Solution

LINES AND ANGLES

Class 09 - Mathematics

Section A

1.

(d) 30^0

Explanation:

Let one angle be x^o

Its supplementary angle will be 180^o - x^o

According to question

$x = \frac{1}{5}(180^o - x)$

$5x + x = 180^o$

$6x = 180^o$

$x = \frac{180}{6}$

$x = 30^0$

2.

(c) 130^o, 50^o

Explanation:

Let the two supplementary angles be x^o and 180^o - x^o

Let 180^o - x be the larger angle

$180^0 - x = 3x - 20^o$

$4x = 200^o$

$x = 50^o$

So the angles are 50^o and 130^o.

3.

(c) three

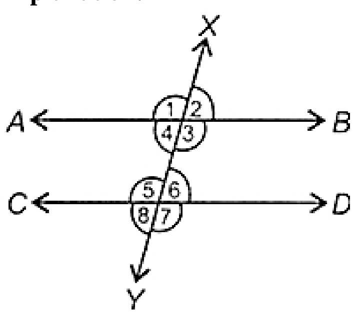
Explanation:

Three because non-collinear points means the point does not lies in a same line.

4.

(b) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

Explanation:



(a) Corresponding angles $\rightarrow \angle 1 = \angle 5$

(b) Alternate interior angles$\rightarrow \angle 4 = \angle 6$

(c) Alternate exterior angles $\rightarrow \angle 1 = \angle 7$

(d) Co-interior angles $\rightarrow \angle 4 + \angle 5 = 180°$

5.

(d) 12 : 3 : 2

Explanation:

Let A be x

B = $\frac{1}{4}$x

C = $\frac{1}{6}$ x

A : B : C

x : $\frac{1}{4}$x : $\frac{1}{6}$ x

LCM of 4 and 6 is 12

12 : 3 : 2

6.

(b) an acute angle

Explanation:

an acute angle

If two angles are complements of each other, that is, the sum of their measures is 90°, then each angle is an acute angle.

7.

(b) 60°

Explanation:

We know that, angle made on the straight line is 180°.

$\therefore$ 3x + 4x + 2x = 180° $\Rightarrow$ 9x = 180° $\Rightarrow$ x = 20°

$\therefore$ $\angle$AOF = 3x = 60°

As, $\angle$DOC = $\angle$AOF (vertically opposite angles)

$\therefore$ $\angle$DOC = 60°

8.

(b) 126°

Explanation:

Let a = 2x and b = 3x

Now, XOY is a straight line.

$\therefore$ 2x + 3x + 90° = 180° $\Rightarrow$ 5x = 90° $\Rightarrow$ x = 18°

$\therefore$ $\angle$XON = $\angle$MOY = $\angle$MOP + $\angle$POY (vertically opposite angles)

= 36°+ 90° = 126°.

9.

(c) (ii) and (iii) are correct

Explanation:

When two straight lines intersect them, Adjacent angles are supplementary and opposite angles are equal.

10. **(a)** 86°

Explanation:

Given,

$\angle$AOC + $\angle$COB + $\angle$BOD = 274^o ......(i)

$\angle$AOD + $\angle$AOC + $\angle$COB + $\angle$BOD = 360^o (Angles at a point)

$\angle$AOD + 274^o = 360^o

$\angle$AOD = 86^o

11.

(c) 36°

Explanation:

Let x and (90^o – x) be two complimentary angles
According to question,

$2x = 3 (90^o – x)$

$2x = 270^o – 3x$

$x = 54^o$

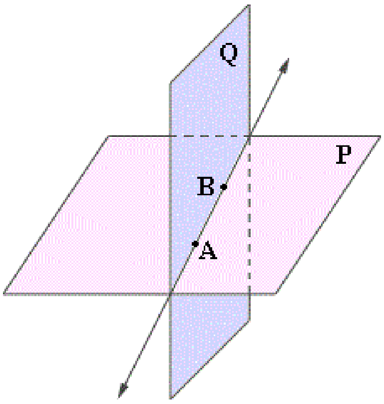
The angles are:

54^o and $90^o – 54^o = 36^o$

Thus, smallest angle is 36^o

12.
(b) 20°
Explanation:
20°

13.
(d) Straight line
Explanation:



As can be seen from the above diagram, the two planes "P" and "Q" are intersecting in a line, which is AB.

14. **(a)** 90°
Explanation:
Given that,
AB and CD intersect at O
$\angle AOC + \angle COB + \angle BOD = 270°$ (i)

$\angle COB + \angle BOD = 180^o$ (Linear pair) (ii)
Using (ii) in (i), we get

$\angle AOC + 180^o = 270^o$

$\angle AOC = 90^o$

15. **(a)** 75°
Explanation:
Let the measure of the required angle be x°
Then, the measure of its complement will be (90 − x)°
$\therefore$ x = 5 (90 - x)
$\Rightarrow$ x = 450 - 5x
$\Rightarrow$ 6x = 450
$\Rightarrow$ x = 75°

16.
(c) 135°
Explanation:

Let the required angle be x

Supplement = 180^o – x

According to question,

x = 3 (180^o – x)

x = 540^o – 3x

x = 135^o

17.

(c) a right triangle

Explanation:

The sum of the angles of triangle is 180^o.

let the angles of triangle be a, b, c

We have given that one angle of a triangle is equal to the sum of the other two angles

so we have

c = a + b

a + b + c = 180^o

Substitute c for a + b

c + c = 180^o

2c = 180^o

c = 90^o

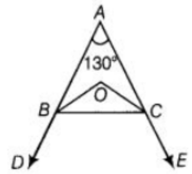
Therefore the triangle is a right triangle.

18.

(b) 155°

Explanation:

Let angles of a triangle be ∠A, ∠B and ∠C.



In △ABC,

$\angle A+\angle B+\angle C=180^{\circ}$ [sum of all interior angles of a triangle is 180^o]

$\Rightarrow \frac{1}{2}\angle A+\frac{1}{2}\angle B+\frac{1}{2}\angle C=\frac{180^{\circ}}{2}=90^{\circ}$ [dividing both sides by 2]

$\Rightarrow \frac{1}{2}\angle B+\frac{1}{2}\angle C=90^{\circ}-\frac{1}{2}\angle A$ [∵ In △OBC, ∠OBC +∠BCO + ∠COB = 180^o]

⇒ Since, $\frac{\angle B}{2}+\frac{\angle C}{2}+\angle BOC=180^{\circ}$ as BO and OC are the angle bisectors of ∠ABC and ∠BCA, respectively

$\Rightarrow 180^{\circ}-\angle BOC=90^{\circ}-\frac{1}{2}\angle A$

∴ ∠BOC = 180^o - 90^o + $\frac{1}{2}\angle A$

= $90^{\circ}+\frac{1}{2}\times 130^{\circ}$ = 90^o + 65^o [∴ ∠A = 130^o (given)]

= 155^o

Hence, the required angle is 155^o.

19.

(d) 20°

Explanation:

We have,

∠AOF = ∠COD = 5y (vertically opposite angles)

Now, BOE is a straight line.

∴ 3 y + 5 y + y = 180° ⇒ 9y = 180° ⇒ y = 20°

20.

(d) 50°, 77°

Explanation:

Since AB || CD

$\therefore$ x = $\angle$APQ = 50° (alternate angles)

In $\triangle$PQR, x + y = 127° (exterior angle property)

$\Rightarrow$ y = 127°-50°$\Rightarrow$ y = 77°

21.

(d) 70°

Explanation:

x + 40° + x + x + 20° = 180° (Linear pair)

$\Rightarrow$ 3x + 60° = 180° $\Rightarrow$ 3x = 120° $\Rightarrow$ x = 40°

Now, $\angle$OCD = $\angle$ODC [$\because$ OD = OC]

Now, $\angle$C + $\angle$D + x = 180°

$\Rightarrow$ $\angle$OCD + $\angle$ODC + 40° = 180°

$\Rightarrow$ 2$\angle$OCD = 140° $\Rightarrow$ $\angle$OCD = 70°

22. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

23.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

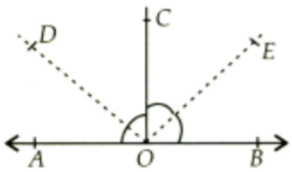
$\angle AOC + \angle BOC = 180^o$ [Linear Pair]

$\frac{1}{2}(\angle AOC + \angle BOC) = \frac{180^o}{2}$

$\frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = 90^o$

$\angle DOC + \angle EOC = 90^o$

The bisectors of the angles of a linear pair are at right angles.



24.

(d) A is false but R is true.

Explanation:

A is false but R is true.

25.

(d) A is false but R is true.

Explanation:

Two adjacent angles do not always form a linear pair. In a linear pair of angles, two non-common arms are opposite rays.

26. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

27. Here, ∠AOC and ∠BOC form a linear pair.
$\therefore$ ∠AOC + ∠BOC = 180°
$\Rightarrow$ x° + 125° = 180°
$\Rightarrow$ x° = 180° - 125° = 55°
Now,
$\angle \mathrm{AOD} = \angle \mathrm{BOC} = 125^\circ$ (Vertically opposite angles)
$\therefore$ y° = 125°
$\angle BOD = \angle AOC = 55^\circ$ (Vertically opposite angles)
$\therefore$ z° = 55°

28. When a ray falls on a mirror, it is reflected and angle of incidence = angle of reflection = x° (say).
QM is drawn normal to AB and therefore,we have,
angle of incidence = ∠PQM,
angle of reflection = ∠MQR
and $\angle AQM = 90^\circ$
Now,we have, $\angle PQM + \angle MQR = \angle PQR = 112^\circ$ (given)
$\therefore 2\angle PQM = 112^\circ$
$\therefore \angle PQM = 56^\circ$
Therefore, $\angle PQA = \angle AQM - \angle PQM = 90^\circ - 56^\circ = 34^\circ$

29. From the given figure,we have,
∠3 + ∠mYz = 180^o(Linear pair)
$\angle 3 + 120^\circ = 180^\circ$
∠3 = 60^o
Now,
Line l ∥ m
∠1 = ∠3 (Corresponding angles)
∠1 = 60^o
Now, m ∥ n, therefore,
∠2 = 120^o (Alternate interior angle)
Therefore,we have,
∠1 = ∠3 = 60^o
∠2 = 120^o

30. r and m are two lines and a transversal p intersects them such that x = y
These angles form a pair of equal corresponding angles.
$\therefore$ r || m . . . . (1)
Similarly, m and n are two lines and a transversal q intersects them such that
a = b
These angles form a pair of equal corresponding angles.
$\therefore$ m || n . . . (2)
r || n . . . [From (1) and (2)]

31. AOB is a straight line. Therefore,by linear pair axiom,
$\angle AOC + \angle COD + \angle BOD = 180^\circ$
$\Rightarrow$ (3x + 7)° + (2x - 19)° + x° = 180°
$\Rightarrow$ 6x = 192°
$\Rightarrow$ x = 32°
Therefore,
$\angle AOC = 3 \times 32^\circ + 7 = 103^\circ$
$\angle COD = 2 \times 32^\circ - 19 = 45^\circ$ and
$\angle BOD = 32^\circ$

32. Let the measure of the angle be x°
$\therefore$ Supplement of x° = (180 - x)°
It is given that,
(180° - x°) - x° = 30°

$\Rightarrow$ 180° - 2x° = 30°

$\Rightarrow$ 2x° = 180° - 30°

$\Rightarrow$ 2x° = 150°

$\Rightarrow$ x° = 75°

Thus, the measure of the angle is 75°

33. $\angle POR + \angle ROQ = 180^0$ [linear pair]

But, ∠POR : $\angle ROQ$ = 5 : 7 [Given]

$\therefore$ ∠ POR = $\frac{5}{12} \times 180^0 = 75^0$

Similarly, ∠ROQ = $\frac{7}{12} \times 180^0 = 105^0$

Now ∠POS = ∠ROQ = 105^0 [vertically opposite angle]

And ∠SOQ = ∠POR = 75^0 [vertically opposite angle]

34. Let the two complementary angles be 2x and 3x.

We know that, sum of complementary angles is 90°.

$\therefore 2x + 3x = 90^\circ$

$\Rightarrow 5x = 90^\circ$

$\Rightarrow \quad x = 18^\circ$

$\therefore$ The angles are $2 \times 18^\circ = 36^\circ$ and $3 \times 18^\circ = 54^\circ$.

35. Let x^o be the required angle.

Then its complementary angle = $90^o - x^o$

$x^0 = \frac{2}{3}(90^0 - x^0)$ . . . [given]

$\therefore 3x^o = 180^o - 2x^o$

$\therefore 3x^o + 2x^o = 180^o$

$\therefore 5x^o = 180^o$

$\therefore x^0 = \frac{180^0}{5} = 36^0$

$\therefore$ there are 36^o in an angle.

36. Let the measure of the required angle be x

Then, measure of its complement = (90 - x)°

given that,

x° = 4(90 - x)°

$\Rightarrow$ x° = 360° - 4x°

$\Rightarrow$ x° + 4x° = 360°

$\Rightarrow$ 5x° = 360°

$\Rightarrow$ x° = 72°

Hence, the value of the required angle is 72°

37. Let the angle be "x"

The, its complement will be $(90^o - x)$

Now, according to question,

angle = $30^o + \frac{1}{2}$ Complement

x = $30^o + \frac{1}{2}(90^o - x)$

x = $30^o + 45^o - \frac{x}{2}$

x + $\frac{x}{2} = 30^o + 45^o$

$\frac{3x}{2} = 75^o$

3x = 150^o

x = 50^0

Thus, the angle is 50°

38. Given angle is 132^o

Since the sum of an angle and its supplement is 180^o

Therefore, its compliment will be:

$180^o - 132^o = 48^o$

39. Since AOB is a straight line and the ray OC stands on it.

$\therefore \angle AOC + \angle BOC = 180^\circ$
$\Rightarrow \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = 90^\circ$
$\Rightarrow \angle COD + \angle COE = 90^\circ$
$\Rightarrow \angle DOE = 90^\circ \ [\because \angle COD + \angle COE = \angle DOE]$

40. Given: AB and CD are two lines intersect each other at O.

To prove:

i. $\angle 1 = \angle 2$
ii. $\angle 3 = \angle 4$

Proof:

$\angle 1 + \angle 4 = 180^0$ ...(i) [By linear pair]
$\angle 4 + \angle 2 = 180^0$ ...(ii) [By linear pair]
$\angle 1 + \angle 4 = \angle 4 + \angle 2$ [By eq (i) and (ii)]
$\angle 1 = \angle 2$
Similarly,
$\angle 3 = \angle 4$

41. $\angle$POC = $\angle$DOQ = 2y . . . . [Vertically opposite angles]

$\angle$AOB = 180^o . . . . . [A straight angle = 180^o]

$\angle$AOB + $\angle$POC + $\angle$BOC = 180^o

$\angle$ 5y + 2y + 5y = 180^o

$\angle$ 12y = 180^o

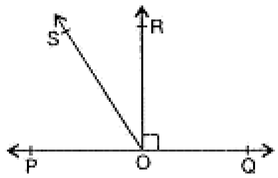
$\angle\, y = \frac{180^0}{12} = 15^0$

42. Let the measure of the required angle be x°,

Then, its complement = (90 - x)°
and its supplement = (180 - x)°.
$\therefore$ 7(90° - x°) = 3(180° - x°) - 10°
$\Rightarrow$ 630° - 7x° = 540° - 3x° -10°
$\Rightarrow$ 4x° = 100°
$\Rightarrow$ x° = 25°
Hence, the measure of the required angle is 25°.

43. AOB will be a straight line if

3x + 20 + 4x - 36 = 180°
$\Rightarrow$ 7x = 196°
$\Rightarrow$ x = 25°
Therefore, x = 28 will make AOB a straight line

44. 

Ray OR is perpendicular to line PQ

$\therefore$ $\angle$QOR = $\angle$POR = 90^o . . . .(1)
$\angle$QOS = $\angle$QOR + $\angle$ROS. . . .(2)
$\angle$POS = $\angle$POR – $\angle$ROS. . . .(3)
From (2) and (3),
$\therefore$ $\angle$QOS – $\angle$POS = ($\angle$QOR – $\angle$POR) + 2$\angle$ROS = 2$\angle$ROS . . . [Using (1)]
$\therefore \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

45. We know that,

The sum of measures of supplementary angles is 180^o

Also,

The sum of measures of complementary angles is 90^o

It is given that pairs of angles are 112^o and 68^o

Therefore,

Sum of the measures of these angles = $112^o + 68^o = 180^o$

As the sum of these angles is equal to 180^o

Therefore,

These angles are supplementary angles.

46. Let the angle measured be x

Compliment angle = $(90^o - x)$

Supplement angle = $(180^o - x)$

Given that,

Supplementary of thrice of the angle = $(180^o - 3x)$

According to question,

$(90^o - x) = (180^o - 3x)$

$2x = 90^o$

$x = 45^o$

Section C

47. In the given figure,

$\angle POY = 90^o$

$\angle POX + \angle POY = 180^o$ (Linear pair)

$\angle POX + 90^o = 180^o$

$\Rightarrow \angle POX = 90^o$

a : b = 2 : 3

Let $a = 2x^o$

and $b = 3x^o$

$\angle POX = a + b = 5x$

$90^o = 5x$

$\Rightarrow x = 18^o$

$\angle MOX = a = 2x = 36^o$

$\angle MOX + \angle NOX = 180^o$ (Linear pair)

$a + c = 180^o$

$36^o + c = 180^o$

$c = 180^o - 36^o = 144^o$

48. $\angle BOD = \angle COA$ . . . [Vertically opposite angles]

$\frac{1}{2}\angle BOD = \frac{1}{2}\angle COA$ . . . [Halves of equals are equal]

$\angle POD = \angle COQ$ . . . .[As OP and OQ are bisectors of $\angle BOD$ and $\angle AOC$ respectively] . . . .(1)

$\angle POQ = \angle POD + \angle DOA + \angle AOQ$

$= \angle COQ + \angle DOA + \angle AOQ$ . . . . [From (1)]

$= \angle COD = 180^o$

OP and OQ are opposite rays.

49. We are given that $\angle POY = 90^\circ$ and a:b = 2:3

We need find the value of c in the given figure.

Let a be equal to 2x and b be equal to 3x.

$\because a + b = 90° \Rightarrow 2x + 3x = 90° \Rightarrow 5x = 90°$

$\Rightarrow x = 18°$

Therefore $b = 3 \times 18^\circ = 54^\circ$

Now b + c = 180° [Linear pair]

$\Rightarrow$ 54° + c = 180°

$\Rightarrow$ c = 180° - 54° = 126°

50. Ray OS stands on the line POQ

$\therefore$ $\angle$POS + $\angle$SOQ = 180^0

But $\angle$PQS = X

$\therefore$ x + $\angle$SOQ = 180°

$\angle$SOQ = 180° - x

Now ray OR bisects $\angle$POS,

Therefore $\angle$ROS = $\frac{1}{2} \times \angle POS = \frac{1}{2} \times x = \frac{x}{2}$

Similarly, $\angle$ SOT = $\frac{1}{2} \times \angle SOQ = \frac{1}{2} \times (180^0 - x) = 90^\circ - \frac{x}{2}$

$\angle ROT = \angle ROS + \angle SOT = \frac{x}{2} + 90^\circ - \frac{x}{2} = 90^0$

51. d = a . . . [Vertically opposite angles]

= 50^o

b + c + d = 180^o . . . [A straight line angle = 180^o]

$\therefore$ 90^o + c + 50^o = 180^o

$\therefore$ c + 140^o = 180^o

$\therefore$ c = 180^o – 140^o

$\therefore$ c = 40^o

e = b . . . [Vertically opposite angles]

= 90^o

f = c . . . [Vertically opposite angles]

= 40^o

52. Given AD is transversal intersect two lines PQ and RS

To prove PQ $\parallel$ RS

Proof: BE bisects ABQ

$\angle 1 = \angle ABE = \angle EBQ = \frac{1}{2}\angle ABQ$ ...(i)

Similarity CG bisects $\angle$ BCS

$\therefore \angle 2 = \frac{1}{2}\angle BCS$ ...(ii)

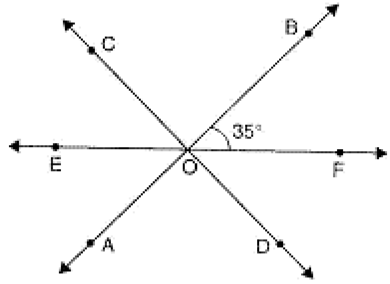
But BE $\parallel$ CG and AD is the transversal

$\therefore$ $\angle$1 = $\angle$2

$\therefore$ $\frac{1}{2}\angle ABQ = \frac{1}{2}\angle BCS$ [by (i) and (ii)]

$\Rightarrow$ $\angle$ABQ = $\angle$BCS [$\because$ corresponding angles are equal]

$\therefore$ PQ $\parallel$ RS

53. 

OF bisects $\angle$BOD . . . [Given]

$\angle$BOF = $\angle$DOF = 35^o

$\angle$COE = $\angle$DOF = 35^o

$\angle$EOF = 180^o . . . . [A straight angle = 180^o]

$\therefore$ $\angle$EOC + $\angle$BOC + $\angle$BOF = 180^o

$\therefore$ 35^o + $\angle$BOC + 35^o = 180^o

$\therefore \angle$BOC = 180^o – 70^o = 110^o

$\angle$AOD = $\angle$BOC . . . [Vertically opposite angles]

= 110^o

54. a + b = 180^o . . . [Linear Pair Axiom] . . . (1)

a = b+$\frac{1}{3}$(a right angle) . . . [Given]

$a = b + \frac{1}{3}(90^0)$ . . . . [right angle = 90^o]

$\therefore$ a + b = 30^o

$\therefore$ a – b = 30^o . . . . . (2)

2a = 180^o + 30^o . . . . . [Adding (1) and (2)]

$\therefore$ 2a = 210^o

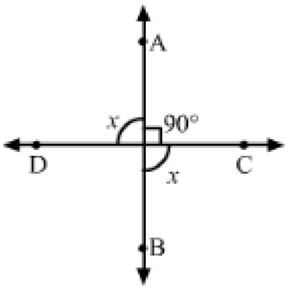
$\therefore a = \frac{210^0}{2} = 105^0$

2b = 180^o – 30^o . . . . . [Subtracting (2) from (1)]

$\therefore$ 2b = 150^o

$\therefore b = \frac{150^0}{2} = 75^0$

55. We know that if two lines intersect, then the vertically-opposite angles are equal.



$\angle AOC = 90^\circ$, Then $\angle AOC = \angle BOD = 90^\circ$

And let $\angle BOC = \angle AOD$ = x°

Also, we know that the sum of all angles around a point is 360°

$\Rightarrow$ 90° + 90° + x° + x° = 360°

$\Rightarrow$ $2x^\circ$ = 180°

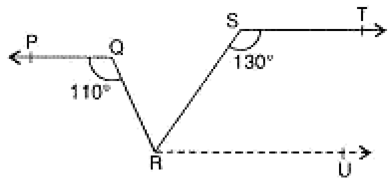
$\Rightarrow$ x° = 90°

Hence, $\angle BOC = \angle AOD$ = x° = 90°

$\therefore \angle AOC = \angle BOD = \angle BOC = \angle AOD = 90^\circ$

Hence, the measure of each of the remaining angles is 90°.

56. Draw a line RU parallel to ST through point R.



$\angle$RST + $\angle$SRU = 180^o

$\therefore$ 130^o + $\angle$SRU = 180^o

$\therefore \angle$SRU = 180^o – 130^o = 50^o . . . (1)

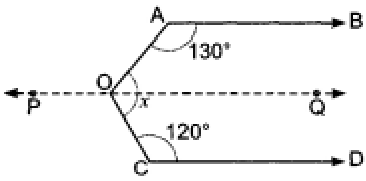
$\angle$QRU = $\angle$PQR = 110^o . . . . [Alternate interior angles]

$\therefore$ $\angle$QRS + $\angle$SRU = 110^o

$\therefore \angle$QRS + 50^o = 110^o . . . [Using (1)]

$\therefore$ $\angle$QRS = 110^o – 50^o = 60^o

57. Through O, draw a line POQ parallel to AB.



From figure , x = $\angle AOQ + \angle COQ$.........(1)

Now $PQ \| AB$ and $CD \| AB$

So, CD$\|$PQ

$\because AB \| PQ$ and AO is a transversal

We have,

$\angle AOQ + \angle OAB = 180^\circ$ (Co interior angles)

$\Rightarrow \angle AOQ + 130^\circ = 180^\circ$

$\Rightarrow \angle AOQ = 180^\circ - 130^\circ = 50^\circ$

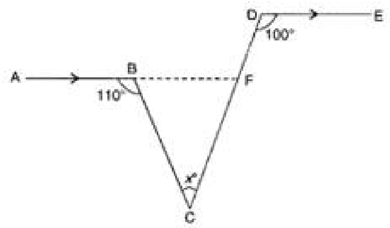
Similarly, $PQ \| CD$ and OC is a transversal

$\therefore \quad \angle QOC + \angle DCO = 180^\circ$ (Co interior angles)

$\Rightarrow \quad \angle QOC + 120^\circ = 180^\circ$

$\Rightarrow \quad \angle QOC = 180^\circ - 120^\circ = 60^\circ$

$\therefore \quad \angle AOC = x = \angle AOQ + \angle QOC = 50^\circ + 60^\circ = 110^\circ$

58. 

Draw AB to meet CD in F.

As AF || DE and transversal DF intersects them

$\therefore \angle DFB = \angle EDF = 100^o$ . . . [Alternate Angles]

$\angle DFB + \angle BFC = 180^o$ . . . [Linear pair axiom]

$\therefore 100^o + \angle BFC = 180^o$

$\therefore \angle BFC = 180^o - 100^o = 80^o$

$\angle ABC + \angle FBC = 180^o$ . . . [Linear pair axiom]

$\therefore 110^o + \angle FBC = 180^o$

$\therefore \angle FBC = 180^o - 110^o = 70^o$

In $\triangle$BFC,

$\angle BCF + \angle BFC + \angle FBC = 180^o$ . . . [Sum of all the angles of a triangle]

$\therefore x^o + 80^o + 70^o = 180^o$

$\therefore x^o + 150^o = 180^o$

$\therefore x^o = 180^o - 150^o = 30^o$

$\therefore x = 30^0$

59. $\angle AOC + \angle BOC = 180^o$ . . . . [Linear pair]

$\angle AOC + \angle BOE + \angle COE = 180^o$ . . . [As $\angle BOC = \angle BOE + \angle COE$]

$\therefore 2x^o + x^o + 90^o = 180^o$

$\therefore 3x^o + 90^o = 180^o$

$\therefore 3x^o = 180^o - 90^o = 90^o$

$\therefore x^o = \frac{90^0}{3} = 30^o \therefore x = 30$

$\angle BOD = \angle AOC$ . . . [Vertically opposite angles]

$\therefore y^o = 2x^o = 2(30^o) = 60^o$

$\therefore y = 60$

$\angle AOD = \angle COB$ . . . [Vertically opposite angles]

$\therefore \angle AOD = \angle COE + \angle EOB$

$\therefore z^o = 90^o + x^o = 90^o + 30^o = 120^o$

$\therefore z = 120$

60. Since AOB is a straight line, the sum of all the angles on the lower side of AOB at a point O on it, is 180°.

$\therefore \angle AOE + \angle BOE = 180^\circ$

$\Rightarrow (3x)^\circ + 72^\circ = 180^\circ$

$\Rightarrow (3x)^{\circ} = (180^{\circ} - 72^{\circ}) = 108^{\circ}$

$\Rightarrow x^{\circ} = \left(\frac{108}{3}\right)^{\circ} = 36^{\circ}$

Again, AOB is a straight line and O is a point on it.

So, the sum of all angles on the upper side of AOB at a point O on it is 180°.

$\therefore \angle AOC + \angle COD + \angle DOB = 180^{\circ}$

$\Rightarrow x^{\circ} + 90^{\circ} + y^{\circ} = 180^{\circ}$ [∵ ∠AOC = x°, ∠COD = 90° and ∠DOB = y°]

$\Rightarrow 36^{\circ} + 90^{\circ} + y = 180^{\circ}$ [∵ x = 36°]

$\Rightarrow 126^{\circ} + y = 180^{\circ}$

$\Rightarrow y = (180^{\circ} - 126^{\circ}) = 54^{\circ}$

$\therefore \angle AOC = x^{\circ} = 36^{\circ}$, $\angle BOD = y^{\circ} = 54^{\circ}$

$\angle AOE = (3x)^{\circ} = (3 \times 36)^{\circ} = 108^{\circ}$

61. Given: In figure, OD ⊥ OE (i.e. $\angle DOE = 90^{0}$), OD and OE are the bisectors of ∠AOC and ∠BOC.

To prove: points A, O and B are collinear i.e., AOB is a straight line.

Proof: From the fig. we have ∠AOB comprising ∠AOC and ∠BOC such that OD and OE are the bisectors of these two angles

∠AOB = ∠AOC + ∠BOC

Since, OD and OE bisect angles ∠AOC and ∠BOC respectively.

$\therefore \angle AOC = 2\angle DOC$ …...(1)

And $\angle COB = 2\angle COE$ …..(2)

On adding equations (1) and (2), we get

$\angle AOC + \angle COB = 2\angle DOC + 2\angle COE$

$\Rightarrow \angle AOC + \angle COB = 2(\angle DOC + \angle COE)$

$\Rightarrow \angle AOC + \angle COB = 2\angle DOE$

$\Rightarrow \angle AOC + \angle COB = 2 \times 90^{\circ}$ $[\because OD \perp OE]$

$\Rightarrow \angle AOC + \angle COB = 180^{\circ}$

$\therefore \angle AOB = 180^{\circ}$

So, $\angle AOC \ and \ \angle COB$ are forming linear pair or AOB is a straight line. Hence, points A, O and B are collinear.

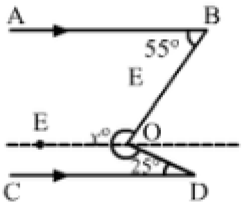
Section D

62. i. 24°
ii. 42°
iii. 180°

OR

2y + z = 90°

Section E

63.


Draw EO ∥ AB ∥ CD

Then, $\angle EOB + \angle EOD = x^{\circ}$

Now, EO ∥ AB and *BO* is the transversal.

$\therefore \angle EOB + \angle ABO = 180^{\circ}$ [Consecutive Interior Angles]

$\Rightarrow \angle EOB + 55^{\circ} = 180^{\circ}$

$\Rightarrow \angle EOB = 125^{\circ}$

Again, EO ∥ CD and DO is the transversal.

$\therefore \angle EOD + \angle CDO = 180^{\circ}$ [Consecutive Interior Angles]

$\Rightarrow \angle EOD + 25^{\circ} = 180^{\circ}$

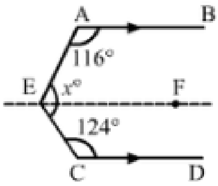
$\Rightarrow \angle EOD = 155^{\circ}$

Therefore,

$x^{\circ} = \angle EOB + \angle EOD$

x° = (125 + 155)°

x° = 280°

64. 

Draw EF ∥ AB ∥ CD

Then, $\angle AEF + \angle CEF = x^\circ$

Now, EF ∥ AB and AE is the transversal

$\therefore \angle AEF + \angle BAE = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle AEF + 116 = 180$

$\Rightarrow \angle AEF = 64^\circ$

Again, EF ∥ CD and CE is the transversal.

$\angle CEF + \angle ECD = 180^\circ$ [Consecutive Interior Angles]

$\Rightarrow \angle CEF + 124 = 180$

$\Rightarrow \angle CEF = 56^\circ$

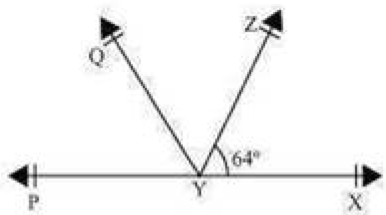
Therefore,

$x^\circ = \angle AEF + \angle CEF$

x° = (64 + 56)°

x° = 120°

65. We are given that ∠XYZ = 64^o, XY is produced to P and YQ bisects ∠ZYP We can conclude the given below figure for the given situation:



We need to find ∠XYQ and reflex ∠QYP

From the given figure, we can conclude that ∠XYZ and ∠ZYP form a linear pair.

We know that sum of the angles of a linear pair is 180^o.

∠XYZ + ∠ZYP = 180^o

But ∠XYZ = 64^o

⇒ 64^o + ∠ZYP = 180^o

⇒ ∠ZYP = 116^o

Ray YQ bisects ∠ZYP,or

∠QYZ = ∠QYP = $\frac{116^o}{2}$ = 58^o

∠XYQ = ∠QYZ + ∠XYZ

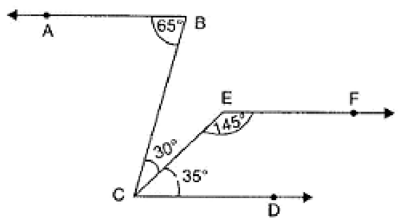
= 58^o + 64^o = 122^o.

Reflex ∠QYP = 360° - ∠QYP

= 360^o - 58^o

= 302^o.

Therefore, we can conclude that ∠XYQ = 122^o and Reflex ∠QYP = 302^o

66. 

∠ABC = 65^o

∠BCD = ∠BCE + ∠ECD = 30^o + 35^o = 65^o

$\therefore \angle ABC = \angle BCD$

These angles form a pair of equal alternate angles

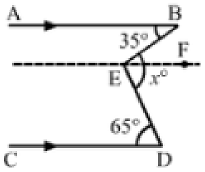
$\therefore$ AB || CD . . . (1)

$\angle FEC + \angle ECD = 145^o + 35^o = 180^o$

These angles are consecutive interior angles formed on the same side of the transversal.

$\therefore$ CD || EF . . . . (2)

AB || EF . . . [From (1) and (2)]

67. 

Draw EF $\parallel$ AB $\parallel$ CD

Now, AB $\parallel$ EF and BE is the transversal.

Then,

$\angle ABE = \angle BEF$ [Alternate Interior Angles]

$\Rightarrow \angle BEF = 35^\circ$

Again, EF $\parallel$ CD and DE is the transversal

Then,

$\angle DEF = \angle FED$

$\Rightarrow \angle FED = 65^\circ$

$\therefore x^\circ = \angle BEF + \angle FED$

$x^\circ = 35^\circ + 65^\circ$

$x^\circ = 100^\circ$

68. PQ intersect RS at O

$\therefore \angle QOS = \angle POR$ [vert'ically opposite angles]

a = 4b ...(1)

Also,

a + b + 75° = 180° [$\because$POQ is a straight lines]

$\therefore$ a + b = 180° - 75°

= 105°

Using, (1)

4b + b = 105°

5b = 105°

Or

b = = $\frac{105^\circ}{5}$ = 21°

Now a=4b

a = 4 $\times$ 21°

a = 84°

Again,$\angle$QOR and $\angle$QOS

$\therefore$ a + 2c = 180°

Using, (2) 84° + 2c = 180°

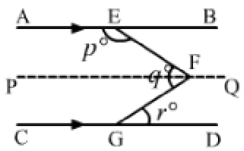
2c = 180° - 84°

2c = 96°

c = $\frac{96^0}{2}$ = 48°c

Hence,

a = 84°, b = 21° and c = 48°

69. 

Draw PFQ $\parallel$ AB $\parallel$ CD

Now, PFQ $\parallel$ AB and EF is the transversal.

Then,

$\angle AEF + \angle EFP = 180^\circ$ ...(i)

[Angles on the same side of a transversal line are supplementary]

Also, PFQ || CD.

$\angle PFG = \angle FGD = r^\circ$ [Alternate Angles]

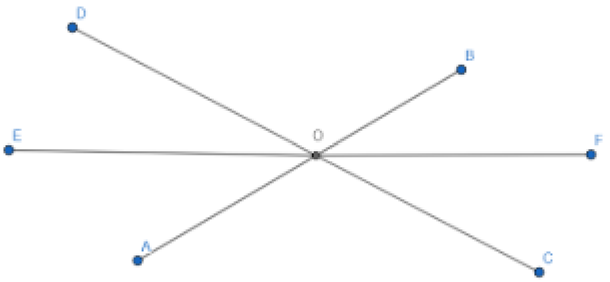
and $\angle EFP = \angle EFG - \angle PFG = q^\circ - r^\circ$

putting the value of ∠EFP in equation (i)

we get,

p° + q °- r° = 180° [∠AEF = p°]

70. AB and CD are straight lines intersecting at O. OE the bisector of angles ∠AOD and OF is the bisector of ∠BOC.



∠AOC = ∠BOD (vertically opposite angles)

Also,

OE is the bisector of *∠AOD* and OF is the bisector of *∠BOC*

To prove: EOF is a straight line.

∠AOD = ∠BOC = 2x (Vertically opposite angle) ...(i)

As OE and OF are bisectors.So *∠AOE = ∠BOF = x .....(ii)*

∠AOD + ∠BOD = 180° (linear pair)

∠AOE + ∠EOD + ∠DOB = 180°

From (ii)

∠BOF + ∠EOD + ∠DOB = 180°

∠EOF = *180^o*

EF is a straight line.

71. i. In △BOD,

∠OBD + ∠BOD + ∠ODB = 180^o

(The sum of the three angles of a triangle is 180^o)

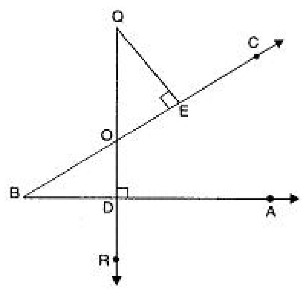
⇒ ∠OBD + ∠BOD + 90^o = 180^o

⇒ ∠OBD + ∠BOD = 90^o .... (1)

In△OEQ,

∠EQO + ∠QOE + ∠OEQ = 180^o .... (2)

(The sum of the three angles of a triangle is 180^o)



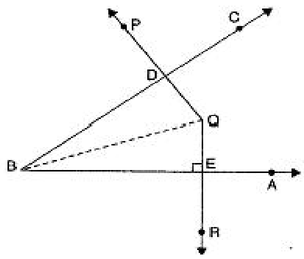
⇒ ∠EQO + ∠QOE + 90^o = 180^o

⇒ ∠EQO + ∠QOE = 90^o .... (2)

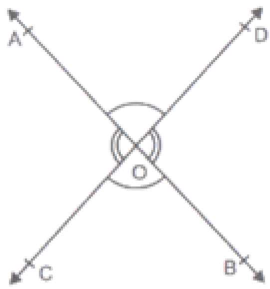
From (1) and (2), we get

∠OBD + ∠BOD = ∠EQO + ∠QOE

But $\angle BOD = \angle QOE$ (Vertically Opposite Angles)
$\therefore \angle OBD = \angle EQO$
ii. Join BQ
In$\triangle BDQ$,
$\angle DBQ + \angle BQD + \angle QDB = 180^o$
(The sum of the three angles of a triangle is 180^o)
$\Rightarrow \angle DBQ + \angle BQD + 90^o = 180^o$
$\Rightarrow \angle DBQ + \angle BQD = 90^o$ .... (1)



In$\triangle BQE$,
$\angle EBQ + \angle BQE + \angle BEQ = 180^o$
(The sum of the three angles of a triangle is 180^o)
$\Rightarrow \angle EBQ + \angle BQE + 90^o = 180^o$
$\Rightarrow \angle EBQ + \angle BQE = 90^o$
Adding (1) and (2), we get
$(\angle DBQ + \angle EBQ) + (\angle BQD + \angle BQE) = 90^o + 90^o = 180^o$
$\Rightarrow \angle DBE + \angle EQD = 180^o$
$\Rightarrow \angle DBE$ and $\angle EQD$ are supplementary.

72. 

Let two lines AB and CD intersect at point O.
To prove: $\angle AOC = \angle BOD$ (vertically opposite angles)
$\angle AOD = \angle BOC$ (vertically opposite angles)
Proof: (i) Since, ray OA stands on the line CD.
$\Rightarrow \angle AOC + \angle AOD$= 180 °…(1)[Linear pair axiom]
Also, ray OD stands on the line AB.
$\angle AOD + \angle BOD = 180^\circ$ …(2) [Linear pair axiom]
From equations (1) and (2), we get
$\angle AOC + \angle AOD = \angle AOD + \angle BOD$
$\Rightarrow \angle AOC = \angle BOD$
Hence, proved.
(ii) Since, ray OD stands on the line AB.
$\therefore \angle AOD + \angle BOD = 180^\circ$ …(3) [Linear pair axiom]
Also, ray OB stands on the line CD.
$\therefore \angle DOB + \angle BOC = 180^\circ$ …(4) [linear pair axiom]
From equations (3) and (4), we get
$\angle AOD + \angle BOD = \angle BOD + \angle BOC$
$\Rightarrow \angle AOD = \angle BOC$
Hence, proved.

73. $\angle AOF + \angle FOG = 180^o$ . . . . [Linear pair axiom]

$\Rightarrow \angle AOG = 180^o$

$\Rightarrow \angle AOB + \angle EOB + \angle FOE + \angle FOG = 180^o$

$\Rightarrow 30^o + 90^o + \angle FOE + 30^o = 180^o$

$\Rightarrow \angle FOE + 150^o = 180^o$

$\Rightarrow \angle FOE = 180^o - 150^o = 30^o$

$\angle AOF + \angle FOG = 180^o$ . . . . [Linear pair axiom]

$\Rightarrow \angle AOG = 180^o$

$\Rightarrow \angle AOB + \angle COB + \angle FOC + \angle FOG = 180^o$

$\Rightarrow 30^o + \angle COB + 90^o + 30^o = 180^o$

$\Rightarrow \angle COB + 150^o = 180^o$

$\Rightarrow \angle COB = 180^o - 150^o = 30^o$

$\angle FOC = 90^o$

$\Rightarrow \angle FOE + \angle DOE + \angle DOC = 90^o$

$\Rightarrow 30^o + \angle DOE + 30^o = 90^o$

$\Rightarrow \angle DOE + 60^o = 90^o$

$\Rightarrow \angle DEO = 30^o$

74. To Prove: $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Given: OR is perpendicular to PQ, or $\angle QOR = 90°$

From the given figure, we can conclude that $\angle POR$ and $\angle QOR$ form a linear pair.

We know that sum of the angles of a linear pair is 180°.

$\therefore \angle POR + \angle QOR = 180°$

or $\angle POR = 90°$

From the figure, we can conclude that

$\angle POR = \angle POS + \angle ROS$

$\Rightarrow \angle POS + \angle ROS = 90°$

$\Rightarrow \angle ROS = 90° - \angle POS$...(i)

Again,

$\angle QOS + \angle POS = 180°$

$\Rightarrow \frac{1}{2}(\angle QOS + \angle POS) = 90°$.(ii)

Substitute (ii) in (i), to get

$\angle ROS = \frac{1}{2}(\angle QOS + \angle POS) - \angle POS$

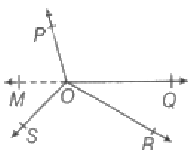
$= \frac{1}{2}(\angle QOS - \angle POS)$.

Therefore, the desired result is proved.

75. Let us produce a ray OQ backwards to a point M, then MOQ is a straight line.

Now, OP is a ray on the line MOQ. Then, by linear pair axiom, we have

$\angle MOP + \angle POQ = 180^o$ ......(i)



Similarly, OS is a ray on the line MOQ. Then, by linear pair axiom, we have

$\angle MOS + \angle SOQ = 180^o$ ....(ii)

Also, $\angle SOR$ and $\angle ROQ$ are adjacent angles.

$\therefore \angle SOQ = \angle SOR + \angle ROQ$ ...(iii)

On putting the value of $\angle SOQ$ from Eq.(iii) in Eq.(ii), we get

$\angle MOS + \angle SOR + \angle ROQ = 180^o$ ....(iv)

Now, on adding Eqs.(i) and (iv), we get

$\angle MOP + \angle POQ + \angle MOS + \angle SOR + \angle ROQ = 180^o + 180^o$

$\Rightarrow \angle MOP + \angle MOS + \angle POQ + \angle SOR + \angle ROQ = 360^o$ ....(iv)

But $\angle MOP + \angle MOS = \angle POS$

Then, from Eq.(v), we get

$\angle POS + \angle POQ + \angle SOR + \angle ROQ = 360^o$

Hence proved.