therefore$ Parallelogram ABCD is a rectangle.

50. The required diagram is shown below:



In $\triangle APD$ and $\triangle BPD$, we have

AP = BP ....[Given]

$\angle 1 = \angle 2$ ....[$\because$ Each equal to 90^{o}]

PD = PD .... [Common side]

So, by SAS Criterion of congruence, we have

$\Delta APD \cong \triangle BPD$

$\therefore \angle A = \angle 3$ ....[CPCT]

But,$\angle 3 = \angle 4$ .... [$\because$ Diagonals bisect opposite angles of a rhombus]

$\Rightarrow \angle A = \angle 3 = \angle 4$ ….........................(1)

Now, AD || BC

SO, $\angle A + \angle ABC = 180^{\circ}$ [$\because$ Sum of consecutive interior angles is 180^{o}]

$\Rightarrow \angle A + \angle 3 + \angle 4 = 180^{\circ}$

$\Rightarrow \angle A + \angle A + \angle A = 180^{\circ}$ [by Using (1)]

$\Rightarrow 3\angle A = 180^{\circ}$

$\Rightarrow \angle A = \frac{180^{\circ}}{3} = 60^{\circ}$

Now, $\angle ABC = \angle 3 + \angle 4$

$= 60^{\circ} + 60^{o}$

$= 120^{\circ}$ .... [$\because$ Opposite angles of a rhombus are equal]

$\therefore \angle ADC = \angle ABC = 120^{\circ}$ .... [$\because$ Opposite angles of a rhombus are equal]

51. Since ABCD is a parallelogram, we can write

AB $\parallel$ DC

Now, AB || DC and transversal BD intersects them at B and D.

$\therefore$ ∠1 = ∠2 ....(Alternate interior angles)

Now, in triangles ABM and CDN, we have

∠1 = ∠2 ....(Proved above)

∠AMB = ∠CND ....(Each 90^{o})

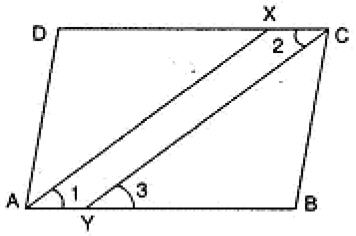
AB = DC ....(Opposite sides of parallelogram)

$\therefore$ △ABM ≅ △CDN ....(By AAS criterion of congruence)

$\therefore$ AM = CN (by CPCT).

Hence proved

52. Given: ABCD is a parallelogram and line segments AX, CY bisect the angles A and C respectively.



To Prove : AX || CY

Proof : ABCD is a parallelogram.

∴ ∠A = ∠C . . .[Opposite ∠s]

⇒ $\frac{1}{2}$∠A = $\frac{1}{2}$ ∠C . . .[As halves of equals are equal]

⇒ ∠1 = ∠2 ...[As AX bisects ∠A and CY bisects ∠C] . .(1)

Now, AB || DC and CY intersects them

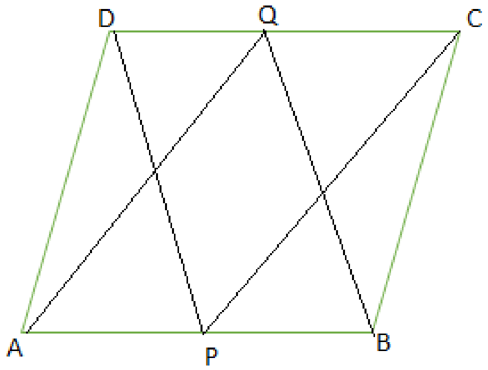
∴ ∠2 = ∠3 . . . .[Alternate interior ∠s] . . . (2)

∠1 = ∠3 . . .[From (1) and (2)]

But these are corresponding angles

∴ AX || CY.

53.



Given, P and Q are mid-points of AB and CD.

Now, AB||CD,

∴AP || QC

Also, AB = DC

$\frac{1}{2}$AB = $\frac{1}{2}$DC

AP = QC

Now, AP || QC and AP = QC

∴APCQ is a parallelogram.

AQ || PC or SQ || PR

Again,

AB || DC ⇒ $\frac{1}{2}$ AB = $\frac{1}{2}$DC

∴BP = QD

Now, BP||QD and BP = QD

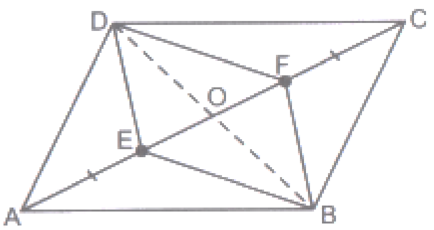
∴BPDQ is a parallelogram

So, PD||BQ or PS||QR

Thus, SQ||RP and PS||QR

∴PQRS is a parallelogram.

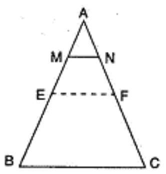
54. Given: A parallelogram ABCD: E and F are points of diagonal AC of parallelogram ABCD such that AE = CF.



To prove: BFDE is parallelogram.
Proof: ABCD is a parallelogram.
∴ OD = OB ...(1) [∵ Diagonals of parallelogram bisect each other]
OA = OC ...(2) [∵ Diagonals of parallelogram bisect each other]
AE = CF …(3) [Given]
By subtracting (3) from (2), we obtain
OA – AE = OC – CF
∴ OE = OF …(4)
Therefore, BFDE is a parallelogram. [Because OD = OB and OE = OF]

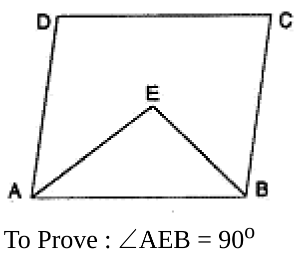
55. ABCD is a rhombus.
We know that rhombus is type of parallelogram whose all sides are equal.
So in △DCB,
DC = BC
∴ ∠CDB = ∠CBD = y^o base angles of isosceles triangle are equal.
Now, x = ∠CAB …alternate angles with transversal AC
∴ x = $\frac{1}{2}$ ×∠BAD
∴ x = $\frac{1}{2} \times 62^o$
∴ x = 31^o
In △DOC,
We know sum of angles of triangle is 180^o
∠CDO + ∠DOC + ∠OCD = 180^o
∴ ∠CDO + 90^o + 31^o = 180^o
∴ ∠CDO = 59^o
∴ y = 59^o
Hence the required values of x = 31^o and y = 59^o

56. Given: In triangle ABC, points M and N on the sides AB and AC respectively are taken so that $AM = \frac{1}{4}AB$ and $AN = \frac{1}{4}AC$.
To Prove : $MN = \frac{1}{4}BC$
Construction: Join EF where E and F are the mid points of AB and AC respectively.



Proof: E is the mid-point of AB and F is the mid-point of AC.
EF || BC and $EF = \frac{1}{2}BC$
Now $\mathrm{AE} = \frac{1}{2}\mathrm{AB}$ and $AM = \frac{1}{4}AB$
$\therefore \mathrm{AM} = \frac{1}{2}\mathrm{AE}$
Similarly, $AN = \frac{1}{2}AF$
M and N are the mid-points of AE and AF respectively.
MN || EF and $\mathrm{MN} = \frac{1}{2}\mathrm{EF} = \frac{1}{2}\left(\frac{1}{2}\mathrm{BC}\right)$ . . . .[From (1)]
$= \frac{1}{4}BC$

57. Given: ABCD is a parallelogram. The angle bisectors AE and BE of adjacent angles A and B meet at E.



To Prove : ∠AEB = 90^o
Proof : AD || BC . . . [Opposite sides of || gm]
∴ ∠DAB + ∠CBA = 180^o . . .[As the sum of interior angles on the same side of a transversal is 180^o]

$\Rightarrow$ 2$\angle$EAB + 2$\angle$EBA = 180^{o} . . .[As AE and BE are the bisectors of $\angle$DAB and $\angle$CBA respectively]

$\Rightarrow$ $\angle$EAB + $\angle$EBA = 90^{o}

In $\triangle$EAB,

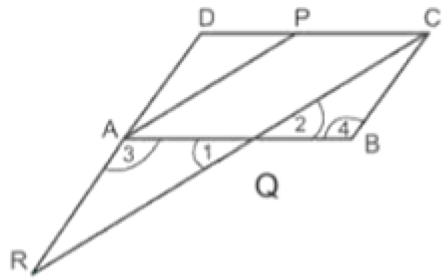
$\angle$EAB + $\angle$EBA + $\angle$AEB = 180^{o} . . .[As the sum of three angles of a triangle is 180^{o}]

$\Rightarrow$ 90^{o} + $\angle$AEB =180^{o} . . . [From (1)]

$\Rightarrow$ $\angle$AEB = 90^{o}

58. ABCD is a parallelogram. P is the mid-point of CD. CR which intersects AB at Q is parallel to AP

In $\triangle \mathrm{DCR}, \mathrm{P}$ is the mid-point of CD and AP || CR,



$\therefore$ A is the mid-point of DR, i.e., AD = AR.

[$\because$The line drawn through the mid-point of one side of a triangle parallel to another side intersects the third side at its mid-point.]

In $\triangle \mathrm{ARQ}$ and $\triangle \mathrm{BCQ}$, we have

AR = BC [$\because$ AD = AR [proved above) and AD = BC]

$\angle 1 = \angle 2$ [Vertically opposite angles]

$\angle 3 = \angle 4$ [Alt. $\angle s$]

$\therefore \triangle \mathrm{ARQ} \cong \triangle \mathrm{BCQ}$ [By AAS Congruence rule]

CQ = QR [CPCT]

CQ = QR

Hence, DA = AR and CQ = QR

Section D

59. i. By joining mid points of sides of a quadrilateral one can make parallelogram.

S and R are mid points of sides AD and CD of ΔADC, P and Q are mid points of sides AB and BC of ΔABC, then by mid-point theorem SR || AC and SR = $\frac{1}{2}$AC similarly PQ || AC and PQ = $\frac{1}{2}$AC.

Therefore SR || PQ and SR = PQ

A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

Hence PQRS is parallelogram.

ii. $\angle$RQP. = 30^{o}, Opposite angles of a parallelogram are equal.

iii. Adjacent angles of a parallelogram are supplementary.

Thus, $\angle$RSP + $\angle$SPQ = 180^{o}

50^{o} + $\angle$SPQ = 180^{o}

$\angle$SPQ = 180^{o} - 50^{o}

= 130^{o}

OR

RQ = 3 cm

Opposite side of a parallelogram are equal.

60. i. We know that line joining mid points of two sides of triangle is half and parallel to third side.

Hence RQ is parallel to BC and half of BC.

RQ = $\frac{28}{2}$ = 14 cm

Length of RQ = 14 cm

ii. By mid-point theorem we know that line joining mid points of two sides of triangle is half and parallel to third side.

PQ = $\frac{AB}{2} = \frac{25}{2}$ = 12.5 cm

QR = $\frac{BC}{2} = \frac{28}{2}$ = 14 cm

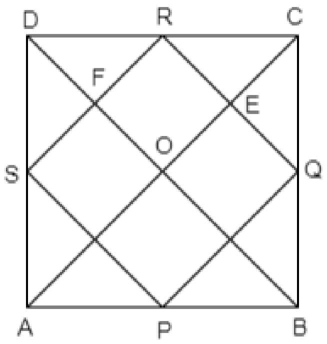
RP = $\frac{AC}{2} = \frac{26}{2}$ = 13 cm

Length of garland = PQ + QR + RP = 12.5 + 14 + 13 = 39.5 cm
Length of garland = 39.5 cm.

iii. As R and P are mid-points of sides AB and BC of the triangle ABC, by mid point theorem, RP || AC Similarly, RQ || BC and PQ || AB. Therefore ARPQ, BRQP and RQCP are all parallelograms. Now RQ is a diagonal of the parallelogram ARPQ, therefore, $\triangle ARQ \cong \triangle PQR$ Similarly $\triangle CPQ \cong \triangle RQP$ and $\triangle BPR \cong \triangle QRP$ So, all the four triangles are congruent.
Therefore Area of $\triangle ARQ$ = Area of $\triangle CPQ$ = Area of $\triangle BPR$ = Area of $\triangle PQR$
Area $\triangle ABC$ = Area of $\triangle ARQ$ + Area of $\triangle CPQ$ + Area of $\triangle BPR$ + Area of $\triangle PQR$
Area of $\triangle$ ABC = 4 Area of $\triangle PQR$
$\triangle PQR = \frac{1}{4} ar(ABC)$
OR
As R and Q are mid-points of sides AB and AC of the triangle ABC. Similarly, P and Q are mid points of sides BC and AC by mid-point theorem, RQ || BC and PQ || AB. Therefore BRQP is parallelogram.

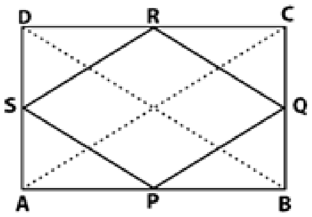
Section E

61. It is given that ABCD is parallelogram and $\angle DAB = 80^o$ and $\angle DBC = 60^o$
We need to find measure of $\angle CDB$ and $\angle ADB$
In ABCD, AD || BC, BD as transversal,
$\angle DBC = \angle ADB = 60^o$ … Alternate interior angles
$\Rightarrow \angle ADB = 60^o$ .....(i)
As $\angle DAB$ and $\angle ADC$ are adjacent angles,
$\angle DAB + \angle ADC = 180^o$
$\Rightarrow \angle ADC = 180^o - \angle DAB$
$\Rightarrow \angle ADC = 180^o - 80^o = 100^o$
Also,
$\angle ADC = \angle ADB + \angle CDB$
$\therefore \angle ADC = 100^o$
$\angle ADB + \angle CDB = 100^o$ ..... (ii)
From (i) and (ii), we get:
$60^o + \angle CDB = 100^o$
$\Rightarrow \angle CDB = 100^o - 60^o = 40^o$
Hence, $\angle CDB = 40^o$ and $\angle ADB = 60^o$

62.


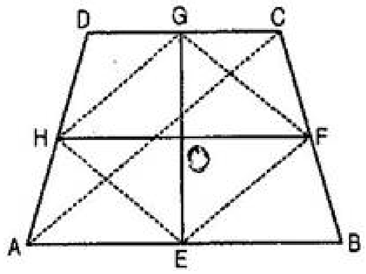
Let ABCD be the square and P, Q, R and S be the midpoints of AB, BC, CD and DA, respectively.
Join diagonals of the square.
In $\triangle ABC$, we have, by midpoint theorem,
$\therefore$ PQ || AC and PQ = $\frac{1}{2}$ AC
Similarly, SR || AC and SR = $\frac{1}{2}$ AC.
As, PQ || AC and SR || AC, then also PQ || SR
Also, PQ = SR, each equal to $\frac{1}{2}$ AC …(1)
So, PQRS is a parallelogram
Now, in $\triangle SAP$ and $\triangle QBP$, we have,
AS = BQ
$\angle A = \angle B = 90°$
AP = BP

$\therefore$ By SAS test of congruency,
$\triangle SAP \cong \triangle QBP$
Hence, PS = PQ …by cpct …(2)
Similarly, $\triangle SDR \cong \triangle QCR$
$\therefore$ SR = RQ … by cpct …(3)
Hence, from 1, 2 and 3 we have,
PQ = PQ = SR = RQ
We know that the diagonals of a square bisect each other at right angles.
$\therefore \angle EOF = 90^o$
Now, RQ || DB
$\Rightarrow$ RE || FO
Also, SR || AC
$\Rightarrow$ FR || OE
$\therefore$ OERF is a parallelogram.
So, $\angle FRE = \angle EOF = 90^o$ (Opposite angles are equal)
Thus, PQRS is a parallelogram with $\angle R = 90^o$ and PQ = PS = SR = RQ.
This means that PQRS is square.
Hence, the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a square is a square.

63. 

Consider $\triangle ABC$
We know that P and Q are the midpoints of AB and BC
So we get PQ || AC and
PQ = $\frac{1}{2}$ AC ... (1)
Consider $\triangle BCD$
We know that Q and R are the midpoints of BC and CD
So we get QR || BD and
QR = $\frac{1}{2}$ BD ... (2)
Consider $\triangle ADC$
We know that S and R are the midpoints of AD and CD
So we get RS || AC and
RS = $\frac{1}{2}$ AC ... (3)
Consider $\triangle ABD$
We know that P and S are the midpoints of AB and AD
So we get SP || BD and
SP = $\frac{1}{2}$ BD ... (4)
Using all the equations
PQ || RS and QR || SP
Thus, PQRS is a parallelogram
It is given AC = BD
We can write it as
$\frac{1}{2}$AC = $\frac{1}{2}$BD
From the equations we get
PQ = QR = RS = SP
PQRS is a rhombus
Therefore, it is proved that the quadrilateral formed by joining the midpoints of its sides is a rhombus.

64. Given: A quadrilateral ABCD in which EG and FH are the line-segments joining the mid-points of opposite sides of a quadrilateral.



To prove: EG and FH bisect each other.

Construction: Join AC, EF, FG, GH and HE.

Proof: In ABC, E and F are the mid-points of respective sides AB and BC.

∴ EF || AC and EF = $\frac{1}{2}$ AC ……….(i)

Similarly, in ADC,

G and H are the mid-points of respective sides CD and AD.

∴HG || AC and HG = $\frac{1}{2}$ AC ……….(ii)

From eq. (i) and (ii), we get,

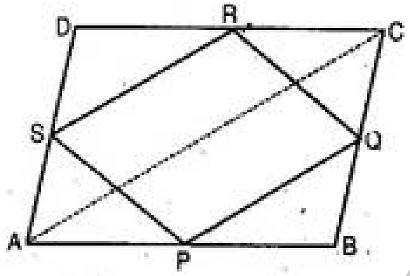
EF || HG and EF = HG

∴ EFGH is a parallelogram.

Since the diagonals of a parallelogram bisect each other, therefore line segments (i.e. diagonals) EG and FH (of parallelogram EFGH) bisect each other.

Hence Proved.

65. Given: A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To prove: PQRS is a rhombus.

Construction: Join AC.

Proof: In △ABC, P and Q are the mid-points of sides AB, BC respectively.

∴ PQ || AC and PQ = $\frac{1}{2}$ AC……….(i)

In △ADC, R and S are the mid-points of sides CD, AD respectively.

∴ SR || AC and SR = $\frac{1}{2}$ AC……….(ii)

From eq. (i) and (ii), PQ || SR and PQ = SR……….(iii)

∴ PQRS is a parallelogram.

Now ABCD is a rectangle.[Given]

∴ AD = BC

⇒ $\frac{1}{2}$ AD = $\frac{1}{2}$ BC ⇒AS = BQ……….(iv)

In triangles APS and BPQ,

AP = BP [P is the mid-point of AB]

∠PAS = ∠PBQ [Each 90^{o}]

And AS = BQ [From eq. (iv)]

∴ △APS ≅ △BPQ[By SAS congruency]

⇒ PS = PQ [By C.P.C.T.]………(v)

From eq. (iii) and (v), we get that PQRS is a parallelogram.

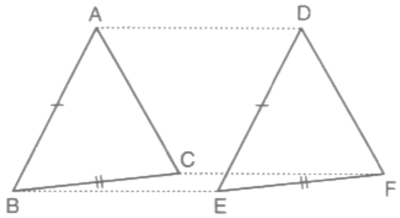
Since, PS = PQ

⇒ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

66. Two triangles ABC and DEF such that AB = DE and AB || DE

Also, BC = EF and BC || EF



TO PROVE AC = DF and AC || DF

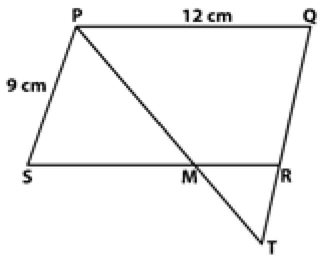
i. In quadrilateral ABED, we have
   AB = DE and AB || DE
   ⇒ One pair of opposite sides are equal and parallel
   ⇒ ABED is a parallelogram
   ⇒ AD = BE and AD || BE

ii. In quadrilateral BCFE, we have
   BC = EF and BC || EF
   ⇒ One pair of opposite sides are equal and parallel
   ⇒ BCFE is a parallelogram
   ⇒ CF = BE and CF || BE ...(ii)

iii. From (i) and (ii), we have
   AD = CF and AD || CF
   ⇒ ACFD is a parallelogram
   ⇒ AC = DF and AC || DF

iv. In $\Delta's$ ABC and DEF, we have
   AB = DE [Given]
   BC = EF (Given) and AC = DF [Proved in (iii)]

So, by SSS congruence criterion, we obtain
$\Delta ABC \cong \Delta DEF$

67. 

From the figure we know that PM is the bisector of ∠P
So we get
∠QPM = ∠SPM ... (1)
We know that PQRS is a parallelogram
From the figure we know that PQ || SR and PM is a transversal ∠QPM and ∠PMS are alternate angles
∠QPM = ∠PMS ... (2)
Consider equation (1) and (2)
∠SPM = ∠PMS ... (3)
We know that the sides opposite to equal angles are equal
MS = PS = 9 cm
∠RMT and ∠PMS are vertically opposite angles
∠RMT = ∠PMS ... (4)
We know that PS || QT and PT is the transversal
∠RTM = ∠SPM
It can be written as
∠RTM = ∠RMT
We know that the sides opposite to equal angles are equal
RT = RM
We get
RM = SR – MS

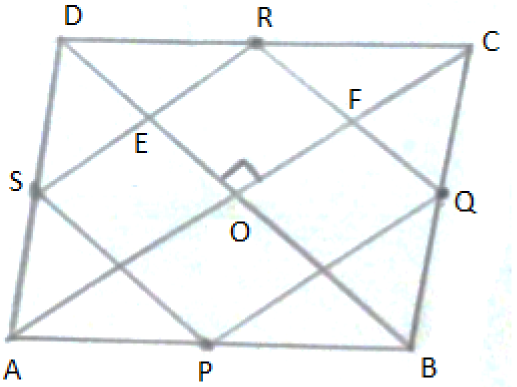
By substituting the values

RM = 12 - 9

RM = 3 cm

RT = RM = 3 cm

Therefore, the length of RT is 3 cm.

68. 

Given, P, Q, R and S are mid-points of the sides AB, BC, CD and DA, respectively.

Also, AC = BD and AC is perpendicular to BD.

In $\triangle$ADC, by mid-point theorem,

SR || AC and SR = $\frac{1}{2}$AC

In $\triangle$ABC, by mid-point theorem,

PQ || AC and PQ = $\frac{1}{2}$AC

$\therefore$PQ || SR and PQ = SR = $\frac{1}{2}$AC

Now, in $\triangle$ABD, by mid-point theorem,

SP || BD and SP = $\frac{1}{2}$BD = $\frac{1}{2}$AC

In $\triangle$BCD, by mid-point theorem,

RQ || BD and RQ = $\frac{1}{2}$BD = $\frac{1}{2}$AC

$\therefore$SP = RQ = $\frac{1}{2}$AC

$\therefore$PQ = SR = SP = RQ

Thus, all four sides are equal.

Now, in quadrilateral EOFR,

OE || FR, OF || ER

$\therefore\angle$EOF = $\angle$ERF = 90°(Opposite angles of parallelogram)

$\therefore\angle$QRS = 90°

Hence, PQRS is a square.

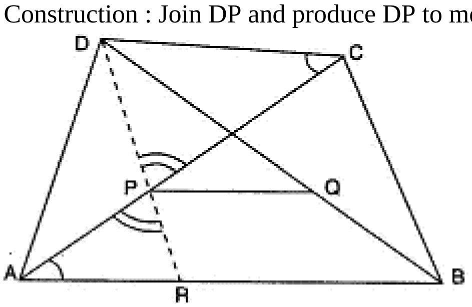
69. Given : ABCD is trapezium. P and Q are the mid-points of the diagonals AC and BD respectively.

To Prove :

i. PQ || AB or DC

ii. PQ = $\frac{1}{2}$ (AB – DC)

Construction : Join DP and produce DP to meet AB in R.



In $\triangle$APR and $\triangle$CPD,

$\angle$PAR = $\angle$PCD ....[Alternate angles]

$\angle$APR = $\angle$CPD ....[Vertically opp. angles]

AP = CP ...[Given]

$\therefore$ $\triangle$APR$\cong$ $\triangle$CPD . . .[By ASA axiom]

$\therefore$ PR = PD ....[c.p.c.t.]

and AR = CD ....[c.p.c.t.]

In △ DRB,

As P and Q are the mid-points of DR and BD respectively.

△ PQ || RB or AB or DC

and PQ = $\frac{1}{2}$RB = $\frac{1}{2}$(AB – AR) = $\frac{1}{2}$(AB – DC) .....[As AR = DC]

70. In △ADQ, we have

∠A + ∠ADQ + ∠Q = 180° (sum of the angles of △ADQ)

⇒ ∠A + $\frac{1}{2}$∠D + ∠Q = 180° ...(1)

In △CBP, we have

∠C + ∠CBP + ∠P = 180° (sum of the angles of △CBP)

⇒ ∠C + $\frac{1}{2}$∠B + ∠P = 180° ...(2)

Adding (1) and (2), we get

∠A + ∠C$\frac{1}{2}$ + ∠B$\frac{1}{2}$ + ∠D + ∠P + ∠Q = 360°

⇒ ∠A +∠C + ∠B + ∠D + ∠P + ∠Q = 360° + $\frac{1}{2}$∠B + $\frac{1}{2}$∠D ... [adding $\frac{1}{2}$∠B + $\frac{1}{2}$∠D) on both sides]

⇒ 360° + ∠P + ∠Q = 360° + $\frac{1}{2}$(∠B+ ∠D)

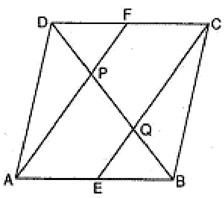
[∵ sum of all the angles of a quadrilateral is 360°⇒∠A +∠C + ∠B + ∠D = 360°]

⇒ ∠P + ∠Q = $\frac{1}{2}$(∠B+ ∠D)

Hence proved

71. Given: In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively.

To Prove: Line segments AF and EC intersect the diagonal BD.



Proof: AB || CD . . .[Opp. sides of || gm ABCD]

∴ AE || FC . . . (1)

As AB = DC . . .[Opp. sides of || gm ABCD]

∴ $\frac{1}{2}AB = \frac{1}{2}DC$ . . .[Halves of equals are equal]

∴ AE = CF . . . (2)

According to (1) and (2)

AECF is a parallelogram . . [A quadrilateral is a parallelogram if a pair of opp. sides is parallel and of equal length]

∴ EC || AF . . . [Opp. sides of || gm AECF] . . .(3)

In △DQC,

As F is the mid-point of DC and FP || CQ . . .[As EC || AF]

P is the mid-point of DQ . . . [By converse of mid-point theorem]

∴ DP = PQ . . . . (4)

Similarly, In △BAP,

BQ = PQ . . . .(5)

DP = PQ = BQ . . . From (4) and (5)

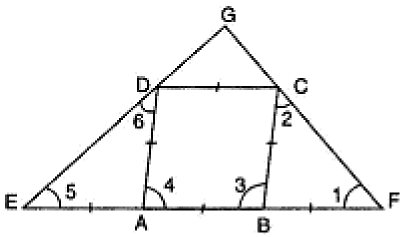
∴ Line segments AF and EC trisect the diagonal BD.

72. Given ABC is a △right angle at C

i. M is mid-point of AB
   And MD$\|$BC
   ∴ D is mid-Point of AC [a line through midpoint of one side of a parallel to another side bisect the third side.
ii. ∵ MD$\|$BC
   $\angle ADM = \angle DCB$∠ADM = ∠DCB [Corresponding angles]
   $\angle ADM = 90^\circ$ ∠ADM = 90°
iii. In △ADM and △CDM
   AD = DC [∵ D is mid-point of AC]
   DM = DM [Common]
   ADM CDM [By SAS]

∴ △AM = △CM [By C.P.C.T]
∴ AM = CM = MB [∵ M is mid-point of AB]
∴ CM = MA= $\frac{1}{2}$ AB.

73. Given: △ABC in which D and E are mid-points of the side AB and AC respectively
To Prove: DE ∥ BC
Construction: Draw CF ∥ BA,which cut extended DE at point F.
Proof: In△ADE and △CFE
∠1 = ∠2 [Vertically opposite angles]
AE = CE [Given]
And∠3 = ∠4 [Alternate interior angles and AC is transversal and AB ∥ CF,by construction]
$\therefore \triangle ADE \cong \triangle CFE$ [By ASA]
∴ DE = FE and AD = FC [By C.P.C.T]
But DA = DB
∴ DB = FC
Now DB ∥ FC
∴ DBCF is a parallelogram
∴ DE ∥ BC
Also DE = EF = $\frac{1}{2}$ BC

74. Given: ABCD is a rhombus and AB is produced to E and F such that AE = AB = BF.
To Prove ED ⊥ FC.
Proof :


AB = BF .... [By construction]
AB = BC ....[As ABCD is a rhombus]
BC = BF
∠1 = ∠2 ....[∠s opposite to equal side of a triangle]
In △BCF,
∠3 = ∠1 + ∠2 = ∠1 + ∠1 ...[From (1)]
= 2∠1 ....(2)
AB = AE ....[By construction]
AB = AD ....[As ABCD is rhombus]
AD = AE
∠5 = ∠6 . . .[∠s opposite to equal side of a triangle] . . .(3)
In △ADE,
∠4 = ∠5 + ∠6 = ∠5 + ∠5 . . . [From (3)]
= 2∠5 . . . .(4)
As AD ∥ BC
and transversal AB intersects them
∴ ∠3 + ∠4 = 180^o . . . .[Consecutive interior angles on the same side of a transversal are supplementary]
⇒ 2∠1 + 2∠5 = 180^o . . .[From (2) and (4)]
⇒ ∠1 + ∠5 = 90^o . . . .(5)
In GEF,
∠1 + ∠5 + ∠EGF = 180^o . . .[Sum of three angles of a triangle]
⇒ 90^o + ∠EGF = 180^o . . . [From (5)]
⇒ ∠EGF = 90^o
⇒ EG ⊥ GF
⇒ ED ⊥ FC

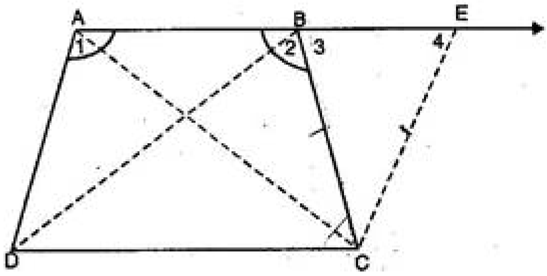
75. Given: ABCD is a trapezium.

AB || CD and AD = BC

To prove:

i. ∠A = ∠ B
ii. ∠C = ∠D
iii. △ABC ≅ △BAD
iv. Diag. AC = Diag. BD

Construction: Draw CE parallel to AD and extend AB to intersect CE at E.



Proof:

i. As AECD is a parallelogram. [By construction]
∴ AD = EC
But AD = BC [Given]
∴ BC = EC
⇒ ∠ 3 = ∠4 [Angles opposite to equal sides are equal]
Now ∠1 +∠ 4 = 180° [Interior angles]
And ∠2 + ∠3 =180° [Linear pair]
⇒ ∠1 +∠ 4 = ∠ 2 + ∠3
⇒ ∠1 = ∠ 2 [∵ ∠3 = ∠4]
⇒ ∠A =∠ B

ii. ∠3 = ∠ C [Alternate interior angles]
And ∠D = ∠4 [Opposite angles of a parallelogram]
But ∠3 = ∠4 [BCE is an isosceles triangle]
∴ ∠C =∠ D

iii. In △ABC and △BAD,
AB = AB [Common]
∠1 = ∠ 2 [Proved]
AD = BC [Given]|
∴ △ABC ≅ △BAD [By SAS congruency]
⇒ AC = BD [By C.P.C.T.]