Solution

INTRODUCTION TO ELUCID’S GEOMETRY

Class 09 - Mathematics

Section A

1.
   (b) lines and curves
   Explanation:
   lines and curves.

2.
   (c) Solids – surfaces – lines – points
   Explanation:
   In each step we lose one dimension. solid- 3 dimension. surface- 2 dimension. lines- 1 dimension. point- has no dimension

3.
   (c) 2
   Explanation:
   A line segment has two end points.

4. **(a)** MC + CN = MN
   Explanation:
   Since, P lies between M and N, MN = MP + PN .....(i)
   Now, C is the mid-point of MP, so, MP = MC + CP
   $\Rightarrow$ MN = MC+CP+PN
   $\Rightarrow$ MN = MC +CN (CP+PN =CN)

5.
   (b) Greece
   Explanation:
   Thales, a Greek mathematician cum philosopher lived in Pre-Socratic times around 620-625 BC. He is commonly known as Thales of Miletus as he belonged to Miletus in Asian region.

6. **(a)** If two things are equal, then their sum is equal to $\frac{1}{3}$ of the one thing.
   Explanation:
   If two things are equal, then their sum is equal to $\frac{1}{3}$ of the one thing.

7.
   (c) 0
   Explanation:
   A point has 0 dimensions.

8.
   (d) An axiom
   Explanation:
   This is Euclid's third axiom stating subtraction of equals. An algebraic version of Euclid’s second axiom would read “if x = y, and if a = b, then x – a = y – b.”

9. **(a)** 1
   Explanation:
   A unique plane passes through 3 given noncollinear points.

10.

(c) Concurrent

Explanation:

When three or more lines intersect in one point, they are concurrent. The point at which they intersect is the point of concurrency.

11.

(c) squares and circles

Explanation:

In ancient India, squares and circular altars were used for household rituals. The geometry of the Vedic period originated with the construction of altars (or vedis) and fireplaces for performing Vedic rites. Square and circular altars were used for household rituals, while altars, whose shapes were combinations of rectangles, triangles and trapeziums, were required for public worship.

12.

(b) Thales

Explanation:

Pythagoras was a student of Thales.

13.

(d) Circle

Explanation:

A rectilinear figure is a figure all of whose edges meet at right angles. A circle has no edge.

14. **(a)** AB

Explanation:

AB

15.

(c) 1

Explanation:

Since the three points are collinear, they lie on the same line,so only one line can be determined by them.

16.

(c) three or more line segments

Explanation:

Polygons are 2-dimensional shapes. They are made of straight lines, and the shape is "closed" (all the lines connect up). Polygon comes from the Greek words, Poly- means "many" and -gon means "angle".

17.

(c) 13

Explanation:

Euclid divided his book 'The Elements' into 13 chapters.

18. **(a)** Surfaces

Explanation:

A solid has shape, size, position and can be moved from one place to another. Its boundaries are called surfaces. They separate one part of the space from the other.

19.

(c) 6

Explanation:

If the four points are A,B,C and D, we can drawthe lines: A–B, A–C, A–D, B–C, B–D, C–D

20.

(d) 49 cm^2

Explanation:

Since, AB = 10cm, C = 3 cm, therefore BC = AB - AC = 10 - 3 = 7cm. Hence, BC^2 = 49 cm^2

21.

(d) Greece

Explanation:

Euclid, sometimes called Euclid of Alexandria to distinguish him from Euclides of Megara, was a Greek mathematician, often referred to as the "father of geometry".

22. **(a)** 3

Explanation:

You need two points to draw a line segment. If the points A,B and are non-collinear,we can draw the lines :AB , AC, BA , BC, CA , CB. Now, line AB is the same as line BA, same for lines AC and CA and BC and CB. So,the lines are: AB, BC and AC ; 3 lines only.

23.

(d) Weight of A = Weight of B

Explanation:

Let the weights of A and B be x kgs. If both of them gain weight by 3 kgs, their new weight would be 'x + 3' kgs.
According to Euclid's axiom if equals are added in equals, then whole are equal.
Hence, Weight of A = Weight of B.

24.

(d) Nine

Explanation:

The Sriyantra (in the Atharvaveda) consists of nine interwoven isosceles triangles.

25.

(b) C is an interior point of AB such that $\overrightarrow{AC} = \overrightarrow{CB}$

Explanation:

A point C is called the midpoint of line segment $\overrightarrow{AB}$, if C is an interior point of $\overrightarrow{AB}$ such that $\overrightarrow{AC} = \overrightarrow{CB}$

26.

(c) Equal

Explanation:

Equal

27. **(a)** Same

Explanation:

Same

28.

(b) Unique

Explanation:

Unique

29.

(d) Triangles

Explanation:

because when we connect base edge and apex it forms a triangle.

30. **(a)** equal

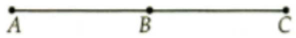
Explanation:

According to an Euclidian axiom, The things which are double of the same things are equal to one another. Example : if 2x = 2y, then x = y.

31.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:



AB + BC = AC

Section B

32. We have AB = BC [Given]

Now, by Euclid's axiom 7, we have things which are halves of the same things are equal to one another.

$\therefore\ \frac{1}{2}$AB = $\frac{1}{2}$BC

Hence, BX = BY. [$\because$ BX = $\frac{1}{2}$AB and BY = $\frac{1}{2}$BC (Given)]

33. We are given that,

x + y = 10 ...(i)

and x = z ...(ii)

According to Euclid's axioms, if equals are added to equals, the wholes are equal.

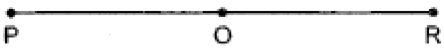
Therefore, From Eq.(ii),

x + y = z + y ...(iii)

From Equations (i) and (iii)

z + y = 10.

34.



Given : In the fig. PR is a line segment such that, PO = OR,

Proof: From Fig. we have

PO + OR = PR ...(i)

PO = OR (Given) ...(ii)

PO + PO = PR [Using (ii) in (i)]

2PO = PR

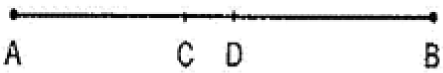
Therefore $PO = \frac{1}{2}PR$

35. It is known that, if two lines intersect each other, then the vertically opposite angles are equal. It is a theorem, therefore,given Statement I is false and not an axiom.

Also, we know that, if a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°. It is an axiom. Therefore, given statement parallel is true and an axiom.

Thus, in given statements, first is false and second is an axiom. Therefore, given system of axioms is not consistent.

36.



Let a line AB have two mid-points, say, C and D. Then

AB = AC + CB = 2AC . . . . (i) . . . [As C is the mid-point of AB]

and AB = AD + DB = 2AD . . . . (ii) [As D is the mid-point of AB]

From equation (i) and (ii)

AC = AD and CB = DB

But this will possible only when D lies on point C. So every line segment has one and only one mid-point.

37. We need to prove that Euclid's fifth axiom is considered as a universal truth.

Euclid's fifth axiom states that "the whole is greater than the part."

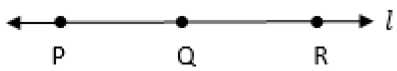
The above given axiom is a universal truth. We can apply the fifth axiom not only mathematically but also universally in daily life.

Mathematical proof:

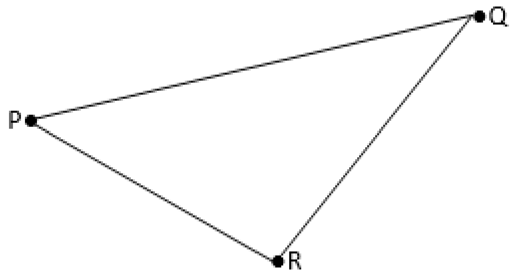
Let us consider a quantity z, which has different parts as a, b, x and y. .
Therefore, we can conclude that z will always be greater than its corresponding parts a, b, x and y.
Universal proof:
We know that Mumbai is located in Maharashtra and Maharashtra is located in India.
In other words, we can conclude that Mumbai is a part of Maharashtra and Maharashtra is a part of India.
Therefore, we can conclude that whole India will be greater than Mumbai or Maharashtra or both.
Therefore, we can conclude that Euclid's fifth axiom is considered as a 'Universal truth'.

38. The equivalent versions of Euclid's fifth postulate are mentioned as follows:
    i. For every line l and for every point P not lying on Z, there exists a unique line m passing through P and parallel to Z.
    ii. Two distinct intersecting lines cannot be parallel to the same line.

    From above two statements it follows given statements is not an equivalent version to the Euclid's fifth postulate.
39. Proof: Let us prove this statement by contradiction method. Let us assume that the line segment PT has two midpoints R and S.
    

    $\Rightarrow \quad PR = \frac{1}{2}PT$ ....(1)
    $PS = \frac{1}{2}PT$ .... (2) ($\because$ R and S are mid-points according to assumption)
    from (1) and (2) , we get
    $\Rightarrow$ PR = PS
    But this is possible only if R and S coincide. { which may not be possible if R and S are two different points }
    Therefore,Our contradiction is incorrect.
    i.e. there is only one mid-point of every line segment.
    Hence, proved
40. We have,
    a - 15 = 25.
    On adding 15 to both sides, we have
    a - 15 + 15 = 25 + 15 = 40 (using Euclid's second axiom).or a = 40
41. We have AX = CY [Given]
    Now, by Euclid's axiom 6, we have things which are double of the same thing are equal to one another,
    so 2AX = 2CY
    Hence, AC = BC. [$\because$ X and Y are the mid- points of AC and BC]
42. We have AB = BC …(1) [Given]
    And BX = BY …(2) [Given]
    Subtracting (2) from (1), we get
    AB - BX = BC - BY
    Now, By Euclid's axiom 3, we have
    If equals are subtracted from equals, the remainder are equal.
    Hence, AX = CY.
43. We have
    $\Rightarrow \angle$ABC = $\angle$ACB …(1) [(Given)]
    And $\angle 4 = \angle 3$ …(2) [(Given)]
    Now, subtracting (2) from (1), we get
    Now, by Euclid's axiom 3, if equals are subtracted from equals, the remainders are equal.
    $\angle \text{ABC} - \angle 4 = \angle \text{ACB} - \angle 3$
    Hence, $\angle 1 = \angle 2$.
44. In the given equation, we have
    u - 5 = 15
    On adding 5 to both sides, we have
    u - 5 + 5 = 15 + 5 { on applying Euclid's axiom }
    we get, u = 20.
45. One if they are collinear and three if they are non-collinear
    For Collinear Points:
    

    For three non-collinear points:



From the above two figures,it follows that only one line can be drawn if three points are collinear and three lines can be drawn if three points are non-collinear.

46. Let x kg be the weight each of Ram and Ravi. On gaining 2 kg, the weight of Ram and Ravi will be (x + 2) each. By Euclid’s second axiom, when equals are added to equals, the wholes are equal. Therefore, the weight of Ram and Ravi are again equal.
47. Euclid's Axiom 5 states that "The whole is greater than the part. Since this is true for anything in any part of the world. So, this is a universal truth.
48. We have
$\angle 1 = \angle 2$ [Given]
$\angle 2 = \angle 3$ [Given]
Now, by Euclid’s axiom 1, things which are equal to the same thing are equal to one other.
Hence, $\angle 1 = \angle 3$.
49. We have $\angle 1 = \angle 3$ …(1) [Given]
And $\angle 2 = \angle 4$ …(2) [Given]
Now, by Euclid’s axiom 2, we have if equal are added to equals, the whole are equal.
Adding (1) and (2), we get
$\angle 1 + \angle 2 = \angle 3 + \angle 4$
Hence, $\angle A = \angle C$.
50. In the above figure, we have
AB = AC + BC = AC + AC = 2AC (Since, C is the mid-point of AB) ..(1)
XY = XD + DY = XD + XD = 2XD (Since, D is the mid-point of XY) ..(2)
Also, AC = XD (Given) ..(3)
From (1),(2)and(3), we get
AB = XY, According to Euclid, things which are double of the same things are equal to one another.
51. AC = BD . . . . [Given] . . . (1)
AC = AB + BC . . . . [Point B lies between A and C] . . . . (2)
BD = BC + CD . . . . [Point C lies between B and D] . . . . (3)
Substituting (2) and (3) in (1), we get
AB + BC = BC + CD
$\Rightarrow$ AB = CD . . . . [Subtracting equals from equals]

Section C

52. We have OX = PX [Given]
Now
$\because$ OX = $\frac{1}{2}$XY (Given) and PX = $\frac{1}{2}$XZ [Given]
$\therefore$ $\frac{1}{2}$XY = $\frac{1}{2}$XZ [$\because$ Things which are halves of the same thing are equal to one another (Euclid’s axiom 7)]
$\therefore$ XY = XZ [$\because$ Things which are double of the same thing are equal to one another (Euclid’s axiom 6)]
53. We have AC = DC …(1) [Given]
And CB = CE…(2) [Given]
Now, by axiom 2, if equals are added to equals, the wholes are equal.
Adding (1) and (2), we get
AC + CB = DC + CE
Hence, AB = DE.
54. The terms need to be defined are:
Polygon: A simple closed figure made up of three or more line segments.
Line segment: Part of a line with two end points.
Line: Undefined term.

Point: Undefined term
Angle: A figure formed by two rays with a common initial point.
Ray: Part of a line with one end point.
Right angle: Angle whose measure is 90°.
Undefined terms used are: line, point.
Euclid's fourth postulate says that "all right angles are equal to one another."
In a square, all angles are right angles, therefore, all angles are equal (From Euclid's fourth postulate).
Three line segments are equal to fourth line segment (Given).
Therefore, all the four sides of a square are equal. (by Euclid's first axiom "things which are equal to the same thing are equal to one another.")

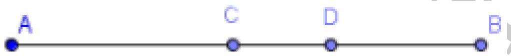
55. From the above Fig. we have
$\angle 3 = \angle 4, \quad \angle 1 = \angle 3$ and $\angle 2 = \angle 4$ [Given] ..(1)
According to Euclid's first axiom, the things which are equal to equal things are equal to one another.
Therefore, $\angle 1 = \angle 2$ [from (1)]

Section D

56. i. Five line segments are: $\overline{\mathrm{PQ}}, \overline{\mathrm{PN}}, \overline{\mathrm{RS}}, \overline{\mathrm{ND}}, \overline{\mathrm{TL}}$
ii. Five rays are: $\underset{QC}{\longrightarrow}, \underset{PM}{\longrightarrow}, \underset{RB}{\longrightarrow}, \underset{DF}{\longrightarrow}, \underset{LH}{\longrightarrow}$
iii. Four Collinear points are: A, P, R, B
iv. Two pairs of non-intersecting line segments are: PN, RS and PQ, TL

57. i. $\overleftrightarrow{EF}$, $\overleftrightarrow{GH}$ and their corresponding point of intersection is R.
$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and their corresponding point of intersection is P.
ii. $\overleftrightarrow{AB}$, $\overleftrightarrow{EF}$, $\overleftrightarrow{GH}$ and their point of intersection is R.
iii. Three rays are: $\overrightarrow{\mathrm{RB}}$, $\overrightarrow{\mathrm{RH}}$, $\overrightarrow{\mathrm{RG}}$
iv. Two line segments are: $\overline{RQ}, \overline{RP}$.

58. We need to prove that every line segment has one and only one mid-point. Let us consider the given below line segment AB and assume that C and D are the mid-points of the line segment AB



If C is the mid-point of line segment AB, then
AC = CB.
An axiom of the Euclid says that “If equals are added to equals, the wholes are equal.”
AC + AC = CB + AC....(i)
From the figure, we can conclude that CB + AC will coincide with AB.
An axiom of the Euclid says that “Things which coincide with one another are equal to one another.” AC + AC = AB....(ii)
An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”
Let us compare equations (i) and (ii), to get
AC + AC = AB, or 2AC = AB.(iii)
If D is the mid-point of line segment AB, then
AD = DB.
An axiom of the Euclid says that “If equals are added to equals, the wholes are equal.”
AD + AD = DB + AD....(iv)
From the figure, we can conclude that DB + AD will coincide with AB.
An axiom of the Euclid says that “Things which coincide with one another are equal to one another.”
AD + AD = AB....(v)
An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”
Let us compare equations (iv) and (v), to get
AD + AD = AB, or
2AD = AB....(vi)
An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.” Let us compare equations (iii) and (vi), to get
2AC = 2AD.

An axiom of the Euclid says that “Things which are halves of the same things are equal to one another.” AC = AD.
Therefore, we can conclude that the assumption that we made previously is false and a line segment has one and only one mid-point.

59. i. From the above figure, We have AB = BC…(1) [Given]
Now, A, M, B are the three points on a line, and M lies between A and B such that M is the mid point of AB [Given], then
AM + MB = AB …(2) Also B, N, C are three points on a line such that N is the mid point of BC [Given]
Similarly, BN + NC = BC....(3)
So, we get AM + MB = BN + NC
From (1), (2), (3) and Euclid’s first axiom
Since M is the mid-point of AB and N is the mid-point of BC, therefore
2AM = 2NC i.e. AM = NC
Hence, AM = NC. Proved
Using axiom 6, things which are double of the same thing are equal to one another.
ii. From the above figure, We have BM = BN …(1) [Given]
As M is the mid-point of AB [Given] , so that
BM = AM…(2)
And N is the mid-point of BC [Given]
BN = NC…(3)
From (1), (2) and (3) and Euclid’s first axiom, we get
AM = NC…(4)
Adding (4) and (1), we get
AM + BM = NC + BN
Hence, AB = BC Proved
[By axiom 2 if equals are added to equals, the wholes are equal]

60.
- Six points: A,B,C,D,E,F
- Five line segments: $\overline{\mathrm{EG}}, \overline{\mathrm{FH}}, \overline{\mathrm{EF}}, \overline{\mathrm{GH}}, \overline{\mathrm{MN}}$
- Four rays: $\overrightarrow{\mathrm{EP}}, \overrightarrow{\mathrm{GR}}, \overrightarrow{\mathrm{GB}}, \overrightarrow{\mathrm{HD}}$
- Four lines: = $\overleftrightarrow{AB}, \overleftrightarrow{CD}$, $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$
- Four collinear points: M,E,G,B