Solution

POLYNOMIALS

Class 10 - Mathematics

Section A

1.

(c) 1

Explanation:

Since α and β are the zeros of quadratic polynomial

$f(x) = x^2 - p(x + 1) - c$

or $f(x) = x^2 - px - p - c$

$\alpha + \beta = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^2} = -\left(\frac{-p}{1}\right)$ = p

$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-p-c}{1}$ = -p-c

We have

$0 = (\alpha + 1)(\beta + 1)$

$0 = \alpha\beta + (\alpha + \beta) + 1$

0 = -c+1

c = 1

Therefore, value of c is1.

2.

(c) 0

Explanation:

p(-2) = $(-2)^2$ + 5(-2) + 6

p(-2) = 4 - 10 + 6

= 0

3. **(a)** $\frac{-b}{a}$

Explanation:

Let $\alpha,\ \beta$ are the zeroes of the given polynomial.

Given: $\alpha = 0\ \therefore \alpha + \beta = \frac{-b}{a} \Rightarrow 0 + \beta = \frac{-b}{a} \Rightarrow \beta = \frac{-b}{a}$

Therefore the other zero is $\frac{-b}{a}$.

4. **(a)** has no linear term and the constant term is negative.

Explanation:

Given quadratic polynomial is, $p(x) = x^2 + ax + b$

Let one of the zero of the polynomial be 'a' then the other zero will be −a.

Sum of zeros = $a + (-a) = \frac{-a}{1}$

$\Rightarrow 0 = \frac{-a}{1} \Rightarrow a = 0$

and product of the zeros $a(-a) = \frac{b}{1}$

$\Rightarrow b = -a^2$

$\therefore$ Given polynomial has no linear term and constant term 'b' has to be negative.

5. **(a)** 5

Explanation:

The Given polynomial is $f(x) = 5x^2 + 13x + k$.

Product of roots = k/5

$1 = \frac{k}{5}$

$\Rightarrow$ k = 5

6. **(a)** infinite

Explanation:

Let α = -1 and β = 2

$\alpha + \beta$ = -1 + 2 = 1

$\alpha\beta$ = (-1) (2) = -2

Required polynomial = k(x^2 - $(\alpha + \beta)$x + $\alpha\beta$)

= k(x^2 - x - 2)

There are infinite values of k is possible.

Hence, number of Polynomial whose zeroes are -1 and 2 are infinite.

7.

(c) x^2 - x - 12

Explanation:

A quadratic polynomial is always in the form of

x^2 - (sum of zeros)x + (product of Zeros)

hence the required polynomial is

x^2 - (1)x + (-12)

= x^2 - x - 12

8.

(c) no real zeroes

Explanation:

The polynomial $9x^2$ + 6x + 4 has no real zeroes because it can not be factorized.

D = b^2-4ac, D = 36 - 4 $\times$ 9 $\times$ 4 = -108

D < 0, roots are imaginary and unequal

9.

(b) more than 3

Explanation:

Let the zeroes of the polynomial be α and β

then, α = -5 and β = -3

The general form of the polynomial with zeroes α and β is given by:

k[x^2 - $(\alpha + \beta)$x + $\alpha\beta$0, where k is any real number

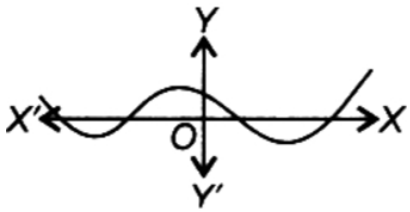
= k[x^2 - (-5 - 3)x + (-5)(-3)]

= k[x^2 + 8x + 15]

Hence, more than 3 polynomials can have zeroes the -3 and -5.

10.

(c)



Explanation:

For more than three distinct real roots the graph must cut x-axis at least four times.

11. **(a)** -36

Explanation:

p(-4) = 0(since - 4 is root of p(x))

$(-4)^2$ - 5(-4) + k = 0

$\Rightarrow$ 16 + 20 + k = 0

36 + k = 0

k = -36

12.

(c) $\frac{-b}{c}$

Explanation:

Explanation here: Here, $\frac{1}{\alpha}+\frac{1}{\beta}$

$=\frac{\alpha+\beta}{\alpha\beta}$

$=\frac{\frac{-b}{a}}{\frac{c}{a}}=\frac{-b}{c}$

13.

(d) $\frac{2}{3}, \frac{-1}{7}$

Explanation:

$7x^2-\frac{11x}{3}-\frac{2}{3}=\frac{21x^2-11x-2}{3}$

Now, $21x^2$ - 11x - 2 = $21x^2$ - 14x + 3x - 2

= 7x(3x - 2) + (3x - 2) = (3x - 2)(7x + 1)

$\therefore$ the zeros are $\frac{2}{3}, \frac{-1}{7}$

14.

(b) 10

Explanation:

p(x) = $-x^2$ + 8x + 9

finding zero of p(x)

$-x^2$ + 9x - x + 9

= -x(x - 9) -1(x - 9)

= (x - 9)(-x - 1)

$x=\underset{\alpha}{\uparrow}9 \quad x=\underset{\beta}{\uparrow}-1$

Now,

$\alpha-\beta$ = 9 - (-1)

= 9 + 1 = 10

15.

(b) $\frac{3}{2}$, -1

Explanation:

$2x^2$ - x - 3

$2x^2$ - 3x + 2x - 3

x(2x - 3) + 1 (2x - 3)

(2x - 3)(x + 1)

Zeroes are $\frac{3}{2}$ and -1

16. **(a)** more than 3

Explanation:

Since, 1 and -2

Sum of Roots = 1+(-2) = -1

Product of roots = (1) (-2) = -2

Therefore, the polynomial (p(x)) is: [p(x) =K[x^2 - (sum of roots)x + product of roots]

p(x) = K [x^2 - (-1)x+ (-2)]

Therefore, There are infinitely many polynomials that can have (1) and (-2) as their zeros. We can multiply or divide the polynomial by any nonzero constant(k), and the zeros will remain the same. So, the required number of polynomials is **infinite**!

17.

(d) $\frac{5}{4}$

Explanation:

$P(x) = 2x^2 - x - 1$

$\alpha + \beta = -(\frac{-1}{2}) = \frac{1}{2}$

$\alpha\beta = \frac{-1}{2}$

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$(\frac{1}{2})^2 = \alpha^2 + \beta^2 + 2(\frac{-1}{2})$

$\frac{1}{4} + 1 = \alpha^2 + \beta^2$

$\alpha^2 + \beta^2 = \frac{5}{4}$

18.

(b) $\frac{1}{3}$, -4

Explanation:

Let $f(x) = 3x^2 + 11x - 4$

$f(x) = 3x^2 + 12x - x - 4$

$f(x) = 3x(x + 4) - 1 (x + 4)$

$f(x) = (x + 4)(3x - 1)$

Put both the factors equal to zero.

$x + 4 = 0, x = -4$

$3x - 1 = 0, x = \frac{1}{3}$

The zeroes of the polynomial $3x^2 + 11x - 4$ are $\frac{1}{3}$ and - 4.

19.

(c) $\frac{-3}{2}, \frac{4}{3}$

Explanation:

$x^2 + \frac{1}{6}x - 2 = \frac{6x^2+x-12}{6}$

$6x^2 + x - 12 = 6x^2 + 9x - 8x - 12 = 3x(2x + 3) - 4(2x + 3)$

$= (2x + 3)(3x - 4)$

$\therefore$ the zeros are $\frac{-3}{2}$ and $\frac{4}{3}$

20.

(b) $x^2 - 3x - 10$

Explanation:

Given, sum and the product of the zeros of a quadratic polynomial are 3 and -10 respectively

i,e. $(\alpha + \beta) = 3$ and $\alpha\beta = -10$

Therefore, required polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 3x - 10$

21.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Reason is true as we know that Sum of zeroes = $-\frac{b}{a}$

Also, we know that Product of zeroes = $\frac{c}{a}$

$\Rightarrow \frac{5k}{1} = -10 \Rightarrow k = -2$

So, the Assertion is true. But Reason is not the correct explanation of assertion.

22.

(d) Assertion is wrong statement but reason is correct statement.

Explanation:

Assertion is wrong statement but reason is correct statement.

23.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation:

Let α, $\frac{1}{\alpha}$ = be the zeros of p(x), then

$\alpha \cdot \frac{1}{\alpha} = \frac{4k}{k^2+4}$

$1 = \frac{4k}{k^2+4}$

$k^2 - 4k + 4 = 0$

$(k - 2)^2 = 0$

$k = 2$

Assertion is true Since, Reason is not correct for Assertion.

24.

(b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

Explanation:

Assertion and reason both are correct statements but reason is not correct explanation for assertion.

25.

(d) A is false but R is true.

Explanation:

A is false but R is true.

26.

(d) A is false but R is true.

Explanation:

Reason is correct. If α and β be the zeroes of the required polynomial f(x),

then $(\alpha + \beta) = 8$ and $\alpha\beta = 12$

$\therefore f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$\Rightarrow f(x) = x^2 - 8x + 12$

So, Assertion is not correct

27. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

28.

(c) Assertion is correct statement but reason is wrong statement.

Explanation:

Assertion is correct statement but reason is wrong statement.

29. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$\alpha = 2$ and $\beta = 3 \Rightarrow \alpha + \beta = 5$ and $\alpha\beta = 6$

So, polynomial is $x^2 - 5x + 6$.

30. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

As irrational roots/zeros always occur in pairs, therefore, when one zero is 3 - $2\sqrt{5}$ then other will be 3 + $2\sqrt{5}$.
So, both A and R are correct and R explains A.

Section B

31. -21

Explanation:

The given polynomial f(x) = $2x^2$ + x + k
If 3 is zero of f(x) then f(3) = 0
i,e. $2(3)^2$ + 3 + k = 0
$\Rightarrow 2 \times 9 + 3 + k = 0$
$\Rightarrow$ 18 + 3 + k = 0
$\therefore$ 21 + k = 0
$\Rightarrow$ k = -21
Thus, for k = -21, 3 is a zero of the polynomial.

32. -2

Explanation:

Given polynomial is p(x) = $(a + 1)x^2 + (2a + 3)x + (3a + 4)$
Sum of zeroes of polynomial = $\frac{-Coefficient(x)}{Coefficient(x^2)}$
$\Rightarrow -1 = \frac{-(2a+3)}{a+1}$
$\Rightarrow$ -a - 1 = -2a - 3
$\Rightarrow$ a = -2

33. -2

Explanation:

Let α and β be the zeros of the polynomial,
p(x) = $6x^2$ + x + k, then,
$\alpha + \beta = -\frac{1}{6}$ and $\alpha\beta = \frac{k}{6}$
We are given that, $\alpha^2 + \beta^2 = \frac{25}{36}$
Consider $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$
$\Rightarrow (-\frac{1}{6})^2 = \frac{25}{36} + \frac{2k}{6}$
$\Rightarrow \frac{1}{36} = \frac{25}{36} + \frac{2k}{6}$
Multiplying by 36, we get,
1 = 25 + 12k
12k = -24
k = -2

34. -5

Explanation:

Given polynomial is p(x) = $5x^2$ + 5x + 1
and α *and* β are zeroes of polynomial
Here, a = 5, b = 5, c = 1
Sum of zeros= $\alpha + \beta = -\frac{b}{a} = -\frac{5}{5} = -1$
And product of zeros = $\alpha\beta = \frac{c}{a} = \frac{1}{5}$
Now, $\alpha^{-1} + \beta^{-1}$
$= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$
$= \frac{-1}{\frac{1}{5}}$
= -5

35. -16

Explanation:

The given polynomial is p(x) = x^2 - 6x + a
And α, β are zeros of given polynomial
$\alpha + \beta = 6$
And $\alpha\beta = a$

Now $3\alpha + 2\beta = 20$ ..... (i)

$\Rightarrow \alpha + 2\alpha + 2\beta = 20$

$\Rightarrow \alpha + 2(\alpha + \beta) = 20$

$\Rightarrow \alpha + 2 \times 6 = 20$

$\Rightarrow \alpha$ = 20 - 12 = 8

Put $\alpha = 8$ in (i), we get

$3(8) + 2\beta$ = 20

$\therefore \beta = -2$

Now, $\alpha\beta = a$

$\therefore$ 8 $\times$(-2) = a

$\Rightarrow$ a = -16

36. 7

Explanation:

Given, x^2 - (k + 6)x + 2(2k - 1)

Here, a = 1 , b = -(k + 6) and c = 2(2k - 1)

Now,

Sum of zeroes = $-\frac{b}{a} = -\frac{-(k\ +\ 6)}{1} =$ k + 6

Product of zeroes = $\frac{c}{a} = \frac{2(2k\ -\ 1)}{1} =$ 2(2k - 1)

According to question, k + 6 = $\frac{1}{2} \times$ 2(2k - 1)

$\Rightarrow$ k = 7

37. Let the polynomial be ax^2 + bx + c,

and it's zeroes be α and β.

Then, $\alpha + \beta = 4 = -\frac{b}{a}$ and $\alpha\beta = 1 = \frac{c}{a}$

If a = 1, then b = -4 and c = 1.

So one quadratic polynomial which fits the given conditions is x^2 - 4x + 1.

38. Sum of zeroes = $\alpha + \beta = \frac{-8}{3}$ and Product of zeroes = $\alpha \cdot \beta = \frac{4}{3}$ [Given]

Required quadratic polynomial = x^2- (Sum of zeroes)x + Product of zeroes

$\therefore$ f(x) = x^2 – (α + β)x + αβ [Formula]

$= x^2 - \left(\frac{-8}{3}\right)x + \frac{4}{3}$

$= x^2 + \frac{8}{3}x + \frac{4}{3}$

So, f(x) = $3x^2$ + 8x + 4 [Multiplying by LCM 3]

Hence $3x^2$ + 8x + 4 is the required quadratic polynomial.

39. p(x) = $2x^2$ - 7x - 15 = 0

$\Rightarrow$ (2x + 3)(x - 5) = 0

$\Rightarrow \alpha$ = x = $-\frac{3}{2}$, β = x = 5.

$\therefore \alpha\beta = -\frac{3}{2} + 5 = \frac{7}{2} = -\frac{(-7)}{2} = \frac{-\text{ coefficient of } x}{\text{coefficient of } x^2}$

$\alpha\beta = -\frac{3}{2} \times 5 = -\frac{15}{2} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

40. $\alpha + \beta = \frac{7}{5}$ and $\alpha\beta = \frac{1}{5}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{7}{5}\right)^2 - 2\times\frac{1}{5}}{\frac{1}{5}}$

$= \frac{39}{5}$ or 7.8

41. Given polynomial is

p(y) = $4\sqrt{3}y^2$ + 5y - $2\sqrt{3}$

For zeroes of p(y),

p(y) = 0

$\Rightarrow 4\sqrt{3}y^2 + 5y - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}y^2 + 8y - 3y - 2\sqrt{3} = 0$

$\Rightarrow 4y(\sqrt{3}y+2)-\sqrt{3}(\sqrt{3}y+2)=0$

$\Rightarrow \quad (\sqrt{3}y+2)(4y-\sqrt{3})=0$

$\Rightarrow y=\frac{-2}{\sqrt{3}}, y=\frac{\sqrt{3}}{4}$

p(y) = $4\sqrt{3}y^2$ + 5y - $2\sqrt{3}$

So, a = $4\sqrt{3}$, b = 5 and c = -2

Sum of zeroes = $\frac{-2}{\sqrt{3}}+\frac{\sqrt{3}}{4}=\frac{-5}{4\sqrt{3}}=-\frac{b}{a}$

Product of zeroes = $\frac{(-2)(\sqrt{3})}{(\sqrt{3})(4)}=\frac{-1}{2}=\frac{-2\sqrt{3}}{4\sqrt{3}}=\frac{c}{a}$

42. Let the two zeroes of x^2-8x+k be $\alpha, \alpha+2$

$\therefore 2\alpha+2=8$

$\Rightarrow \alpha=3,$ other zero is 5

$\therefore \mathrm{k}=15$

43. Let α, β be the zeroes of the polynomial. then

$\alpha+\beta=\sqrt{2}$ and $\alpha\beta=-\frac{3}{2}$

The quadratic polynomial

$f(x)=(x-\alpha)(x-\beta)$

= x^2 - $(\alpha+\beta)$x + $\alpha\beta$

= $x^2-\sqrt{2}x-\frac{3}{2}$

Now, f(x) = $x^2-\sqrt{2}x-\frac{3}{2}$

$\Rightarrow f(x)=\frac{1}{2}\left(2x^2-2\sqrt{2}x-3\right)$

$\Rightarrow f(x)=\frac{1}{2}\left(2x^2-3\sqrt{2}x+\sqrt{2}x-3\right)$

$f(x)=\frac{1}{2}\{\sqrt{2}x(\sqrt{2}x-3)+(\sqrt{2}x-3)\}$

$\Rightarrow f(x)=\frac{1}{2}(\sqrt{2}x-3)(\sqrt{2}x+1)$

Now for f(x) = 0, we get

$x=\frac{3}{\sqrt{2}}$ or, $x=-\frac{1}{\sqrt{2}}$

Hence, the zeroes of f(x) are $\frac{3}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$.

44. The given quadratic polynomial is $2x^2$ + 5x + k.

If α, β are zeroes of quadratic polynomial

$\alpha+\beta=\frac{-b}{a}=\frac{-5}{2}$

$\alpha\beta=\frac{c}{a}=\frac{k}{2}$

Putting these values in $(\alpha+\beta)^2-\alpha\beta$ = 24,

we get $\left(\frac{-5}{2}\right)^2-\frac{k}{2}=24$

$\Rightarrow \frac{25}{4}-\frac{k}{2}=24$

$\Rightarrow \frac{-k}{2}=24-\frac{25}{4}$

$\Rightarrow \frac{-k}{2}=\frac{96-25}{4}$

$\Rightarrow k=\frac{-71}{4}\times 2=\frac{-71}{2}$

45. Let p(x) = (a + 5)x^2 + 13x + 6a

Let b be the one zero of p(x), then other zero is $\frac{1}{b}$

Now, product of zeroes of p(x) = $\frac{6a}{5+a}$, then,

$b\times\frac{1}{b}=\frac{6a}{5+a}$

$\frac{6a}{5+a}$ = 1

5 + a = 6a

5a = 5

$\therefore$ a = 1

46. One zero = 5,

Product of zeroes = 30

$\therefore$ Other zero = $\frac{30}{5}$ = 6

The polynomial

p(x)=(x-5)(x-6)

= x^2 - 6x-5x+30

= x^2 - 11x + 30

Section C

47. Here, $\alpha + \beta = -2\sqrt{3}$ and $\alpha\beta = -9$

f(x) = x^2 – $(\alpha + \beta)$x + $\alpha\beta$ [Formula]

$= x^2 - (-2\sqrt{3})x + (-9)$

$\Rightarrow f(x) = x^2 + 2\sqrt{3}x - 9$

For zeroes of polynomial f(x), f(x) = 0

$\Rightarrow x^2 + 2\sqrt{3}x - 9 = 0$

$\Rightarrow x^2 + 3\sqrt{3}x - 1\sqrt{3}x - 9 = 0$

$\Rightarrow x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$

$\Rightarrow (x + 3\sqrt{3})(x - \sqrt{3}) = 0$

$\Rightarrow x + 3\sqrt{3} = 0$ or $(x - \sqrt{3}) = 0$

$\Rightarrow x = -3\sqrt{3}$ or $x = \sqrt{3}$

$\therefore\ \alpha = -3\sqrt{3}$ and $\beta = \sqrt{3}$

Hence the polynomial is $x^2 + 2\sqrt{3}$ x - 9 and its zeros are $-3\sqrt{3}$ and $\sqrt{3}$.

48. We know, quadratic polynomial = x^2 - (Sum of zeroes)x + Product of zeroes

Given, Sum of zeroes = $-\frac{21}{8}$ and Product of zeroes = $\frac{5}{16}$

$\therefore$ Quadratic Polynomial = $x^2 + \frac{21}{8}x + \frac{5}{16}$

$= \frac{1}{16}(16x^2 + 42x + 5)$

$\Rightarrow$ Quadratic polynomial is $16x^2$ + 42x + 5

Now, we rewrite the polynomial as $16x^2 + 2x + 40x + 5$

$= 2x \cdot (8x + 1) + 5 \cdot (8x + 1)$

$= (2x + 5) \cdot (8x + 1)$

Now, for Zeros, $(8x + 1) \cdot (2x + 5) = 0$

$\Rightarrow x = \frac{-1}{8}, \frac{-5}{2}$

49. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

$= \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$

$= \frac{1}{3}(3y - 2)(7y + 1)$

$\Rightarrow y = \frac{2}{3}, \frac{-1}{7}$ are zeroes of the polynomial.

If Given polynimoal is $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Then a = 7 , b = $-\frac{11}{3}$ and c = $-\frac{2}{3}$

Sum of zeroes = $\frac{2}{3} + \frac{-1}{7} = \frac{14-3}{21} = \frac{11}{21}$ ..... (i)

Also, $\frac{-b}{a} = \frac{-\left(\frac{-11}{3}\right)}{7} = \frac{11}{21}$ ..... (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

Now, product of zeroes = $\frac{2}{3} \times \frac{-1}{7} = \frac{-2}{21}$ ..... (iii)

Also, $\frac{c}{a} = \frac{\frac{-2}{3}}{7} = \frac{-2}{21}$ ..... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

50. The given quadratic polynomial is p(x) = $2x^2$ - 3x + p

Since, 3 is a root (zero) of p(x)

$\Rightarrow 2(3)^2 - 3 \times 3 + p = 0$

$\Rightarrow$18 - 9 + p = 0

$\Rightarrow$9 + p = 0

$\Rightarrow$ p = - 9

Now p(x) = $2x^2$ - 3x - 9

= $2x^2$- 6x + 3x - 9

= 2x (x - 3) + 3 (x - 3)

= (x - 3) (2x + 3)

For roots of polynomial, p(x) = 0

$\Rightarrow$(x - 3) (2x + 3) = 0

$\Rightarrow$ x = 3 or x = $-\frac{3}{2}$

Hence the other root is $-\frac{3}{2}$.

51. Sum of the zeroes: (2 + β) = (-1)

Product of the zeroes : 2β = -20

So, required Quadratic polynomial

= $\left[x^2 + (\alpha + \beta)x + 2\beta\right]$

= $\left[x^2 + (-1)x + (-20\right]$

= $x^2 - x - 20$

$\Rightarrow$ $x^2 - x - 20$ = 0 is the polynomial

52. Since,

Sum of the zeroes of polynomial = α + β = $\frac{-b}{a}$

and product of zeroes of polynomial = αβ = $\frac{c}{a}$

Simplify the given expression and substitutie the values, we obtain

$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$

$= \frac{\alpha\beta^2+b\beta+\alpha^2a+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= \frac{a\alpha^2+b\beta^2+b\alpha+b\beta}{a^2\times\frac{c}{a}+ab(\alpha+\beta)+b^2}$

$= \frac{a\left[(\alpha+\beta)^2+b(\alpha+\beta)\right]}{ac}$

$= \frac{a\left[(a+\beta)^2-2\alpha\beta\right]-\frac{b^2}{a}}{ac}$

$= \frac{a\left[\frac{b^2}{a}-\frac{2c}{a}\right]-\frac{b^2}{a}}{ac}$

$= \frac{a\left[\frac{b^2-ac}{a}\right]-\frac{b^2}{a}}{ac}$

$= \frac{a\left[\frac{b^2-ac-b^2}{a}\right]}{ac}$

$= \frac{b^2-2c-b^2}{ac}$

$= \frac{-2c}{ac} = \frac{-2}{a}$

53. We know that, if x = a is a zero of a polynomial then x - a is a factor of quadratic polynomials.

Since $\frac{-1}{4}$ and 1 are zeros of polynomial.

Therefore $\left(x + \frac{1}{4}\right)$ (x - 1)

$= x^2 + \frac{1}{4}x - x - \frac{1}{4}$

$= x^2 + \frac{1}{4}x - \frac{4}{4}x - \frac{1}{4}$

$= x^2 + \frac{1-4}{4}x - \frac{1}{4}$

$= x^2 - \frac{3}{4}x - \frac{1}{4}$

Hence, the family of quadratic polynomials is f(x) = $k\left(x^2 - \frac{3}{4}x - \frac{1}{4}\right)$, where k is any non-zero real number.

54. Let f(x) = $6x^2$ + x - 2

a = 6, b = 1 and c = -2

And α and β are the zeros of polynomial,

$\alpha + \beta = -\frac{b}{a} = -\frac{1}{6}$

$\alpha\beta = \frac{c}{a} = \frac{-2}{6} = \frac{-1}{3}$

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$= \frac{\left(-\frac{1}{6}\right)^2-2\left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)}$

$= -\frac{\frac{1}{36}+\frac{2}{3}}{\frac{1}{3}}$

$= -\frac{\frac{25}{36}}{\frac{1}{3}}$

$= -\frac{25}{36} \times \frac{3}{1}$

$= -\frac{25}{12}$

55. Sum of zeroes = $\frac{-q}{p}$ <0 [as zeroes are negative means sum of zeroes is negative]

So that $\frac{q}{p}$ > 0

$\Rightarrow\ q > 0,\ p > 0\ or\ q < 0,\ \ p <\ 0$ …….. (i)

Product of zeros = $\frac{r}{p}$ > 0 [as zeroes are negative means product of zeroes is positive]

$\Rightarrow\ r > 0,\ p > 0\ or\ r < 0,\ p\ < 0$ ……. (ii)

∴ From (i) and (ii), p, q and r will have same signs i.e.

Either $p > 0,\ q > 0,\ \ r > 0$

Or $p\ < 0,\ q < 0,\ r\ < 0.$

56. Let the given polynomial is p(x) =x^2 + 7x + 7

Here,a = 1, b = 7, c = 7

$\therefore \alpha, \beta$ are both zeroes of p(x)

$\therefore\quad \alpha + \beta = \frac{-b}{a}$ = -7.....(i)

$\alpha\beta = \frac{c}{a} = 7$ .....(ii)

Now,

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$

$= \frac{-7}{7} - 2 \times 7$

= -1 - 14

= -15

Hence the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is - 15.

Section D

57. i. Graph of y = f(x) intersects X-axis at two distinct points. So we can say that no of zeros of y = f(x) is 2.

ii. There will not be any zero if graph of f(x) does not intersect x- axis.

iii. x^2 + (a + 1) x + b is the quadratic polynomial.

2 and -3 are the zeros of the quadratic polynomial.

Thus, 2 + (-3) = $\frac{-(a+1)}{1}$

$\Rightarrow \frac{(a+1)}{1}$ = 1

$\Rightarrow$ a + 1 = 1

$\Rightarrow$ a = 0

Also, 2 $\times$ (-3) = b

$\Rightarrow$ b = -6

OR

If -4 is zero of given polynomial then,

$(-4)^2$ - 2(-4) - (7p + 3) = 0

$\Rightarrow$ 16 + 8 - 7p - 3 = 0

$\Rightarrow$ 7p = 21

$\Rightarrow$ p = 3

58. i. Zeroes are -2 and 8

$\alpha + \beta$ = -2 + 8 = 6

$\alpha\beta$ =-2 $\times$ 8 = -16

expression of polynomial

$x^2 - (\alpha + \beta)x + \alpha\beta$

x^2 - 6x - 16

ii. P(x) = x^2 - 6x - 16

P(4) = 4^2 - 6(4) - 16

= 16 - 24 - 16

= -24

iii. P(x) = $-x^2$ + 3x - 2

$\alpha + \beta = \frac{-3}{-1}$

$\alpha + \beta$ = 3 ...(i)

$\alpha\beta = \frac{-2}{-1}$

$\alpha\beta$ = 2 ...(ii)

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$(\alpha - \beta)^2$ = $(3)^2$ - 4(2)

$(\alpha - \beta)^2$ = 9 - 8

$\alpha - \beta = \pm\sqrt{1}$

$\alpha - \beta = \pm 1$

Taking

$\alpha - \beta = 1$

$\alpha + \beta = 3$

2α = 4

α = 2

Put α = 2 in, $\alpha - \beta = 1$

2 - β = 1

β = 1

OR

$\alpha + \beta = \frac{-3}{-1} = 3$

$\alpha\beta = \frac{-2}{-1} = 2$

59. i. Point of intersection of graph of polynomial, gives the zeroes of the polynomial.

∴ zeroes = -4 and 7

ii. Since, zero's are α = -4, β = 7

$\alpha + \beta$ = -4 + 7 = 3

$\alpha\beta$ = -4 × 7 = -28

P(x) = x^2 - (Sum of zeroes)x + product of zeroes

P(x) = x^2 - 3x + (-28)

P(x) = x^2 - 3x - 28

iii. Product of zeroes = -4 × 7

= -28

OR

a is a non-zero real number, b and c are any real numbers c.

60. i. Parabola

ii. As the curve cuts x-axis two times

∴ No of zero's = 2

iii. ∵ The parabola cuts x-axis at x = -2 and x = 4

∴ The zero's are = -2, 4

OR

2

61. i. 2

ii. 81.2 m

iii. quadratic polynomial

OR

(x - 3) and (x - 2)

62. i. Two

ii. 7 and -7

iii. -(a + 1) = 2 + (-3) $\Rightarrow$ a = 0

b = 2 $\times$ (–3) $\Rightarrow$ b = -6

OR

Let α and β be the zeroes of given polynomial

Here, $\alpha + \beta$ = -p and $\alpha\beta$ = 45

$(\alpha - \beta)^2$ = 144

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 144$

$\Rightarrow (-p)^2 - 4 \times 45 = 144$

$\Rightarrow p = \pm 18$

63. i. Parabola

ii. a > 0

iii. $\because$ The graph cut the x-axis thrice

$\therefore$ No of zeroes = 3

OR

a < 0

64. i. a is a non zero real number and b and c are any real numbers.

ii. D = 0

iii. $2x^2 - x + 8k$

$\alpha \times \frac{1}{\alpha} = \frac{8k}{2}$

1 = 4k

$k = \frac{1}{4}$

OR

$\alpha + \beta = \frac{-b}{a}$ i.e., $\left(\frac{-\text{coefficient of x}}{\text{coefficient of } x^2}\right)$

$\alpha\beta = \frac{c}{a}$ i.e., $\left(\frac{\text{constant term}}{\text{coeff of } x^2}\right)$

65. i. Zeroes of the polynomial are 0 and 5

ii. Maximum height achieved by ball

$= 25 \times \frac{5}{2} - 5 \times \left(\frac{5}{2}\right)^2$

$= \frac{125}{4}$ or 31.25 m

iii. a. $-5t^2 + 25t = 30$

$\Rightarrow t^2 - 5t + 6 = 0$

$\Rightarrow$ (t - 2)(t - 3) = 0

$t \neq 3, t = 2$

OR

b. $-5t^2 + 25t = 20$

$\Rightarrow t^2 - 5t + 4 = 0$

$\Rightarrow$ (t - 4)(t - 1) = 0

$\Rightarrow$ t = 4, 1

Section E

66. Given quadratic polynomial is

$f(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3}$

For zeroes of f(y), put f(y) = 0

$\Rightarrow 7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$

$\Rightarrow 21y^2 - 11y - 2 = 0$

$\Rightarrow 21y^2 - 14y + 3y - 2 = 0$ (by splitting the middle term method)

$\Rightarrow 7y(3y - 2) + 1(3y - 2) = 0$

$\Rightarrow (3y - 2)(7y + 1) = 0$

Therefore, either 3y - 2 = 0 or 7y + 1 = 0

$\Rightarrow y = \frac{2}{3}$ or $y = \frac{-1}{7}$

Now Verification of the relations between α, β ,a, b, and c:

We have $\alpha = \frac{2}{3}$, $\beta = \frac{-1}{7}$, a = 7, $b = -\frac{11}{3}$, $c = \frac{-2}{3}$

$\Rightarrow \alpha + \beta = \frac{-b}{a}$

$\Rightarrow \left(\frac{2}{3}\right) - \frac{1}{7} = \frac{+\frac{11}{3}}{7}$

$\Rightarrow \frac{14-3}{21} = \frac{11}{3} \times \frac{1}{7}$

$\Rightarrow \frac{11}{21} = \frac{11}{21}$

$\Rightarrow$ LHS = RHS

Hence verified.

Also, we know that $\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow \left(\frac{2}{3}\right) \times \left(\frac{-1}{7}\right) = \frac{\frac{-2}{3}}{7}$

$\Rightarrow \frac{-2}{21} = \frac{-2}{3} \times \frac{1}{7}$

$\Rightarrow \frac{-2}{21} = \frac{-2}{21}$

$\Rightarrow$ LHS = RHS

Hence verified.

67. Here the given polynomial is

f(s) = $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

$= s(2s - 1) - \sqrt{2}(2s - 1)$

$= (2s - 1)(s - \sqrt{2})$

Hence f(s) = 0 if 2s - 1 = 0 or $s - \sqrt{2} = 0$

$s = \frac{1}{2}$ or $s = \sqrt{2}$

Verification of the relation between α, β, a, b and c

$\alpha = \frac{1}{2}, \beta = \sqrt{2}$,, a = 2, $b = -(1 + 2\sqrt{2})$, $c = \sqrt{2}$

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{+(1+2\sqrt{2})}{2}$

$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \frac{2\sqrt{2}}{2}$

$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \sqrt{2}$

⇒ LHS = RHS

Now, $\alpha \times \beta = \frac{c}{a}$

$\Rightarrow \left(\frac{1}{2}\right)(\sqrt{2}) = \frac{\sqrt{2}}{2}$

$\Rightarrow \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

⇒ LHS = RHS

Hence verified.

68. Here, $f(v) = v^2 + 4\sqrt{3}v - 15$

For zeroes of f(v), put f(v) = 0

$\Rightarrow v^2 + 4\sqrt{3}v - 15 = 0$

$\Rightarrow v^2 + 5\sqrt{3}v - 1\sqrt{3}v - 15 = 0$ (By splitting the middle term)

$\Rightarrow v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3}) = 0$

$\Rightarrow (v + 5\sqrt{3})(v - \sqrt{3}) = 0$

$\Rightarrow (v + 5\sqrt{3}) = 0$ or $(v - \sqrt{3}) = 0$

Therefore, either $v = -5\sqrt{3}$ or $v = \sqrt{3}$

Verification of relations between α, β, a, b, c:

we have, $\alpha = -5\sqrt{3}$, $\beta = \sqrt{3}$, a = 1, $b = 4\sqrt{3}$, and c = -15

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow -5\sqrt{3} + \sqrt{3} = \frac{-4\sqrt{3}}{1}$

$\Rightarrow -4\sqrt{3} = -4\sqrt{3}$

$\Rightarrow$ LHS = RHS

Hence, verified.

Also we know that

$\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow (-5\sqrt{3})(\sqrt{3}) = \frac{-15}{1}$

$\Rightarrow$ -5 × 3 = -15

$\Rightarrow$ -15 = -15

$\Rightarrow$ LHS = RHS

Hence, verified.

69. Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$.

Hence, $\alpha + \beta = -\frac{2}{3}$

and $\alpha\beta = \frac{1}{3}$

Now, for the new polynomial,

Sum of zeroes = $\frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$

$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$

$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$

$\therefore$ Sum of zeroes = $\frac{4/3}{2/3} = 2$

Product of zeroes = $\left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$

$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$

$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$

$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{3}{3}} = 3$

Hence, Required polynomial = x^2 - (Sum of zeroes)x + Product of zeroes

= x^2 - 2x + 3

70. According to the question, α and β are zeroes of $p(x) = 6x^2 - 5x + k$

So, Sum of zeroes = $\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ .....(i)

$\alpha - \beta = \frac{1}{6}$ (Given) .....(ii)

Adding equations (i) and (ii) , we get

$2\alpha = 1$

or, $\alpha = \frac{1}{2}$

On putting the value of α in equation (ii), we get

$\frac{1}{2} - \beta = \frac{1}{6}$

$\beta = \frac{1}{2} - \frac{1}{6}$

$\beta = \frac{2}{6} = \frac{1}{3}$

$\therefore \quad \alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Hence, k = 1

71. Zeroes are -2, -3

factors are (x + 2), (x + 3)

g(x) = (x + 2) (x + 3) = $x^2 + 5x + 6$

$\frac{x^4+2x^3-7x^2-8x+12}{x^2+5x+6}$ = x^2 - 3x + 2

x^2 - 3x + 2 = (x - 2)(x - 1)

Other zeroes are 2, 1

72. According to the question, $\alpha$and $\beta$are zeroes of $p(x) = 6x^2 - 5x + k$

So, Sum of zeroes =$\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ .....(i)

$\alpha - \beta = \frac{1}{6}$ (Given).....(ii)

Adding equations (i) and (ii), we get

2α= 1

or, $\alpha = \frac{1}{2}$

On putting the value of α in equation (ii), we get

$\frac{1}{2} - \beta = \frac{1}{6}$

$\beta = \frac{1}{2} - \frac{1}{6}$

$\beta = \frac{2}{6} = \frac{1}{3}$

$\therefore \quad \alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$

Hence, k = 1

73. Sum of zeroes of a quadratic polynomial is $\frac{-b}{a}$ and the product is $\frac{c}{a}$

So a + b = $\frac{-2}{5}$ and ab = $\frac{-3}{5}$

According to question

Sum of zeroes of the polynomial is $\frac{1}{a}$ + $\frac{1}{b}$

= $\frac{a+b}{ab}$

= $\frac{\frac{-2}{5}}{\frac{-3}{5}}$

= $\frac{2}{3}$

Product of zeroes of the polynomial is $\frac{1}{ab}$

= $\frac{1}{\frac{-3}{5}}$

= $\frac{-5}{3}$

We know that a quadratic equation is of the form $ax^2 + bx + c$

= $x^2 - \frac{2}{3}x - \frac{5}{3}$

74. Since α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$

$\alpha + \beta = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a}$

$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$

We have,

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\alpha^3+\beta^3}{\alpha\beta}\right) + b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}\right) + \left(\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right)$

By substituting $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ we get,

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\left(\frac{-b}{a}\right)^3 - 3\times\frac{c}{a}\left(\frac{-b}{a}\right)}{\frac{c}{a}}\right) + b\left(\frac{\left(\frac{-b}{a}\right)^2 - 2\times\frac{c}{a}}{\frac{c}{a}}\right)$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\frac{-b^3}{a^3} + \frac{3bc}{a^2}}{\frac{c}{a}}\right) + b\left(\frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}\right)$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{-b^3+3bca}{a^3} \times \frac{a}{c}\right) + b\left(\frac{b^2-2ca}{a^2} \times \frac{a}{c}\right)$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{-b^3+3abc}{a^2c}\right) + b\left(\frac{b^2-2ca}{ac}\right)$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{-b^3+3abc}{ac} + \frac{b^3-2abc}{ac}$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{-b^3+3abc+b^3-2abc}{ac}$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{3abc-2abc}{ac}$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{abc}{ac}$

$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = b$

Hence, the value of $a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$ is b.

75. Since α and β are the zeroes of the quadratic polynomial x^2+ 4x + 3

So, $\alpha + \beta = -4$

and $\alpha\beta = 3$

Sum of zeroes of new polynomial $= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$

$= \frac{\alpha\beta+\beta^2+\alpha\beta+\alpha^2}{\alpha\beta}$

$= \frac{\alpha^2+\beta^2+2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$

Product of zeroes = $\left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)$

$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$

$= \frac{(\alpha+\beta)^2}{\alpha\beta}$

$= \frac{(-4)^2}{3} = \frac{16}{3}$

So required polynomial = x^2 - (Sum of the zeroes)x + Product of the zeroes

$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$

$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$

$= \frac{1}{3}\left(3x^2 - 16x + 16\right)$