Solution

POLYNOMIAL

Class 09 - Mathematics

Section A

1.

(d) 7 and - 18

Explanation:

It is given (x + 2) and (x - 1) are the factors of the polynomial

$f(x) = x^3 + 10x^2 + mx + n$

i.e., f(-2) = 0 and f(1) = 0

Now

$f(-2) = (-2)^3 + 10(-2)^2 + m(-2) + n = 0$

-8 + 40 - 2m + n = 0

$\Rightarrow$ -2m + n = -32

$\Rightarrow$ 2m - n = 32 ..... (i)

$f(1) = (1)^3 + 10(1)^2 + m(1) + n = 0$

1 + 10 + m + n = 0

m + n = -11 ..... (ii)

Solving equation (i) and (ii) we get

m = 7 and n = -18

2.

(d) ₹ 60

Explanation:

Amount of money Vikas has

$= ₹(x^3 + 2ax + b)$

Now, he can buy exactly (x - 1) jeans or (x + 1) shirts.

$\therefore$ (x - 1) and (x + 1) are factors of $x^3 + 2ax + b$.

$\therefore (1)^3 + 2a(1) + b = 0 \Rightarrow 2a + b = -1$ ...(i)

and $(-1)^3 + 2a(-1) + b = 0 \Rightarrow 2a - b = -1$ ...(ii)

Adding (i) and (ii), we get

$4a = -2 \Rightarrow a = \frac{-1}{2}$

$\therefore -1 + b = -1 \Rightarrow b = 0$

So, amount of money he has $= ₹(x^3 - x)$

$= ₹(4^3 - 4) = ₹(64 - 4) = ₹60$

3.

(b) 0

Explanation:

$x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 - y^2)(x^2 + y^2)$

$= (x - y)(x + y)(x^2 + y^2)$

Since, (x - y) is a factor of $x^4 - y^4$.

Using factor theorem, the remainder will be 0.

4.

(b) $p\left(\frac{b}{a}\right)$

Explanation:

Given: Divisor = b - ax

For getting the remainder we have to find value of x, which is put in p(x).

Then, b - ax = 0

$\Rightarrow x = \frac{b}{a}$

Therefore, the required remainder is p(x) = $p\left(\frac{b}{a}\right)$

5. **(a)** $3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)$

Explanation:

$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

Here,

$a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$

Therefore,

$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

$= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$ [Since $x^3 + y^3 + z^3 = 3xyz$ if $x + y + z = 0$]

$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

$= 3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)$

6.

(c) $x^2 - 9$

Explanation:

$x^2 - 9$

$= x^2 - 3^2$

$= (x + 3)(x - 3)$ [Using identity $a^2 - b^2 = (a + b)(a - b)$]

Then the zeroes are x + 3 = 0 and x - 3 = 0

$\Rightarrow$ x = -3 and x = 3

7.

(c) -2

Explanation:

x = 1 is a zero

p(1) = 0

$k(1)^2 + 4(1) + k = 0$

k + 4 + k = 0

2k = -4

k = -2

8.

(d) not defined

Explanation:

not defined

9.

(d) -a

Explanation:

x + a = 0 $\Rightarrow$ x = -a

By the remainder theorem, we know that when p(x) is divided by (x + a), the remainder is p (−a).

Thus, we have: $p(-a) = (-a)^3 + a \times (-a)^2 + 2 \times (-a) + a$

$= -a^3 + a^3 - 2a + a$

= -a

10. **(a)** x^6 - 1

Explanation:

$(x^2 - 1)(x^4 + x^2 + 1)$

= $x^2\left(x^4+x^2+1\right)$ - 1 $\left(x^4+x^2+1\right)$

= $x^6+x^4+x^2 - x^4-x^2-1$

= x^6-1

11.

(c) -2

Explanation:

$x^2+kx-3=(x-3)(x+1)$

=> $x^2+kx-3=x^2+(-3+1)x+(-3)\times 1$

=> $x^2+kx-3=x^2-2x-3$

On comparing the term, we get $k=-2$

12.

(d) $(x - 1 + y)(x^2 + 1 + y^2 + x + y - xy)$

Explanation:

The given expression to be factorized is $x^3 - 1 + y^3 + 3xy$

This can be written in the form

$x^3 - 1 + y^3 + 3xy = (x)^2 + (-1)^3 + (y)^3 - 3(x).(-1).(y)$

Recall the formula $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Using the above formula, we have,

$x^3 - 1 + y^3 + 3xy$

$= (x + (-1) + y) \{(x)^2 + (-1)^2 + (y)^2 - (x).(-1) - (-1).(y) - (y).(x)\}$

$= (x - 1 + y)(x^2 + 1 + y^2 + x + y - xy)$

13. **(a)** a = b = c

Explanation:

Given: $a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow 2(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow (2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) = 0$

$\Rightarrow (\{a^2 + b^2 - 2ab\} + \{b^2 + c^2 - 2bc\} + \{c^2 + a^2 - 2ca\}) = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

Now, since the sum of all squares is zero

$\Rightarrow$ a - b = 0 $\Rightarrow$ a = b

$\Rightarrow$ b - c = 0 $\Rightarrow$ b = c

$\Rightarrow$ c - a = 0 $\Rightarrow$ c = a

$\Rightarrow$ a = b = c

14.

(d) 3

Explanation:

If $x+1$ is a factor of p(x) = $2x^2+kx+1$, then

p(-1) = 0

$\Rightarrow 2x^2+kx+1=0$

$\Rightarrow 2(-1)^2+k(-1)+1=0$

$\Rightarrow 2-k+1=0$

$\Rightarrow k=3$

15.

(b) -6

Explanation:

p(x) = 5x - 4x^2 + 3

Putting x = -1 in p(x), we get

p(-1) = 5 × (-1) -4 × $(-1)^2$ + 3 = - 5 - 4 + 3 = -6

16.

(c) (x + 3) (x - 3)

Explanation:

x^2 - 9

= x^2 - 3^2

Using identity a^2 - b^2 = (a + b)(a - b)

= (x + 3)(x - 3)

17.

(c) $3\sqrt{3}$

Explanation:

$\left(x^4+\frac{1}{x^4}\right)=623$

$\Rightarrow\left(x^2\right)^2+\left(\frac{1}{x^2}\right)+2\times x^2\times\frac{1}{x^2}=623+2\times x^2\times\frac{1}{x^2}$

$\Rightarrow\left(x^2+\frac{1}{x^2}\right)^2=625$

$\Rightarrow x^2+\frac{1}{x^2}=\sqrt{625}=25$

Now ,

$\Rightarrow\left(x^2\right)+\left(\frac{1}{x^2}\right)+2\times x\times\frac{1}{x}=25+2\times x\times\frac{1}{x}$

$\Rightarrow\left(x+\frac{1}{x}\right)^2=27$

$\Rightarrow x+\frac{1}{x}=\sqrt{27}=3\sqrt{3}$

18. **(a)** $5x^3$ - $3x^2$ - $\sqrt{x}$ + 2

Explanation:

$5x^3$ - $3x^2$ - $\sqrt{x}$ + 2 is not a polynomial because each term of a polynomial should be a product of a constant and one or more variable raised to a positive, zero or integral power. Here $\sqrt{x}$ does not satisfy the condition of being a polynomial.

19. **(a)** 0

Explanation:

$x^3-8y^3-36xy-216$

Putting x = 2y + 6,

$(2y+6)^3-8y^3-36\,(2y+6)\,y-216$

= $8y^3+216+3\times2y\times6\,(2y+6)-8y^3-36\,(2y+6)\,y-216$

= $8y^3+216+72y^2+216y-8y^3-72y^2-216y-216$

= 0

20.

(c) 50

Explanation:

x^{51} + 51

If x^{51} + 51 is divided by x + 1, then using remainder theorem

$(-1)^{51}$ + 51

= -1 + 51

= 50

21.

(d) A is false but R is true.

Explanation:

The highest power of x in the polynomial P(x) = $4x^3 - x^2 + 5x^4 + 3x - 2$ is 4

Therefore, the degree of the polynomial P(x) is 4

A is false but R is true.

22. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Relation is true as we know that Sum of zeroes = $-\frac{b}{a}$

$\Rightarrow \frac{-(-2k)}{1} = 2 \Rightarrow k = 1$

So, Assertion is true.

23.

(d) A is false but R is true.

Explanation:

A is false but R is true.

24.

(c) A is true but R is false.

Explanation:

Degree of zero polynomial is not defined.

25. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Maximum number of real zeroes of any polynomial is equal to its degree.

26.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation:

Assertion and reason both are correct statements but reason is not correct explanation for assertion.

27. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

28.

(d) A is false but R is true.

Explanation:

Number of zeroes of linear polynomial is always only.

29.

(d) A is false but R is true.

Explanation:

A is false but R is true.

30. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

31. Let k be added to $2x^4 - 5x^3 + 2x^2 - x - 3$ so that the result is exactly divisible by (x - 2). Here, k is a constant.

$\therefore$ f(x) = $2x^4 - 5x^3 + 2x^2 - x - 3 + k$ is exactly divisible by (x - 2)

Using factor theorem, we have

f(2) = 0

$\Rightarrow 2 \times 2^4 - 5 \times 2^3 + 2 \times 2^2 - 2 - 3 + k = 0$

$\Rightarrow 32 - 40 + 8 - 5 + k = 0$

$\Rightarrow -5 + k = 0$

$\Rightarrow k = 5$

32. $x^3 + x^2 + x + 1$

While applying the factor theorem, we get

$p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$ =-1+1-1+1

= 0

We conclude that on dividing the polynomial by $(x+1)$ we get the remainder as 0.

Therefore, we conclude that $(x+1)$ is a factor of $x^3 + x^2 + x + 1$

33. Given, f(x) = $x^3 - 6x^2 + 2x - 4$

By remainder theorem, when f(x) is divided by g(x) = 1 - 3x, the remainder is equal to f(1/3)

Now, f(x) =$x^3 - 6x^2 + 2x - 4$

$\Rightarrow f(\frac{1}{3}) = (\frac{1}{3})^3 - 6(\frac{1}{3})^2 + 2 \times \frac{1}{3} - 4 = \frac{1}{27} - \frac{6}{9} + \frac{2}{3} - 4 = \frac{1}{27} - \frac{2}{3} + \frac{2}{3} - 4 = \frac{-107}{27}$

Hence, the required remainder = $\frac{-107}{27}$

34. $27y^3 + 125z^3$

= $(3y)^3 + (5z)^3 = (3y + 5z)\{(3y)^2 - (3y)(5z) + (5z)^2\}$

= $(3y + 5z)(9y^2 - 15yz + 25z^2)$

35. p(x) = $2x^3 + x^2 - 15x - 12$, g(x) = x + 2

By remainder theorem, when $p(x)$ is divided by (x + 2), then the remainder = p(-2)

Putting x = -2 in p(x), we get

$p(-2) = 2 \times (-2)^3 + (-2)^2 - 15 \times (-2) - 12 = -16 + 4 + 30 - 12 = 6$

$\therefore$ Remainder = 6

36. $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ = $\left(z + \frac{3}{2}\right)\left(z - \frac{3}{2}\right)$ (Where $y^2 = z$)

= $(z)^2 - (\frac{3}{2})^2$

{Using Identity $a^2 - b^2 = (a - b)(a + b)$}

$(z)^2 - \frac{9}{4} = (y^2)^2 - \frac{9}{4}$

(Substituting the value of z)

= $y^4 - \frac{9}{4}$

37. $a^3b - a^2b + 5ab - 5b$

By taking a^2b common in the first term and 5b as common in the second term

= $a^2b(a - 1) + 5b(a - 1)$

So we get,

= $(a - 1)(a^2b + 5b)$

By taking b as common = $(a - 1)\, b(a^2 + 5)$

So we get,

= $b(a - 1)(a^2 + 5)$

This is the required factorisation.

38. Area : $35y^2 + 13y - 12$

The expression $35y^2 + 13y - 12$ can also be written as, $35y^2 + 28y - 15y - 12$

= $35y^2 + 28y - 15y - 12 = 7y(5y + 4) - 3(5y + 4)$

= $(7y - 3)(5y + 4)$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2 + 13y - 12$ is Length = 7y - 3 and Breadth = 5y + 4

39. On dividing(x + 3) by (x + 4), we get 1 as quotient and -1 as remainder.

$$\begin{array}{r|l} x+4 & x+3\ (1 \\ & \underline{x+4} \\ & \quad - \ - \\ & \quad -1 \end{array}$$

$\therefore$ we may write,

$\frac{x+3}{x+4} = 1 - \frac{1}{(x+4)}$ = 1 - (x + 4)$^{-1}$, which is not a polynomial as power of (x+4) is negative.

Hence, r(x) = $\frac{x+3}{x+4}$ is not a polynomial.

40. Finding a zero of p(x), is the same as solving the equation p(x) = 0

Now, 2x + 1 = 0 gives us $x = -\frac{1}{2}$

So, $-\frac{1}{2}$ is a zero of the polynomial 2x + 1

41. We have, g(x) = 2x + 1 which give $x = -\frac{1}{2}$

Remainder of $p\left(-\frac{1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 - 11\left(\frac{-1}{2}\right)^2 - 4\left(\frac{-1}{2}\right) + 5$

$= 2\left(\frac{-1}{8}\right) - 11\left(\frac{1}{4}\right) + 2 + 5 = \frac{-1}{4} - \frac{11}{4} + 7$

$= \frac{-1-11+28}{4} = \frac{16}{4} = 4$

Since remainder $\neq$ 0, therefore, p(x) is not a multiple of g(x).

42. Let p(x) = $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

The zero of x + 1 is -1

p(-1) = $(-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} = -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2 + 2^{\sqrt{2}} \neq 0$

$\therefore$ By factor theorem, x + 1 is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

43. i. 6

The maximum exponent of the variable is 6. Therefore, the degree is 6.

ii. Given that: $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$

$= \frac{x^3}{5} + \frac{2x+1}{5} - \frac{7}{2}x^2 - x^6$

The coefficient of x^3 is $\frac{1}{5}$.

iii. -1

The coefficient of x^6 is -1.

iv. Given that: $\frac{x^3+2x+1}{5} - \frac{7}{2}x^2 - x^6$

$= \frac{x^3}{5} + \frac{2x}{5} + \frac{1}{5} - \frac{7}{2}x^2 - x^6$

Therefore, the constant term is $\frac{1}{5}$.

44. g(x) = 0 $\Rightarrow$ x - 2 = 0 $\Rightarrow$ x = 2

By the remainder theorem, we know that when p(x) is divided by (x - 2) then the remainder is p(2)

Now, p(2) = $(2^4 + 2 \times 2^3 - 3 \times 2^2 + 2 - 1)$

= (16 +16 - 12 + 2 - 1) = 21

Therefore, the remainder is 21.

45. $p\left(x\right) = 3x + 1, x = -\frac{1}{3}$

We need to check whether $p\left(x\right) = 3x + 1$ at $x = -\frac{1}{3}$ is equal to zero or not.

$p\left(-\frac{1}{3}\right) = 3x + 1 = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$

Therefore, we can conclude that $x = -\frac{1}{3}$ is a zero of the polynomial $p\left(x\right) = 3x + 1$

Section C

46. Let p(y) = $2y^3 + y^2 - 2y - 1$

p(1) = $2(1)^3 + (1)^2 - 2(1) - 1$

= 2 + 1 - 2 - 1 = 0

$\therefore$ By Factor Theorem, (y - 1) is a factor of p(y).

$2y^3 + y^2 - 2y - 1$

= $(y - 1)(2y^2 + 3y + 1) = (y - 1)(2y^2 + 2y + y + 1)$

= (y - 1){2y(y + 1) + 1(y + 1)} = (y - 1)(y + 1)(2y + 1)

47. Here, $f(x) = 3x^3 + x^2 - 20x + 12$

$g(x) = 3x - 2$

Now, $3x-2=0$

$\Rightarrow x=\frac{2}{3}$

Therefore,to show g(x) is a factor of f(x),we must have,

$f(\frac{2}{3}) = 0$

Substitute the value of x in f(x)

$f(\frac{2}{3}) = 3(\frac{2}{3})^3 + (\frac{2}{3})^2 - 20(\frac{2}{3}) + 12$

$= 3(\frac{8}{27}) + \frac{4}{9} - \frac{40}{3} + 12$

$= \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$

$= \frac{12}{9} - \frac{40}{3} + 12$

Taking L.C.M

$= \frac{12-120+108}{9}$

$= \frac{120-120}{9}$

$= 0$

Since, the result is 0 g(x) is the factor of f(x)

48. Let $p(x) = x^3 + mx^2 - nx + 10$

x – 1 and x – 2 exactly divide p(x)

$\therefore p(1) = 0 \; and \; p(2) = 0$

$p(1) = 1^3 + m \times 1^2 - n \times 1 + 10 = 0$

1+m-n+10=0

m-n+11=0

m-n= -11 -----(1)

$p(2) = 2^3 + m \times 2^2 - n \times 2 + 10 = 0$

8+4m-2n+10=0

4m-2n=-18

2m-n=-9 ----{dividing by 2}

Subtracting eq. (2) form (1). We get

-m=-2

m=2

Putting m = 2 in eq. (1). We get

2-n=-11

-n=-11-2

+n=+13

Therefore, n=13 and m=2

49. $x^3 - 3x^2 - 9x - 5$

We need to consider the factors of -5 which are $\pm 1, \pm 5$

Let us substitute 1 in the polynomial $x^3 - 3x^2 - 9x - 5$,to get

$(-1)^3 - 3(-1)^2 - 9(-1) - 5 = - 1 - 3 + 9 - 5 = 0$

Thus, according to factor theorem, we can conclude that $(x + 1)$ is a factor of the polynomial $x^3 - 3x^2 - 9x - 5$

$$
\begin{array}{r|l}
 & x^2 - 4x - 5 \\
x+1 & x^3 - 3x^2 - 9x - 5 \\
 & \underline{x^3 + x^2} \\
 & \quad\ - \quad -\\
 & -4x^2 - 9x \\
 & \underline{-4x^2 - 4x} \\
 & \quad + \quad\ + \\
 & -5x - 5 \\
 & \underline{-5x - 5} \\
 & \quad + \quad\ + \\
 & \underline{\quad 0 \quad}
\end{array}
$$

$x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5)$

$= (x + 1)(x^2 + x - 5x - 5)$

$= (x + 1)[x(x + 1) - 5(x + 1)]$

$= (x + 1)(x - 5)(x + 1)$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 3x^2 - 9x - 5$, we get $(x + 1)(x - 5)(x + 1)$

50. The given expression may be rewritten as,

$(x + 2 + x - 2)((x + 2)^2 - (x + 2)(x - 2) + (x - 2)^2)$

$\therefore [a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$

$= 2x(x^2 + 4x + 4 - (x + 2)(x - 2) + x^2 - 4x + 4)$

$= 2x(2x^2 + 8 - (x^2 - 2^2))$

$[\therefore (a + b)(a - b) = a^2 - b^2]$

$= 2x(2x^2 + 8 - x^2 + 4)$

$= 2x(x^2 + 12)$

$\therefore (x + 2)^3 + (x - 2)^3 = 2x(x^2 + 12)$

51. $p(x) = kx^2 - 3x + k$

We know that according to the factor theorem

$p(a) = 0$, if $x - a$ is a factor of $p(x)$

We conclude that if $(x - 1)$ is a factor of $p(x) = kx^2 - 3x + k$ then $p(1) = 0$

$p(1) = k(1)^2 - 3(1) + k$, or $2k - 3 = 0 \Rightarrow k = \frac{3}{2}$

Therefore, we can conclude that the value of k is $\frac{3}{2}$.

52. Let $a = \frac{1}{2}, b = \frac{1}{3}, c = -\frac{5}{6}$

$\therefore\ a + b + c = \frac{1}{2} + \frac{1}{3} - \frac{5}{6}$

$= \frac{3+2-5}{6} = \frac{0}{6} = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$\therefore\ \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(-\frac{5}{6}\right)^3$

$= 3 \times \frac{1}{2} \times \frac{1}{3}\left(-\frac{5}{6}\right) = -\frac{5}{12}$

53. $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as $= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$

$= (2a)^3 - (b)^3 - 3 \times 2a \times b\,(2a - b)$.

Using identity $(x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$ with respect to the expression

$(2a)^3 - (b)^3 - 3 \times 2a \times b\,(2a - b)$, we get $(2a - b)^3$

Therefore, after factorizing the expression $8a^3 - b^3 - 12a^2b + 6ab^2, we get (2a - b)^3$

54. $\frac{3}{2}x^2 - x - \frac{4}{3}$

Here $\frac{3}{2} \times \frac{-4}{3} = -2$

We factories by splitting middle term $-2 + 1 = -1$

$\frac{3}{2}x^2 - 2x + x - \frac{4}{3} = \frac{3}{2}x\left(x - \frac{4}{3}\right) + 1\left(x - \frac{4}{3}\right) = \left(\frac{3}{2}x + 1\right)\left(x - \frac{4}{3}\right)$

55. $27-125a^3-135a+225a^2$

The expression $27-125a^3-135a+225a^2$ can be written as

$=(3)^3-(5a)^3-3\times3\times3\times5a+3\times3\times5a\times5a$

$=(3)^3-(5a)^3+3\times3\times5a(3-5a)$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$

$(3)^3-(5a)^3+3\times3\times5a(3-5a), weget(3-5a)^3$

Therefore, after factorizing the expression $27-125a^3-135a+225a^2, weget(3-5a)^3$

56. Given, p(x) = $2x^3 - 9x^2 + x + 12$

Now, $2x-3=0$

$\Rightarrow x=\frac{3}{2}$

Now, $p\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^3-9\left(\frac{3}{2}\right)^2+\frac{3}{2}+12$

$=2\times\frac{27}{8}-9\times\frac{9}{4}+\frac{3}{2}+12$

$=\frac{54}{8}-\frac{81}{4}+\frac{3}{2}+12$

$=\frac{54-162+12+96}{8}$

$=\frac{162-162}{8}$

$=\frac{0}{8}$

$p\left(\frac{3}{2}\right)=0$

As (2x - 3) divides p(x) completely, therefore p(x) is a multiple of (2x -3).

57. $\left(\frac{1}{2}a-\frac{1}{3}b+1\right)^2$

Using identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yx+2zx$

$\left(\frac{1}{2}a-\frac{1}{3}b+1\right)^2 = \left(\frac{1}{2}a\right)^2+\left(\frac{-1}{3}b\right)^2+(1)^2+2\times\frac{1}{2}a\times\frac{-1}{3}b+2\times\frac{-1}{3}b\times1+2\times1\times\frac{1}{2}a$

$=\frac{a^2}{4}+\frac{b^2}{9}+1-\frac{ab}{3}-\frac{2b}{3}+a$

58. Let f(x) = $px^2 + 5x + r$ be the given polynomial. Since x - 2 and $x-\frac{1}{2}$ are factors of f(x).

$\therefore$ f(2) = 0 and $f\left(\frac{1}{2}\right)$ = 0

$\Rightarrow$ p $\times$ 2^2 + 5 $\times$ 2 + r = 0 and p$(\frac{1}{2})^2$ + 5 $\times\frac{1}{2}$ + r = 0

$\Rightarrow$ 4p + 10 + r = 0 and $\frac{p}{4}+\frac{5}{2}$ + r = 0

$\Rightarrow$ 4p + r = -10 and $\frac{p+4r+10}{4}=0$

$\Rightarrow$ 4p + r = -10 and p + 4r + 10 = 0

$\Rightarrow$ 4p + r = -10 and p + 4r = -10

$\Rightarrow$ 4p + r = p + 4r [RHS of the two equations are equal]

$\Rightarrow$ 3p = 3r $\Rightarrow$ p = r

59. We have,

$(2x - 5y)^3 - (2x + 5y)^3$

$= \{(2x - 5y) - (2x + 5y)\}\{(2x - 5y)^2 + (2x - 5y)(2x + 5y) + (2x + 5y)^2\}$ [$\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= (2x - 5y - 2x - 5y)(4x^2 + 25y^2 - 20xy + 4x^2 - 25y^2 + 4x^2 + 25y^2 + 20xy)$

$= (-10y)(2x^2 + 25y^2)$

$= - 120x^2y - 250y^3$

60. given $(a+b)^3=a^3+b^3+3ab(a+b)$

$\therefore a^3+b^3=(a+b)^3-3ab(a+b)$

$=(a+b)[(a+b)^2-3ab]$

$=(a+b)[a^2+b^2+2ab-3ab]$

$=(a+b)(a^2+b^2-ab)$

$=(a+b)(a^2-ab+b^2)$

Section D

61. The given polynomial is,

$f(x) = x^4 + ax^3 - 7x^2 - 8x + b$

Now, x + 2 = 0 $\Rightarrow$ x = -2

By the factor theorem, we can say: f(x) will be exactly divisible by (x + 2) if f(-2) = 0

Therefore, we have:

$f(-2) = [(-2)^4 + a \times (-2)^3 - 7 \times (-2)^2 - 8 \times (-2) + b]$

$= (16 - 8a - 28 + 16 + b)$

$= (4 - 8a + b)$

$\therefore f(-2) = 0 \Rightarrow 8a - b = 4$ ...(i)

Also, $x + 3 = 0 \Rightarrow x = -3$

By the factor theorem, we can say: f(x) will be exactly divisible by (x + 3) if f(-3) = 0

Therefore, we have:

$f(-3) = [(-3)^4 + a \times (-3)^3 - 7 \times (-3)^2 - 8 \times (-3) + b]$

$= (81 - 27a - 63 + 24 + b)$

$= (42 - 27a + b)$

$\therefore f(-3) = 0 \Rightarrow 27a - b = 42$ ...(ii)

Subtracting (i) from (ii), we have:

$\Rightarrow 19a = 38$

$\Rightarrow a = 2$

Putting the value of a, we get the value of b, i.e., 12

$\therefore a = 2$ and $b = 12$

62. If $g(x) = x^2 + 2x + k$ is a factor of $f(x) = 2x^4 + x^3 - 14x^2 + 5x + 6$, then remainder is zero when f(x) is divided by g(x).

Let quotient = Q and remainder = R

Let us now divide f(x) by g(x).

$$x^2 + 2x + k \,\big)\, 2x^4 + x^3 - 14x^2 + 5x + 6 \,\big(\, 2x^2 - 3x - 2(k+4)$$

$$2x^4 + 4x^3 + 2kx^2$$

$$- \quad - \quad -$$

$$-3x^3 - 2x^2(k+7) + 5x + 6$$

$$-3x^3 - 6x^2 \quad - 3kx$$

$$+ \quad + \quad +$$

$$-2x^2(k+4) + x(5+3k) + 6$$

$$-2x^2(k+4) - 4x(k+4) - 2k(k+4)$$

$$+ \quad + \quad +$$

$$x(7k+21) + (2k^2 + 8k + 6)$$

$R = x(7k + 21) + (2k^2 + 8k + 6)$ -------(1) and $Q = 2x^2 - 3x - 2(k + 4)$.------------(2)

Now, R = 0.

$\Rightarrow x(7k + 21) + 2(k^2 + 4k + 3) = 0$

$\Rightarrow 7x(k + 3) + 2(k+1)(k+3) = 0$

$\Rightarrow (k+3)[7x + 2(k+1)] = 0$

$\Rightarrow k + 3 = 0$

$\Rightarrow k = -3$

Thus, polynomial f(x) can be written as,

$2x^4 + x^3 - 14x^2 + 5x + 6 = (x^2 + 2x + k)[2x^2 - 3x - 2(k + 4)] = (x^2 + 2x - 3)(2x^2 - 3x - 2)$

Zeros of $x^2 + 2x - 3$ are,

$x^2 + 2x - 3 = 0$

$\Rightarrow (x + 3)(x - 1) = 0$

$\Rightarrow x = -3$ or $x = 1$

Zeros of $(2x^2 - 3x - 2)$ are,

$2x^2 - 3x - 2 = 0$

$\Rightarrow 2x^2 - 4x + x - 2 = 0$
$\Rightarrow 2x(x - 2) + 1(x - 2) = 0$
$\Rightarrow (x - 2)(2x + 1) = 0$
$x = 2$ or $x = -\frac{1}{2}$
Thus, the zeros of f(x) are: -3 ,1, 2 and $-\frac{1}{2}$

63. Given, $f(x) = x^3 + 2x^2 – x – 2$
The constant term in f(x) is -2
The factors of (-2) are $\pm 1, \pm 2$
Let, $x – 1 = 0$
$\Rightarrow x = 1$
Substitute the value of x in f(x)
$f(1) = (1)^3 + 2(1)^2 – 1 – 2$
$= 1 + 2 – 1 – 2$
$= 0$
Similarly, (x + 1) and (x + 2) are factors of f(x).
Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.
$\therefore f(x) = k(x – 1)(x + 2)(x + 1)$
$x^3 + 2x^2 – x – 2 = k(x – 1)(x + 2)(x + 1)$
Substitute x = 0 on both the sides
$0 + 0 – 0 – 2 = k(-1)(1)(2)$
$\Rightarrow – 2 = - 2k$
$\Rightarrow k = 1$
Substitute k value in $f(x) = k(x – 1)(x + 2)(x + 1)$
$f(x) = (1)(x – 1)(x + 2)(x + 1)$
$\Rightarrow f(x) = (x – 1)(x + 2)(x + 1)$
So, $x^3 + 2x^2 – x – 2 = (x – 1)(x + 2)(x + 1)$
This is the required factorisation of f(x).

64. Let $f(x) = (2x^3 + x^2 - ax + 2)$ and $g(x) = (2x^3 - 3x^2 - 3x + a)$
By remainder theorem, when f(x) is divided by (x - 2), then the remainder = f(2)
Putting x = 2 in f(x), we have,
$f(2) = 2 \times 2^3 + 2^2 - a \times 2 + 2 = 16 + 4 - 2a + 2 = -2a + 22$
By remainder theorem, when g(x) is divided by (x - 2), then the remainder = g(2)
Putting x = 2 in g(x), we have,
$g(2) = 2 \times 2^3 - 3 \times 2^2 - 3 \times 2 + a = 16 - 12 - 6 + a = -2 + a$
We have,
$f(2) = g(2)$
$\Rightarrow -2a + 22 = -2 + a$
$\Rightarrow -3a = -24$
$\Rightarrow a = 8$
Therefore, the value of a is 8.

65. Let, $p(x) = x^3 - 6x^2 - 15x + 80$
$q(x) = x^2 + x - 12$
by division algorithm, when p(x) is divided by q(x) the remainder is a linear expression in x.
so, let r(x) = ax + b is subtracted from p(x), so that p(x) - r(x) is divisible by q(x)
let f(x) = p(x) - r(x)
$q(x) = x^2 + x - 12$
$= x^2 + 4x - 3x - 12$
$= x(x + 4)(-3)(x + 4)$
$= (x + 4) (x - 3)$
clearly, (x - 3) and (x + 4) are factors of q(x)
Therefore, f(x) will be divisible by q(x) if (x - 3) and (x + 4) are factors of f(x)

From factor theorem,we must have,

f(-4) = 0 and f(3) = 0

$\Rightarrow$ f(3) = 33 - 6(3)^2 - 3(a + 15) + 80 - b = 0

=> 27 - 54 - 3a - 45 + 80 - b=0

=> -3a - b + 80 ....(1)

Similarly,we have,

f(- 4) = 0

$\Rightarrow$ (- 4)^3 - 6(- 4)^2 - (- 4)(a + 15) + 80 - b = 0

$\Rightarrow$ - 64 - 96 - 4a + 60 + 80 - b = 0

$\Rightarrow$ 4a - b - 20 = 0 .... (2)

Substract eq 1 and 2,we have,

$\Rightarrow$ 4a - b - 20 - 8 + 3a + b = 0

$\Rightarrow$ 7a - 28 = 0

$\Rightarrow$ a = 28/7

$\Rightarrow$ a = 4

Put a = 4 in (1)

$\Rightarrow$ - 3(4) - b = - 8

$\Rightarrow$ - b - 12 = - 8

$\Rightarrow$ - b = - 8 + 12

$\Rightarrow$ b = - 4

Substitute a and b values in r(x)

$\Rightarrow$ r(x) = ax + b

= 4x - 4

Hence, p(x) is divisible by q(x), if r(x) = 4x - 4 is subtracted from it

66. By long division, we have

$$
\begin{array}{l}
x-2\,\big)\,3x^4-4x^3+0x^2-3x-1\,\big(\,3x^3+2x^2+4x+5 \\
\quad 3x^4-6x^3 \\
\quad - \quad + \\
\hline
\qquad 2x^3+0x^2-3x-1 \\
\qquad 2x^3-4x^2 \\
\qquad - \quad + \\
\hline
\qquad\quad 4x^2-3x-1 \\
\qquad\quad 4x^2-8x \\
\qquad\quad - \quad + \\
\hline
\qquad\qquad 5x-1 \\
\qquad\qquad 5x-10 \\
\qquad\qquad - \quad + \\
\hline
\qquad\qquad\quad 9
\end{array}
$$

$\therefore$ quotient, q(x) = 3x^3 + 2x^2 + 4x + 5 and remainder, r(x) = 9

Now, g(x) $\times$ q(x) + r(x) = (x - 2) (3x^3 + 2x^2 + 4x + 5) + 9

= 3x^4 - 4x^3 - 3x - 10 + 9

= 3x^4 - 4x^3 - 3x - 1 = p(x)

$\therefore$ p(x) = g(x) $\times$ q(X) + r(x), where degree r(x) = 0 < 1 = degree g(x)

Therefore, division algorithm is verified.

Also, p(2) = 3 $\times$ 24 - 4 $\times$ 2^3 - 3 $\times$ 2 - 1 = 48 - 32 - 6 - 1 = 9

Since p(2) is not zero, therefore 2 is not a zero of p(x).

67. Given expression can be written as

$\left[\frac{1}{3}(2x+5y)\right]^3+\left[-\frac{5}{3}y+\frac{3}{4}z\right]^3+\left[-\frac{3}{4}z-\frac{2}{3}x\right]^3$

$Let\frac{1}{3}(2x+5y)=a, \frac{-5}{3}y+\frac{3}{4}z=b$

$and\frac{-3}{4}z-\frac{2}{3}x=c$

Here a + b + c = 0, then

$a^3+b^3+c^3=3abc$

Thus,

$\frac{1}{27}(2x+5y)^3+\left(\frac{-5}{3}y+\frac{3}{4}z\right)^3-\left(\frac{3}{4}z+\frac{2}{3}x\right)^3$

$=3\left[\frac{1}{3}(2x+5y)\left(\frac{-5}{3}y+\frac{3}{4}z\right)\left(\frac{-3}{4}z-\frac{2}{3}x\right)\right]$

$=-(2x+5y)\left(\frac{-5}{3}y+\frac{3}{4}z\right)\left(\frac{3}{4}z+\frac{2}{3}x\right)$

$=-(2x+5y)\left(\frac{-20y+9z}{12}\right)\left(\frac{9z+8x}{12}\right)$

$=\frac{1}{144}(2x+5y)(20y-9z)(9z+8x)$

68. Let p(x) = $ax^3 + bx^2 - 5x + 2$, g(x) = x + 2 and h(x) = x - 2. Then, g(x) = 0 ⇒ x + 2 = 0 ⇒ x = -2

h(x) = 0 ⇒ x - 2 = 0 ⇒ x = 2

(x + 2) is a factor of p(x) ⇒ p(-2) = 0

Now ,p(-2) = 0 ⇒ $a \times (-2)^3 + b \times (-2)^2 - 5 \times (-2) + 2) = 0$

⇒ -8a + 4b +12 = 0

⇒ 8a - 4b = 12 ⇒ 2a - b = 3 ...(i)

When p(x) is divided by (x - 2), then the remainder is p(2)

∴ p(2) = 12 ⇒ $(a \times 2^3) + (b \times 2^2) - (5 \times 2) + 2) = 12$

⇒ 8a + 4b = 20 ⇒ 2a + b = 5 ...(ii)

On solving (i) and (ii), we get a = 2 and b = 1

69. Here, f(x) = $x^3 - 6x^2 - 19x + 84$

To prove that (x + 4), (x - 3) and (x - 7) are factors of f(x),

We must have f(- 4) , f(3) and f(7) should be zero

Let, x + 4 = 0

⇒ x = - 4

Substitute the value of x in f(x)

f(-4) = $(-4)^3 - 6(-4)^2 - 19(-4) + 84$

= -64 - (6 × 16) - (19 × (- 4)) + 84

= - 64 - 96 + 76 + 84

= 160 - 160

= 0

Let, x - 3 = 0

⇒ x = 3

Substitute the value of x in f(x)

f(3) = $(3)^3 - 6(3)^2 - 19(3) + 84$

= 27 – (6 × 9) – (19 × 3) + 84

= 27 – 54 – 57 + 84

= 111 – 111

= 0

Let, x - 7 = 0

⇒ x = 7

Substitute the value of x in f(x)

f(7) = $(7)^3 - 6(7)^2 - 19(7) + 84$

= 343 – (6 × 49) – (19 × 7) + 84

= 343 – 294 - 133 + 84

= 427 – 427

= 0

Hence, (x+4),(x-3) and (x-7) are factors of f(x).

70. Let, f(x) = $x^3 - 6x^2 + 3x + 10$

The constant term in f(x) is 10

The factors of 10 are ± 1, ± 2, ± 5, ± 10

Let, x + 1 = 0

⇒ x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^3$- $6(-1)^2$ + $3(-1)$ + 10

= -1 – 6 – 3 + 10

= 0

Similarly, (x – 2) and (x – 5) are other factors of f(x)

Since, f(x) is a polynomial having a degree 3, it cannot have more than three linear factors.

∴ f(x) = k(x + 1)(x – 2)(x – 5)

Substitute x = 0 on both sides

⇒ x^3– $6x^2$ + 3x + 10 = k(x + 1)(x – 2)(x – 5)

⇒ 0 – 0 + 0 + 10 = k(1)(-2)(-5)

⇒ 10 = k(10)

⇒ k = 1

Substitute k = 1 in f(x) = k(x + 1)(x – 2)(x – 5)

f(x) = (1)(x + 1)(x – 2)(x – 5)

so, x^3 – $6x^2$ + 3x + 10 = (x + 1)(x – 2)(x – 5)

This is the required factorisation of f(x)

71. As we know,

$a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$

$=(a+b+c)\left[a^2+b^2+c^2-(ab+bc+ca)\right]$

$=5\left\{a^2+b^2+c^2-(ab+bc+ca)\right\}$

$=5\left(a^2+b^2+c^2-10\right)$

Now, $a+b+c=5$

Squaring both sides, we get

$(a+b+c)^2=5^2$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=25$

$\therefore a^2+b^2+c^2+2(10)=25$

$\Rightarrow a^2+b^2+c^2=25-20=5$

Now, $a^3+b^3+c^3-3abc=5\left(a^2+b^2+c^2-10\right)$

$=5(5-10)=5(-5)=-25$

Hence, proved.

72. Here, f(x) = x^3 − $3x^2$ − 10x + 24

To prove that (x - 2), (x + 3) and (x - 4) are factors of f(x),

We must have f(2), f(-3) and f(4) should be zero

Let, x - 2 = 0

⇒ x = 2

Substitute the value of x in f(x)

f(2) = 2^3 - $3(2)^2$ - 10(2) + 24

= 8 - (3 × 4) - 20 + 24

= 8 - 12 - 20 + 24

= 32 - 32

= 0

Let, x + 3 = 0

⇒ x = -3

Substitute the value of x in f(x)

f(-3) = $(-3)^3$ - $3(-3)^2$ - $10(-3)$ + 24

= -27 - 3(9) + 30 + 24

= -27 - 27 + 30 + 24

= 54 - 54

= 0

Let, x - 4 = 0

⇒ x = 4

Substitute the value of x in f(x)

$f(4) = (4)^3 - 3(4)^2 - 10(4) + 24$
$= 64 - (3 \times 16) - 40 + 24$
$= 64 - 48 - 40 + 24$
$= 84 - 84$
$= 0$
Hence, (x-2),(x+3) and (x-4) are factors of f(x).

73. The given polynomials are,
$f(x) = 2x^3 + ax^2 + 3x - 5$
$p(x) = x^3 + x^2 - 4x + a$
The remainders are f(2) and p(2) when f(x) and p(x) are divided by x - 2
We have,
f(2) = p(2) (given in problem)
we need to calculate f(2) and p(2)
for, f(2)
substitute (x = 2) in f(x)
$f(2) = 2(2)^3 + a(2)^2 + 3(2) - 5$
$= (2 \times 8) + 4a + 6 - 5$
$= 16 + 4a + 1$
$= 4a + 17$ ....(1)
for, p(2)
substitute (x = 2) in p(x)
$p(2) = 2^3 + 2^2 - 4(2) + a$
$= 8 + 4 - 8 + a$
$= 4 + a$ ...(2)
Since, f(2) = p(2)
Equate eqn 1 and 2
$\Longrightarrow 4a + 17 = 4 + a$
$\Longrightarrow 4a - a = 4 - 17$
$\Longrightarrow 3a = -13$
$\Longrightarrow a = -13/3$

74. We have, $f(x) = 2x^3 - 3x^2 + ax + b$
Zeros of f(x) are 0 and -1
Substitute x = 0 in f(x),we get,
$f(0) = 2(0)^3 - 3(0)^2 + a(0) + b$
$= 0 - 0 + 0 + b$
$= b$ .... (1)
Substitute x = (-1) in f(x),we have,
$f(-1) = 2(-1)^3 - 3(-1)^2 + a(-1) + b$
$= -2 - 3 - a + b$
$= -5 - a + b$ .....(2)
We need to equate equations 1 and 2 to zero
$b = 0$ and $-5 - a + b = 0$
since, the value of b is zero
substitute b = 0 in equation 2
$\Longrightarrow -5 - a = -b$
$\Longrightarrow -5 - a = 0$
$a = -5$

75. Given, that $f(x) = x^3 + 6x^2 + 11x + 6$
Clearly we can say that, the polynomial f(x) with an integer coefficient and the highest degree term coefficient which is known as leading factor is 1.
So, the roots of f(x) are limited to integer factor of 6, they are $\pm 1, \pm 2, \pm 3, \pm 6$

Let x = -1

f(-1) = $(-1)^3 + 6(-1)^2 + 11(-1) + 6$

= -1 + 6 -11 + 6

= 0

Let x = – 2

f(-2) = $(-2)^3 + 6(-2)^2 + 11(-2) + 6$

= – 8 – (6 * 4) - 22 + 6

= – 8 + 24 - 22 + 6

= 0

Let x = – 3

f(-3) = $(-3)^3 + 6(-3)^2 + 11(-3) + 6$

= – 27 – (6 × 9) - 33 + 6

= – 27 + 54 - 33 + 6

= 0

But from all the given factors only -1, -2, -3 gives the result as zero.Furher,since f(x) is a polynomial of degree 3, therefore,it has almost 3 roots.

Therefore, the integral roots of $x^3 + 6x^2 + 11x + 6$ are -1, -2, -3.