Solution

INTRODUCTION TO TRIGONOMETRY

Class 10 - Mathematics

Section A

1. **(a)** 1

 Explanation:

 Given: $\frac{\tan 30^\circ}{\cot 60^\circ}$

 $= \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = 1$

2. **(a)** $\frac{b}{\sqrt{b^2-a^2}}$

 Explanation:

 Sin $\theta = \frac{a}{b}$

 cos $\theta = \sqrt{1 - sin^2\theta}$

 $= \sqrt{1 - \frac{a^2}{b^2}}$

 $= \sqrt{\frac{b^2-a^2}{b^2}}$

 Now,

 Sec $\theta = \frac{1}{cos\theta}$

 $= \sqrt{\frac{b^2}{b^2-a^2}}$

 $= \frac{b}{\sqrt{b^2-a^2}}$

3. **(a)** $\frac{m^2-n^2}{m^2+n^2}$

 Explanation:

 Given: tan$\theta = \frac{m}{n}$

 Dividing all the terms of $\frac{m\sin\theta - n\cos\theta}{m\sin\theta + n\cos\theta}$ by cosθ,

 $= \frac{m\tan\theta - n}{m\tan\theta + n}$

 $= \frac{m \times \frac{m}{n} - n}{m \times \frac{m}{n} + n}$

 $= \frac{m^2-n^2}{m^2+n^2}$

4. **(a)** cos A

 Explanation:

 $(\sec A + \tan A)(1 - \sin A)$

 $= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

 $= [\frac{1+SinA}{CosA}] \times (1 - SinA) = \frac{1-Sin^2A}{CosA}$

$= \frac{\cos^2 A}{\cos A} = \cos A$

Hence, the correct choice is cos A

5.

(c) $\sec^2$ A

Explanation:

$\frac{\operatorname{cosec}^2 A - \cot^2 A}{1 - \sin^2 A}$

$\frac{1}{\cos^2 A} = \sec^2 A$

6.

(d) -1

Explanation:

$\frac{1}{\cot^2 \theta} - \frac{1}{\cos^2 \theta}$

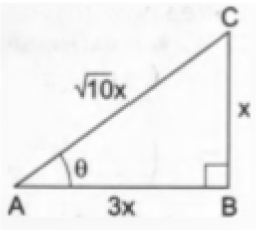
$= \text{Tan}^2\theta - \sec^2\theta$

= -1

7.

(c) $\frac{\sqrt{10}}{3}$

Explanation:



$cosec\theta = \frac{AC}{BC} = \frac{\sqrt{10}}{1} = \frac{\sqrt{10}x}{x} \Rightarrow AC = \sqrt{10x}$ and BC = x.

$\therefore$ $AB^2 = AC^2 - BC^2 = (\sqrt{10}x)^2 - \left(x^2\right) = 10x^2 - x^2 = 9x^2$

$\Rightarrow AB = \sqrt{9x^2} = 3x$

$\therefore \sec\theta = \frac{AC}{AB} = \frac{\sqrt{10}x}{3x} = \frac{\sqrt{10}}{3}$

8.

(d) 1

Explanation:

$\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \cos 45^\circ$

$\theta = 45^\circ$

$\text{Tan}\theta = \text{Tan}45^\circ$

= 1

9. **(a)** 25

Explanation:

Given that,

a cos θ + b sin θ = 4

a sin θ - b cos θ = 3

Squaring and adding, we get

$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta = 16$

$a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta = 9$

$a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = 25$ ($\because \sin^2 \theta + \cos^2 \theta = 1$)

$\Rightarrow a^2 \times 1 + b^2 \times 1 = 25$

$\Rightarrow a^2 + b^2 = 25$

10. **(a)** $x^2 + y^2 + z^2 = r^2$

Explanation:

$x = r\sin\theta\cos\phi \Rightarrow \frac{x}{r} = \sin\theta\cos\phi$ ...(i)

$y = r\sin\theta\sin\phi \Rightarrow \frac{y}{r} = \sin\theta\sin\phi$ ...(ii)

$z = r\cos\theta \Rightarrow \frac{z}{r} = \cos\theta$ ...(iii)

Squaring and adding (i) and (ii)

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi$

$= \sin^2\theta\left(\cos^2\phi + \sin^2\phi\right)$

$= \sin^2\theta \times 1 \quad \left\{\sin^2\theta + \cos^2\theta = 1\right\}$

$= \sin^2\theta$

Now adding (iii) in it

$\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = \sin^2\theta + \cos^2\theta = 1$

Hence $\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = 1$

$\Rightarrow \frac{x^2+y^2+z^2}{r^2} = 1$

$\Rightarrow x^2 + y^2 + z^2 = r^2$

11.

(b) a^2b^2

Explanation:

Given: $x = a \cos\theta$ and $y = b \sin\theta$

$\therefore b^2x^2 + a^2y^2$

$= b^2(a \cos\theta)^2 + a^2(b \sin\theta)^2$

$= b^2a^2 \cos^2\theta + a^2b^2\sin^2\theta$

$\Rightarrow b^2x^2 + a^2y^2$

$= a^2b^2(\cos^2\theta + \sin^2\theta)$

$= a^2b^2$

[$\because \sin^2\theta + \cos^2\theta = 1$]

12. **(a)** $\tan\theta$

Explanation:

Here $\sqrt{\left(1 - \cos^2\theta\right)\sec^2\theta}$

$=\sqrt{\sin^2\theta \times \frac{1}{\cos^2\theta}}$

[$\because 1 - \cos^2\theta = \sin^2\theta$ and $\sec^2\theta = \frac{1}{\cos^2\theta}$

$= \sqrt{\frac{\sin^2\theta}{\cos^2\theta}}$

$= \sqrt{\tan^2\theta}$

$= \tan\theta$

13. **(a)** 2 cosecθ

Explanation:

We have, $\frac{\tan\theta}{\sec\theta - 1} + \frac{\tan\theta}{\sec\theta + 1}$

$= \tan\theta\left(\frac{1}{\sec\theta - 1} + \frac{1}{\sec\theta + 1}\right)$

$= \frac{\tan\theta(\sec\theta + 1 + \sec\theta - 1)}{(\sec\theta - 1)(\sec\theta + 1)}$

$= \frac{\tan\theta \times 2\sec\theta}{\sec^2\theta - 1} = \frac{2\tan\theta\sec\theta}{\tan^2\theta}$

$= \frac{2\sec\theta}{\tan\theta} = \frac{2 \times \cos\theta}{\cos\theta \times \sin\theta} = \frac{2}{\sin\theta}$

$= 2cosec\theta$

14.

(b) 60^o

Explanation:

$\tan^2\theta = 3$

$\tan\theta = \sqrt{3}$

$\tan\theta = \tan 60^\circ$

$\theta = 60^\circ$

15.

(b) 2

Explanation:

Since $\sec\theta = \sqrt{1 + \tan^2\theta}$

$\therefore \sec\theta = \sqrt{1 + \left(\sqrt{3}\right)^2}$

$= \sqrt{1 + 3} = \sqrt{4} = 2$

16.

(d) 0

Explanation:

$= \frac{1 - tan^2 45^0}{1 + tan^2 45^0} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

17.

(d) 0

Explanation:

$5\tan\theta - 4 = 0 \Rightarrow 5\tan\theta = 4$

$\Rightarrow \tan\theta = \frac{4}{5}$

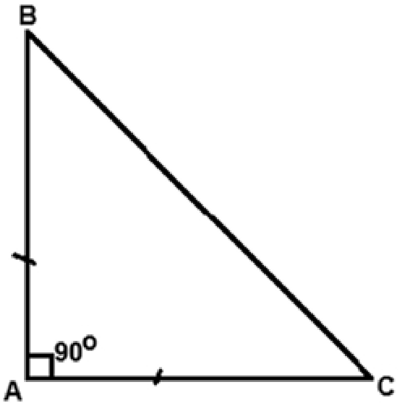
Now, $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 4\frac{\cos\theta}{\cos\theta}}{5\frac{\sin\theta}{\cos\theta} + 4\frac{\cos\theta}{\cos\theta}}$ (Dividing by cos θ)

$= \frac{5\tan\theta - 4}{5\tan\theta + 4}$

$= \frac{5 \times \frac{4}{5} - 4}{5 \times \frac{4}{5} + 4} = \frac{4-4}{4+4} = \frac{0}{8} = 0$

18. **(a)** $\frac{1}{\sqrt{2}}$

Explanation:



Since AB = AC

$\angle B = \angle C$

In $\triangle$ABC

$\angle A + \angle B + \angle C = 180^o$

$90^\circ + \angle C + \angle C = 180^\circ$

$2\angle C = 90^\circ$

$\angle C = 45^\circ$

sin c = sin 45^o

$= \frac{1}{\sqrt{2}}$

19.

(b) $\frac{\sqrt{b^2 - a^2}}{b}$

Explanation:

$\cos^2\theta = \left(1 - \sin^2\theta\right) = \left(1 - \frac{a^2}{b^2}\right) = \frac{b^2 - a^2}{b^2} \Rightarrow \cos\theta = \frac{\sqrt{b^2 - a^2}}{b}$

20.

(b) -5

Explanation:

$\frac{5}{\cot^2\theta} - \frac{5}{\cos^2\theta}$

$= 5\tan^2\theta - 5\sec^2\theta$

$5\left(\tan^2\theta - \sec^2\theta\right)$

= 5(-1)

= -5

21. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

As $\angle$A + $\angle$B + $\angle$C = 180^o and $\angle$B = 90^o

So, $\angle$A + $\angle$C = 90^o

Hence, sin(A + C) = sin 90^o = 1

22.

(d) A is false but R is true.

Explanation:

sin θ cannot be equal to $\frac{1}{Tan\ \theta}$. But tanθ = $\frac{Sin\ \theta}{Cos\ \theta}$

23.

(d) A is false but R is true.

Explanation:

A is false but R is true.

24. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$\tan 30^o = \frac{AB}{BC} = \frac{AB}{20}$

$AB = \frac{1}{\sqrt{3}} \times 20 = \frac{20}{1.73} = 11.56$ m

25. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$\sin\theta = \frac{P}{H} = \frac{4}{3}$

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.

26. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$\because \ \text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \ (\text{cosec}\theta - \cot\theta)(\text{cosec}\theta + \cot\theta) = 1$

$\Rightarrow \ (\text{cosec}\theta - \cot\theta) = \frac{1}{\text{cosec}\,\theta + \cot\,\theta}$

$\therefore$ It is clear that cosecθ - cotθ and cosecθ + cotθ are reciprocal of each other.

27. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$\because \ \text{cosec}^2\,\theta - \cot^2\,\theta = 1$

$\Rightarrow \ (\text{cosec}\,\theta - \cot\,\theta)(\text{cosec}\,\theta + \cot\,\theta) = 1$

$\Rightarrow \ (\text{cosec}\,\theta - \cot\,\theta) = \frac{1}{\text{cosec}\,\theta + \cot\,\theta}$

$\therefore$ It is clear that cosec θ - cot θ and cosec θ + cot θ are reciprocal of each other.

28.

(d) A is false but R is true.

Explanation:

$\cos A + \cos^2 A = 1$

$\cos A = 1 - \cos^2 A = \sin^2 A$

$\sin^2 A + \sin^4 A = \cos A + \cos^2 A = 1$

$\sin^2 A + \sin^4 A = 1$

29.

(d) A is false but R is true.

Explanation:

sin θ and cosec θ are reciprocal of each other so $\sin\theta \times \text{cosec}\,\theta = 1$

$\sin\theta \times \text{cosec}\,\theta \neq \cot\theta$

30. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

Section B

31. 90

Explanation:

Given: $2sin^2\theta - cos^2\theta = 2$

$\Rightarrow 2(1 - cos^2\theta) - cos^2\theta = 2$ [$\because sin^2\theta + cos^2\theta = 1$]

$\Rightarrow 2 - 2cos^2\theta - cos^2\theta = 2$

$\Rightarrow 2 - 3cos^2\theta = 2$

$\Rightarrow -3cos^2\theta = 2 - 2$

$\Rightarrow -3cos^2\theta = 0$

$\Rightarrow cos^2\theta = 0$

$\Rightarrow cos\theta = 0$

$\Rightarrow cos\theta = cos90°$

$\Rightarrow \theta = 90°$

32. 2

Explanation:

We have,.

$\tan\theta + \cot\theta = 2$

$\Rightarrow (\tan\theta + \cot\theta)^2 = 4$ [On squaring both sides]

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 4$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2 = 4$ [$\because \tan\theta\cot\theta = 1$]

$\Rightarrow \tan^2\theta + \cot^2\theta = 2$

33. 48

Explanation:

Here, $\tan\left(\frac{5\theta}{2}\right) = \sqrt{3}$

Also $\tan 60° = \sqrt{3}$

$\Rightarrow \tan\left(\frac{5\theta}{2}\right) = \tan 60°$

$\Rightarrow \frac{5\theta}{2} = 60°$

$\Rightarrow 5\theta = 120^\circ$

$\Rightarrow \theta = 24^\circ$

and $2\theta = 2 \times 24^\circ$

= 48°

34. 5

Explanation:

$= \frac{3\tan^2 30^\circ + \tan^2 60^\circ + cosec 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$

$= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$

$= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1}$

= 1 + 3 + 2 - 1

= 6 - 1

= 5

35. 16

Explanation:

Given:

$\cos\theta = \frac{3}{4}$

$\Rightarrow \frac{1}{\cos\theta} = \frac{4}{3}$

$\Rightarrow \sec\theta = \frac{4}{3}$

We know that

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\frac{4}{3}\right)^2 - \tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{16}{9} - 1$

$\Rightarrow \tan^2\theta = \frac{7}{9}$

Therefore,

$9\tan^2\theta + 9 = 9 \times \frac{7}{9} + 9$

= 7 + 9

= 16

36. 1

Explanation:

Given:

$cosec^2\theta(1 + \cos\theta)(1 - \cos\theta) = \lambda$

$\Rightarrow cosec^2\theta\{(1 + \cos\theta)(1 - \cos\theta)\} = \lambda$

$\Rightarrow \quad cosec^2\theta\left(1 - \cos^2\theta\right) = \lambda$

$\Rightarrow \quad \text{cosec}^2\theta\sin^2\theta = \lambda$

$\Rightarrow \quad \frac{1}{\sin^2\theta} \times \sin^2\theta = \lambda$

$\Rightarrow \quad 1 = \lambda$

$\Rightarrow \quad \lambda = 1$

Thus, the value of λ is 1.

37. 3

Explanation:

$3\,tan\,\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$

Given,

$= \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$

$= \frac{3\tan\theta + 2}{3\tan\theta - 2}$[Dividing numerator and denominator by cos θ]

$= \frac{\left(3 \times \frac{4}{3} + 2\right)}{\left(3 \times \frac{4}{3} - 2\right)} = \frac{6}{2} = 3$

38. 30

Explanation:

As per the question we have,

cos x = cos 60^o cos 30^o + sin 60^o sin 30^o.

$\Rightarrow \quad \cos x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$\Rightarrow \quad \cos x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$\Rightarrow \quad \cos x = \frac{\sqrt{3}}{2} \Rightarrow \cos x = \cos 30^\circ \Rightarrow x = 30^\circ$

39. Here, L.H.S. $= \cos(A + B) = \cos\left(30^\circ + 60^\circ\right) = \cos 90^\circ = 0$

$\text{R. H. S} = \text{cosAcosB} - \text{sinAsinB}$

$= \cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

$\therefore$ LHS = R.H.S

40. LHS

$$= \frac{\sin\theta - 2\sin^3\theta}{2\cos^2\theta - \cos\theta} = \frac{\sin\theta\left(1 - 2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$

$$= \frac{\sin\theta\left(\cos^2\theta + \sin^2\theta - 2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - \cos^2\theta - \sin^2\theta\right)} \quad \because \cos^2\theta + \sin^2\theta = 1$$

$$= \frac{\sin\theta\left(\cos^2\theta - \sin^2\theta\right)}{\cos\theta\left(\cos^2\theta - \sin^2\theta\right)} = \tan\theta$$

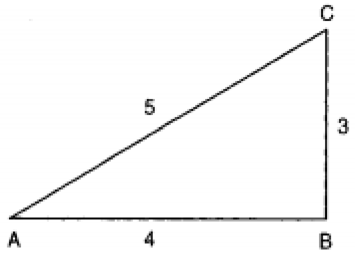
= RHS

41. In $\triangle ABC$, $\angle C = 90^\circ$ (Angle in a semi-circle)

$tanA = \frac{P}{B} = \frac{BC}{AC} = \frac{2}{3}$

and $tanB = \frac{P}{B} = \frac{AC}{BC} = \frac{3}{2}$

$\therefore \tan A.\tan B = \frac{2}{3} \times \frac{3}{2} = 1$

42. 

We have, $AB = 4$ *and* $BC = 3$.

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC = \sqrt{AB^2 + BC^2}$

$\Rightarrow \quad AC = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

When we consider the t-ratios of $\angle A$, we have

$Base = AB = 4$, $perpendicular = BC = 3$ and, $Hypotenuse = AC = 5$.

$\therefore \quad \sin A = \frac{BC}{AC} = \frac{3}{5}$, $\cos A = \frac{AB}{AC} = \frac{4}{5}$, $\tan A = \frac{BC}{AB} = \frac{3}{4}$

$cosecA = \frac{AC}{BC} = \frac{5}{3}$, $\sec A = \frac{AC}{AB} = \frac{5}{4}$ and , $\cot A = \frac{AB}{BC} = \frac{4}{3}$

43. $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

L.H.S. $= \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$

$= \frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} = \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}\left[\because 1 - \sin^2\theta = \cos^2\theta\right]$

$= 2\sec^2\theta$ [$\because \sec(x) = \frac{1}{\cos(x)}$]

= R.H.S. Proved

44. L.H.S

$\frac{1+\sec\theta}{\sec\theta} = \frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$

$= \frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{1}{\cos\theta}}$

$= \frac{1+\cos\theta}{1}$

Multiplying the numerator and denominator by $(1 - \cos\theta)$, we have

$\frac{1+\sec\theta}{\sec\theta} = \frac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)}$

$= \frac{1-\cos^2\theta}{1-\cos\theta}$

$= \frac{\sin^2\theta}{1-\cos\theta}$

45. We have,

L.H.S. = $\frac{(\cot\theta+cosec\theta)-1}{(\cot\theta-cosec\theta+1)}$

$= \frac{(cosec\theta+\cot\theta)-\left(cosec^2\theta-\cot^2\theta\right)}{(\cot\theta-cosec\theta+1)}$ [$\because$ 1 = cosec$^2\theta$ - cot$^2\theta$]

$= \frac{(cosec\theta+\cot\theta)[1-(cosec\theta-\cot\theta)]}{(\cot\theta-cosec\theta+1)}$

$= \frac{(cosec\theta+\cot\theta)(\cot\theta-cosec\theta+1)}{(\cot\theta-cosec\theta+1)}$

$= (cosec\theta + \cot\theta) = \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right) = \frac{1+\cos\theta}{\sin\theta}$ = R.H.S.

Hence, $\frac{\cot\theta+\ cosec\theta-1}{\cot\theta-\ cosec\theta+1} = \frac{1+\cos\theta}{\sin\theta}$.

46. L.H.S. $= \tan(A-B) = \tan\left(60^\circ - 60^\circ\right) = \tan 0^\circ = 0$

R. H. S. $= \frac{\tan A - \tan B}{1+\tan A \tan B}$

$= \frac{\tan 60^\circ - \tan 60^\circ}{1+\tan 60^\circ \tan 60^\circ}$

$= \frac{\sqrt{3}-\sqrt{3}}{1+\sqrt{3}\times\sqrt{3}} = \frac{0}{1+3} = 0$

$\therefore$ L.H.S. = R.H.S.

47. We have,

$\Rightarrow$ L. H. S = $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$

$\Rightarrow$ L.H.S $= \frac{(\sin A-\sin B)(\sin A+\sin B)+(\cos A+\cos B)(\cos A-\cos B)}{(\cos A+\cos B)(\sin A+\sin B)}$

$\Rightarrow$ L. H. S = $\frac{\sin^2 A-\sin^2 B+\cos^2 A-\cos^2 B}{(\cos A+\cos B)(\sin A+\sin B)}$ $\left[\because a^2-b^2=(a+b)(a-b)\right]$

$\Rightarrow$ L.H.S $= \frac{\left(\sin^2 A+\cos^2 A\right)-\left(\sin^2 B+\cos^2 B\right)}{(\cos A+\cos B)(\sin A+\sin B)}$

$\Rightarrow$ L. H. S = $\frac{1-1}{(\cos A+\cos B)(\sin A+\sin B)}$ = 0 = R.H.S

$\therefore \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

Hence proved.

48. Given that: A = B = 60°

L.H.S. $= \cos(A-B) = \cos\left(60^\circ - 60^\circ\right) = \cos 0^\circ = 1$

R.H.S. = cos A cos B + sin A sin B

$= \cos 60^\circ \cos 60^\circ + \sin 60^\circ \sin 60^\circ$

$= \cos^2 60^\circ + \sin^2 60^\circ$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$

$\therefore$ LHS = RHS

Section C

49. Given,

(cotθ + tanθ) = m and (secθ - cosθ) = n

$\Rightarrow \left(\frac{1}{\tan\theta} + \tan\theta\right) = m$ and $\left(\frac{1}{\cos\theta} - \cos\theta\right)$ = n

$\Rightarrow \left(\frac{1+\tan^2\theta}{\tan\theta}\right) = m$ and $\frac{\left(1-\cos^2\theta\right)}{\cos\theta} = n$

$\Rightarrow \left(\frac{\sec^2\theta}{\tan\theta}\right) = m$ and $\frac{\left(1-\cos^2\theta\right)}{\cos\theta} = n$

$\Rightarrow m = \frac{1}{\cos^2\theta \times \frac{\sin\theta}{\cos\theta}}$ and $\frac{\sin^2\theta}{\cos\theta} = n$

$\Rightarrow m = \frac{1}{\cos\theta\sin\theta}$ and $n = \frac{\sin^2\theta}{\cos\theta}$.......(1)

Now, L.H.S.

$= \left(m^2n\right)^{\frac{2}{3}} - \left(mn^2\right)^{\frac{2}{3}}$

$= \left[\frac{1}{\cos^2\theta\sin^2\theta} \times \frac{\sin^2\theta}{\cos\theta}\right]^{\frac{2}{3}} - \left[\frac{1}{\cos\theta\sin\theta} \times \frac{\sin^4\theta}{\cos^2\theta}\right]^{\frac{2}{3}}$. [from (1)]

$= \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

$= \sec^2\theta - \tan^2\theta$

$= 1$ [$\because$ $\sec^2\theta - \tan^2\theta = 1$]

= R.H.S. Hence, Proved.

50. We have,

LHS = $(\text{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)$

$\Rightarrow \quad \text{LHS} = \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)$

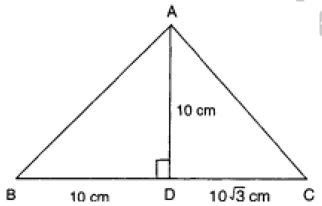
$\Rightarrow \quad \text{LHS} = \frac{1-\sin^2\theta}{\sin\theta} \times \frac{1-\cos^2\theta}{\cos\theta}$

$\Rightarrow \quad \text{LHS} = \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta}$ (since, $\sin^2A+\cos^2A = 1$)

$\Rightarrow \quad \text{LHS} = \frac{\sin\theta\cos\theta}{1} = \frac{\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta} = \frac{1}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}$

$\Rightarrow \quad LHS = \frac{1}{\frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}} = \frac{1}{\tan\theta+\cot\theta} = RHS$

51.



Given that,

ABC is a right triangle, right angled at D in which, AD = 10 cm and BD = 10 cm.

$\therefore \quad \tan\angle BAD = \frac{BD}{AD}$

$\Rightarrow \quad \tan\angle BAD = \frac{10}{10} = 1$

$\Rightarrow \quad \tan\angle BAD = \tan 45^\circ$

$\Rightarrow \quad \angle BAD = 45^\circ$......(i)

ACD is a right triangle right angled at D in which AD = 10 cm and DC $= 10\sqrt{3}\ cm$

$\therefore \quad \tan\angle CAD = \frac{CD}{AD}$

$\Rightarrow \quad \tan\angle CAD = \frac{10\sqrt{3}}{10} = \sqrt{3}$

$\Rightarrow \quad \tan\angle CAD = \tan 60^\circ$

$\therefore \quad \angle CAD = 60^\circ$......(ii)

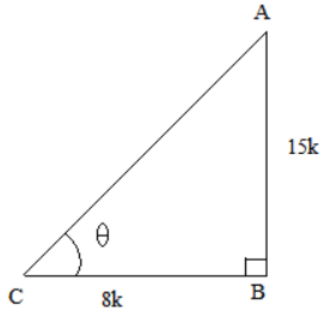
Adding (i) and (ii), we get

$\angle BAD + \angle CAD = 45^\circ + 60^\circ$

$\therefore \angle BAC = 105^\circ$

52. Let us first draw a right $\triangle$ABC, right angled at B and $\angle C = \theta$.

Now, we know that $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BC} = \frac{15}{8}$



So, if BC = 8k, then AB = 15k, where k is a positive number.

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2 = (15k)^2 + (8k)^2$

$\Rightarrow AC^2 = 225k^2 + 64k2 = 289k^2$

$\Rightarrow AC = 17k$

Now, finding the other T-ratios using their definitions, we get:

$\sin\theta = \frac{AB}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\cos\theta = \frac{BC}{AC} = \frac{8k}{17k} = \frac{8}{17}$

$\therefore \cot\theta = \frac{1}{\tan\theta} = \frac{8}{15}, cosec\theta = \frac{1}{\sin\theta} = \frac{17}{15}$ and $\sec\theta = \frac{1}{\cos\theta} = \frac{17}{8}$

53. We have

$cosec\ \theta = \frac{13}{12}$

$\sin\theta = \frac{1}{cosec\ \theta} = \frac{12}{13}$

$\sin^2\theta = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$

We know that,

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \frac{144}{169}$

$\cos^2\theta = \frac{25}{169}$

$\cos\theta = \frac{5}{13}$

Now, $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

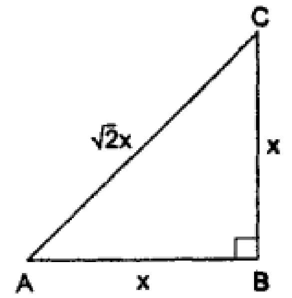
$= \frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}}$

$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$

$= \frac{\frac{9}{13}}{\frac{3}{13}} = \frac{9}{3} = 3$

Hence $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

54. Let us draw a triangle ABC in which ∠B = 90° and AB: AC = 1: $\sqrt{2}$.



Let AB = x.Then, AC =$\sqrt{2}$x.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

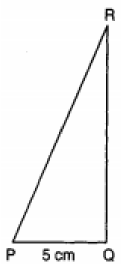
$\Rightarrow BC^2 = (\sqrt{2}x)^2 - (x)^2 = 2x^2 - x^2$

$= x^2$

$\Rightarrow BC = x$

$\therefore \tan A = \frac{BC}{AB} = \frac{x}{x} = 1$

So, the given expression = $\left(\frac{2\tan A}{1+\tan^2 A}\right) = \left(\frac{2\times 1}{1+1}\right) = \frac{2}{2} = 1$

55. 

In $\triangle PQR$, by Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (25 - QR)^2 = 5^2 + QR^2 [\because PR + QR = 25 \text{ cm} \Rightarrow PR = 25 - QR]$

$625 - 50QR + QR^2 = 25 + QR^2$

$\Rightarrow 600 - 50QR = 0$

$\Rightarrow QR = \frac{600}{50} = 12$ cm

Now, PR + QR = 25 cm

$\Rightarrow$ PR = 25 - Q R = 25 - 12 = 13 cm

Hence, $\sin P = \frac{QR}{PR} = \frac{12}{13}$, $\cos P = \frac{PQ}{PR} = \frac{5}{13}$ and, $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

56. We have,

$LHS = 2\left(\sin^6\theta + \cos^6\theta\right) - 3\left(\sin^4\theta + \cos^4\theta\right) + 1$

$= 2\left\{\left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3\right\} - 3\left(\sin^4\theta + \cos^4\theta\right) + 1$

Using $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ and $a^2 + b^2 = (a+b)^2 - 2ab$, we obtain

$LHS = 2\left\{\left(\sin^2\theta + \cos^2\theta\right)^3 - 3\sin^2\theta\cos^2\theta\left(\sin^2\theta + \cos^2\theta\right)\right\} - 3\left\{\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta + 1\right\}$

$= 2\left(1 - 3\sin^2\theta\cos^2\theta\right) - 3\left(1 - 2\sin^2\theta\cos^2\theta\right) + 1$

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1 = 0 = RHS$

57. Let us consider two right triangles LMN and PQR

Such that ∠LNM = ∠A and ∠PRO = ∠B

cos A = cos B ...... Given

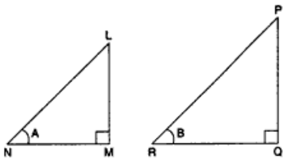
$\Rightarrow \frac{NM}{NL} = \frac{RQ}{RP}$

$\Rightarrow \frac{\text{NM}}{\text{RQ}} = \frac{\text{NL}}{\text{RP}} = \text{k(say)} \text{ .... (1)}$

where k is a positive number

$\Rightarrow$ NM = kRQ

NL = K RP



Now, using Pythagoras theorem,

$\text{ML} = \sqrt{\text{NL}^2 - \text{NM}^2} = \sqrt{(\text{kRP})^2 - (\text{kRQ})^2}$

$= \sqrt{k^2RP^2 - k^2RQ^2} = k\sqrt{RP^2 - RQ^2}$

and $QP = \sqrt{RP^2 - RQ^2}$

So, $\frac{\text{ML}}{\text{QP}} = \frac{\text{k}\sqrt{\text{RP}^2 - \text{RQ}^2}}{\text{RP}^2 - \text{RQ}^2} = \text{k}$

From (1) and (2), we have

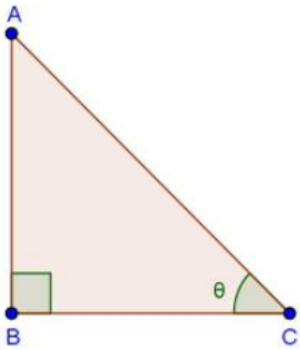
$\frac{NM}{RQ} = \frac{NL}{RP} = \frac{ML}{QP}$

$\therefore \Delta LMN \sim \Delta PQR$ ... SSS similarly criterion

$\therefore \angle\text{LNM} = \angle\text{PRQ}$ $\because$ Corresponding angles of two similar triangles are equal.

$\Rightarrow \angle A = \angle B$

58.



Given $\tan\theta = \frac{24}{7} = \frac{AB}{BC}$ (θ is $\angle C$, see figure)

Let AB = 24K and BC = 7K, where K is positive integer.

In ΔABC, by Pythagoras theorem

$\text{AC}^2 = \text{AB}^2 + \text{BC}^2$

Or, $\text{AC}^2 = (24\text{K})^2 + (7\text{K})^2$

Or, $\text{AC}^2 = 576\text{K}^2 + 49\text{K}^2$

Or, $\text{AC}^2 = 625\text{K}^2$

$\therefore AC = \sqrt{625K^2} = 25K$

Now,

$\sin\theta = \frac{AB}{AC} = \frac{24K}{25K} = \frac{24}{25}$

$\cos\theta = \frac{BC}{AC} = \frac{7K}{25K} = \frac{7}{25}$

$\therefore \sin\theta + \cos\theta$

$= \frac{24}{25} + \frac{7}{25}$

$= \frac{24+7}{25} = \frac{31}{25}$

59. i. (d) $\frac{4}{3}$

ii. (d) $\frac{16}{3}\sqrt{3}$ units

iii. 1

60. i. All triangles have the same value of sin A. The value of sin A does not depend on side lengths but is a ratio of the side lengths.

ii. (b) 1

iii. (b) The values of sin and cos vary from 0 to 1

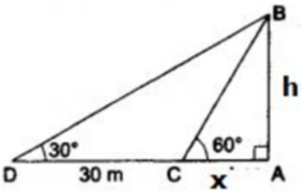
Section D

61. i. Let AB be the tree of height h meter and AC = x be the width of river.

Assuming that C be the position of a man standing on the opposite bank of the river. After moving 30 m away from point C.

Let new position of man be D,

Thus CD = 30 m, $\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$ and $\angle DAB = 90^\circ$



From right ΔABC we have

$\frac{AC}{AB} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)

From right ΔDAB, we have

$\frac{AD}{AB} = \cot 30^\circ = \sqrt{3}$

$\Rightarrow \frac{x+30}{h} = \sqrt{3}$

$\Rightarrow x = \sqrt{3}h - 30$ ...(ii)

Equating the values of x from (i) and (ii), we get

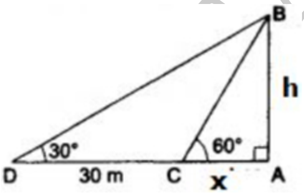
$\Rightarrow 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3} = 15 \times 1.732 = 25.98$ m

Thus the height of the tree h = 25.98 m

ii. Let AB be the tree of height h meter and AC = x be the width of river.

Assuming that C be the position of a man standing on the opposite bank of the river. After moving 30 m away from point C.

Let new position of man be D, Thus CD = 30 m, $\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$ and $\angle DAB = 90^\circ$



From right ΔABC we have

$\frac{AC}{AB} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)

Putting the value of h in (i) we get

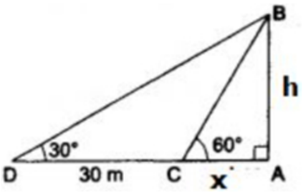
$\boldsymbol{x = \frac{15\sqrt{3}}{\sqrt{3}} = 15\ m}$

Hence the width of the river is 15 m.

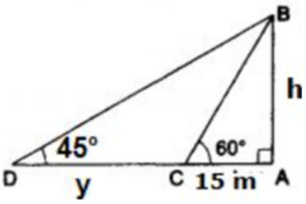
iii. Let AB be the tree of height h meter and AC=x be the width of river.

Assuming that C be the position of a man standing on the opposite bank of the river. After moving 30 m away from point C.

Let new position of man be D, Thus CD = 30 m, $\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$ and $\angle DAB = 90^\circ$



Suppose after moving y meters away from the river the angle of elevation becomes 45^o.



Then in the $\triangle ABD$

$\frac{y+15}{h} = \cot 45^\circ = 1$

y + 15 = h

y = h - 15 = 25.98 - 15

y = 10.98 m

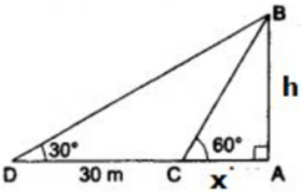
Hence after moving 10.98 meters away from the river the angle of elevation would become.

OR

Let AB be the tree of height h meter and AC = x be the width of river.

Assuming that C be the position of a man standing on the opposite bank of the river. After moving 30 m away from point C.

Let new position of man be D, Thus CD = 30 m, $\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$ and $\angle DAB = 90^\circ$



From right ΔDAB, we have

$\frac{AD}{AB} = \cot 30^\circ = \sqrt{3}$

$\Rightarrow \frac{x+30}{h} = \sqrt{3}$

$\Rightarrow x = \sqrt{3}h - 30$ ...(ii)

From (ii) we are having

x = h$\sqrt{3}$ - 30

Given that width of river x = 40 m

Thus 40 = h$\sqrt{3}$ - 30

$h = \frac{70}{\sqrt{3}}$

h = 40.41 m

Thus in this case height of the tree would be 40.41 m.

62. i. In $\triangle$ ABC

tan $45^o = \frac{5}{BC}$

BC = 5 m

In $\triangle$ DEF

tan $30^o = \frac{6}{EF}$

$\frac{1}{\sqrt{3}} = \frac{6}{EF}$

EF = $6\sqrt{3}$

length of flat part = 30 - (5 + $6\sqrt{3}$)

= 30 - 15.392

= 14.60 m

ii. Upper inclination

iii. Length of slide = AB + DE

AB = $\sqrt{5^2 + 5^2}$

AB = $5\sqrt{2}$ m

DC = $\sqrt{6^2 + 6\sqrt{3}^2}$

= $\sqrt{36 + 108}$

= $\sqrt{144}$

= 12 m

$\therefore$ Length of slide = $5\sqrt{2} + 12$

= 8.66 + 12

= 20.66 m

OR

Length of single slide = $\sqrt{30^2 + 11^2}$

= $\sqrt{900 + 121}$

= $\sqrt{1021}$ m

= 31.95 m

63. i. Sin $\theta = \frac{2}{4}$

Sin $\theta = \frac{1}{2}$

$\theta = 30^o$

i.e., the dock makes an angle of 30^o with the street.

ii. tan $30^o = \frac{2}{BC}$

BC = $\frac{2}{tan 30^o}$

B = $\frac{2}{\frac{1}{\sqrt{3}}}$

BC = $2\sqrt{3}$

BC = 3.5 m

$\therefore$ the length of base of ramp = 3.5 m

iii. tan $45^o = \frac{AB}{BC}$

1 = $\frac{AB}{BC}$

1 $\times$ 3.5 = AB

AB = 3.5 m

$\therefore$ height of ramp becomes 3.5 m

OR

sin $45^o = \frac{3.5}{AC}$

$\frac{1}{\sqrt{2}} = \frac{3.5}{AC}$

AC = 3.5 $\times$ 1.41

= 4.93 m

64. i. cos $\theta = \frac{60}{120}$

cos $\theta = \frac{1}{2}$

$\theta = 60^o$

ii. tan $60^o = \sqrt{3}$

iii. $\tan\theta = \frac{AB}{BC}$

$\tan 60^o = \frac{AB}{60}$

$\sqrt{3} = \frac{AB}{60}$

AB = 60 × 1.732

AB = 103.9 m

OR

$\angle A = 90 - \theta$

$\angle A = 90 - 60$

$\angle A = 30^o$

$\tan 30^o = \frac{1}{\sqrt{3}}$

65. i. In $\triangle BRS$

$\sin\theta = \frac{RS}{RB}$

$\sin\theta = \frac{8}{16} = \frac{1}{2}$

$\therefore \theta = 30^o$

Now, $\cos\theta = \frac{BS}{RB}$

$\cos 30^o = \frac{BS}{16}$

$\frac{\sqrt{3}}{2} = \frac{BS}{16}$

BS = $8\sqrt{3}$ m

ii. In $\triangle BRS$

use, $\sin\theta = \frac{RS}{RB}$

$\sin\theta = \frac{8}{16}$

$\theta = 30^o$

Hence, jib B makes an angle of 30^o with the horizontal.

iii. AS = $\sqrt{AB^2 + BS^2}$

$= \sqrt{24^2 + (8\sqrt{3})^2}$

$= \sqrt{576 + 192}$

$= \sqrt{768}$

= 27.71 m

OR

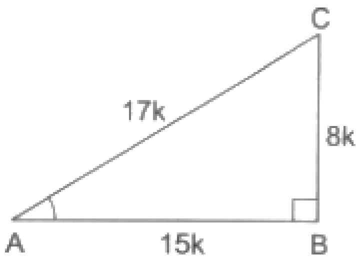
$\tan\alpha = \frac{8\sqrt{3}}{24}$

$\alpha = 30^o$

Section E

66. Let us draw a $\triangle ABC$ in which $\angle B = 90°$

Then, $\sin A = \frac{BC}{AC} = \frac{8}{17}$



Let BC = 8k and AC = 17k, where k is positive.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$\Rightarrow AB^2 = (17k)^2 - (8k)^2 = 289k^2 - 64k^2 = 225k^2$

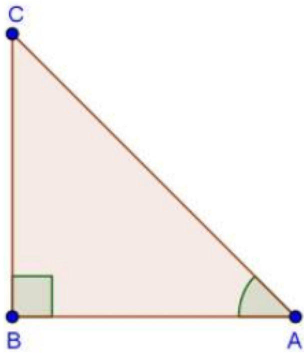
$\Rightarrow AB = \sqrt{225k^2} = 15k$

$\therefore \quad \sin A = \frac{BC}{AC} = \frac{8k}{17k} = \frac{8}{17}; \cos A = \frac{AB}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\tan A = \frac{\sin A}{\cos A} = \left(\frac{8}{17} \times \frac{17}{15}\right) = \frac{8}{15}$

$cosecA = \frac{1}{\sin A} = \frac{17}{8}; \sec A = \frac{1}{\cos A} = \frac{17}{15}$

and $\cot A = \frac{1}{\tan A} = \frac{15}{8}$

67. 

Given $\sec A = \frac{17}{8} = \frac{AC}{AB}$

Let AC = 17K

and, AB = 8K

In ΔABC, by Pythagoras theorem

$BC^2 + AB^2 = AC^2$

$BC^2 + (8K)^2 = (17K)^2$

$BC^2 + 64K^2 = 289K^2$

$BC^2 = 289K^2 - 64K^2$

$BC^2 = 225K^2$

$BC = \sqrt{225K^2} = 15K$

$\therefore \sin A = \frac{BC}{AC} = \frac{15K}{17K} = \frac{15}{17}$

$\cos A = \frac{AB}{AC} = \frac{8K}{17K} = \frac{8}{17}$

$\tan A = \frac{BC}{AB} = \frac{15K}{8K} = \frac{15}{8}$

LHS

$= \frac{3-4\sin^2 A}{4\cos^2 A - 3}$

$= \frac{3-4\times\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2 - 3}$

$= \frac{3-\frac{900}{289}}{\frac{256}{289}-3}$

$= \frac{\frac{867-900}{289}}{\frac{256-867}{289}}$

$= \frac{\frac{-33}{289}}{\frac{-611}{289}}$

$= \frac{-33}{289} \times \frac{289}{-611}$

$= \frac{33}{611}$

RHS

$= \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$

$= \frac{3 - \left(\frac{15}{8}\right)^2}{1 - 3 \times \left(\frac{15}{8}\right)^2}$

$= \frac{3 - \frac{225}{64}}{1 - \frac{675}{64}}$

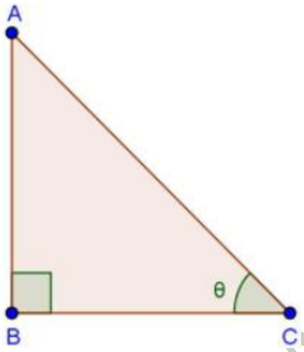
$= \frac{\frac{192 - 225}{64}}{\frac{64 - 675}{64}}$

$= \frac{\frac{-33}{64}}{\frac{-611}{64}}$

$= \frac{-33}{64} \times \frac{64}{-611}$

$= \frac{33}{611}$

Hence verified

68. 

Let us draw a right angled ΔABC ,right angled at B .

Let $\angle C = \theta$

Hence, Given $\cos\theta = \frac{3}{5} = \frac{BC}{AC}$

Let BC = 3K and, AC = 5K , where K is positive integer.

In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

or, $AB^2 + (3K)^2 = (5K)^2$

or, $AB^2 + 9K^2 = 25K^2$

or, $AB^2 = 16K^2$

$\therefore AB = \sqrt{16K^2} = 4K$

$\therefore \sin\theta = \frac{AB}{AC} = \frac{4K}{5K} = \frac{4}{5}$

$\tan\theta = \frac{AB}{BC} = \frac{4K}{3K} = \frac{4}{3}$

Now,

$\dfrac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta}$

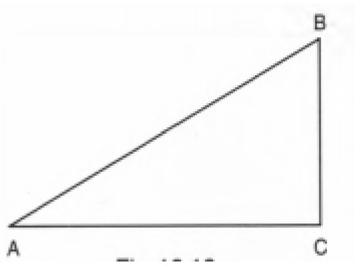
$= \dfrac{\frac{4}{5} - \frac{3}{4}}{2 \times \frac{4}{3}}$

$= \dfrac{\frac{16-15}{20}}{\frac{8}{3}}$

$= \dfrac{\frac{1}{20}}{\frac{8}{3}}$

$= \frac{1}{20} \times \frac{3}{8} = \frac{3}{160}$

69. Let us draw a $\triangle$ ABC, right angled at C in which tan A = $\frac{1}{\sqrt{3}}$



Now, $\tan A = \frac{1}{\sqrt{3}}$

Let, $\frac{BC}{AC} = \frac{1}{\sqrt{3}} \left[\because \tan A = \frac{BC}{AC}\right]$

$\Rightarrow BC = x$ and $AC = \sqrt{3}x$ ....(i)

By Pythagoras theorem, we have

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = (\sqrt{3}x)^2 + x^2$

$\Rightarrow AB^2 = 3x^2 + x^2$

$\Rightarrow AB^2 = 4x^2$

$\Rightarrow AB = 2x$

To find the ratios of $\angle A$, we have

Base =AC = $\sqrt{3}x$, Perpendicular = BC = x and Hypotenuse = AB = 2x

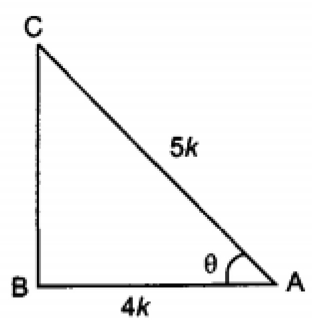
$\therefore \sin A = \frac{BC}{AB} = \frac{x}{2x} = \frac{1}{2}$ and $\cos A = \frac{AC}{AB} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$

When we consider the ratios of $\angle B$, we have

Base = B C = x ,Perpendicular = AC = $\sqrt{3}x$ and, Hypotenuse = AB = 2x

$\therefore \cos B = \frac{BC}{AB} = \frac{x}{2x} = \frac{1}{2}$ and, $\sin B = \frac{AC}{AB} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$

$\therefore \sin A \cos B + \cos A \sin B = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

70. 

If ABC is a triangle, right-angled at B and $\angle A = \theta$, then

$\sec\theta = \frac{AC}{AB} = \frac{5}{4}$

$\Rightarrow$ AC = 5k and AB = 4k

Since, $AC^2 = AB^2 + BC^2$

$\Rightarrow (5k)^2 = (4k)^2 + BC^2$

$\Rightarrow BC^2 = 25k^2 - 16k^2 = 9k^2$

$\Rightarrow BC = \sqrt{9k^2} = 3k$

Now, $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\frac{BC}{AB}}{1+\left(\frac{BC}{AB}\right)^2} = \frac{\frac{3k}{4k}}{1+\left(\frac{3k}{4k}\right)^2} = \frac{\frac{3}{4}}{\left(1+\frac{9}{16}\right)}$

$= \frac{3}{4} \div \frac{25}{16} = \frac{3}{4} \times \frac{16}{25} = \frac{12}{25}$

Also, $\frac{\sin\theta}{\sec\theta} = \frac{\frac{BC}{AC}}{\frac{AC}{AB}} = \frac{\frac{3k}{5k}}{\frac{5k}{4k}}$

$= \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$

Hence, verified

71. LHS = $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$

$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2$

$= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + \frac{1}{\sin^2 A} + 2\frac{\cos A}{\sin A}$

$= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + 2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} + 2\left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right)$

$= 1 + \frac{1}{\sin^2 A \cos^2 A} + \frac{2}{\sin A \cos A}$

$= \left(1 + \frac{1}{\sin A \cos A}\right)^2$

$= (1 + \sec A \operatorname{cosec} A)^2$ = RHS

72. LHS $= \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta \, . \cos^2\theta$

$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right)\sin^2\theta \cos^2\theta$

$= \left(\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right)\sin^2\theta \cos^2\theta$

$= \left(\frac{\cos^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)} + \frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right)\sin^2\theta \cos^2\theta$

$= \left(\frac{\cos^2\theta}{\sin^2\theta(1+\cos^2\theta)} + \frac{\sin^2\theta}{\cos^2\theta \cdot (1+\sin^2\theta)}\right)\sin^2\theta \cos^2\theta$ [Since, $\sin^2 A + \cos^2 A = 1$]

$= \left(\frac{\cos^4\theta(1+\sin^2\theta) + \sin^4\theta(1+\cos^2\theta)}{\sin^2\theta(1+\cos^2\theta)\cos^2\theta(1+\sin^2\theta)}\right)\sin^2\theta \cos^2\theta$

$$= \frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\cos^4\theta + \cos^2\theta\sin^2\theta\left(\sin^2\theta + \cos^2\theta\right) + \sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\cos^4\theta + \cos^2\theta\sin^2\theta + \sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta}{1+1+\sin^2\theta\cos^2\theta}$$

$$= \frac{1-\sin^2\theta\cdot\cos^2\theta}{2+\sin^2\theta\cdot\cos^2\theta} = \text{RHS}$$

Hence, Proved.

73. $\text{LHS} = \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$

$$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{\cos^2\theta\left(1-\sin^4\theta\right) + \sin^2\theta\left(1-\cos^4\theta\right)}{\left(1-\cos^4\theta\right)\left(1-\sin^4\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{\cos^2\theta\left(1-\sin^2\theta\right)\left(1+\sin^2\theta\right) + \sin^2\theta\left(1-\cos^2\theta\right)\left(1+\cos^2\theta\right)}{\left(1-\cos^2\theta\right)\left(1+\cos^2\theta\right)\left(1-\sin^2\theta\right)\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{\cos^2\theta\cdot\cos^2\theta\left(1+\sin^2\theta\right) + \sin^2\theta\sin^2\theta\left(1+\cos^2\theta\right)}{\sin^2\theta\left(1+\cos^2\theta\right)\cos^2\theta\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= \left(\frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\sin^2\theta\cos^2\theta\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right)$$

$$= \left(\frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right)$$

$= \left(\dfrac{\cos^4\theta + \sin^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta\cos^2\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)} \right)$

$= \dfrac{\left(\cos^2\theta + \sin^2\theta\right)^2 - 2\cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta\left(\cos^2\theta + \sin^2\theta\right)}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$

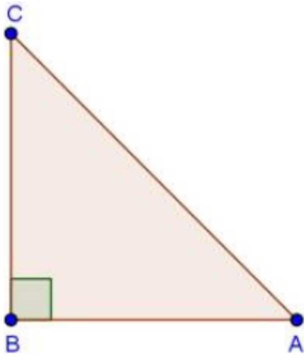
$= \dfrac{1 - 2\cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta \times 1}{1 + \sin^2\theta + \cos^2\theta + \cos^2\theta\sin^2\theta} \left[\sin^2\theta + \cos^2\theta = 1\right]$

$= \dfrac{1 - \cos^2\theta\sin^2\theta}{1 + 1 + \cos^2\theta\sin^2\theta}$

$= \dfrac{1 - \cos^2\theta\sin^2\theta}{2 + \cos^2\theta\sin^2\theta}$

= RHS

Hence proved

74. 

Given $\text{cosec}A = \dfrac{2}{1} = \dfrac{AC}{BC}$

Let AC = 2K

and, BC = 1K

In ΔABC, by Pythagoras theorem

$BC^2 + AB^2 = AC^2$

$(1K)^2 + AB^2 = (2K)^2$

$K^2 + AB^2 = 4K^2$

$AB^2 = 4K^2 - K^2 = 3K^2$

$AB = \sqrt{3K^2} = \sqrt{3}K$

$\therefore \tan A = \dfrac{BC}{AB} = \dfrac{1K}{\sqrt{3}K} = \dfrac{1}{\sqrt{3}}$

$\sin A = \dfrac{BC}{AC} = \dfrac{1K}{2K} = \dfrac{1}{2}$

$\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{3}K}{2K} = \dfrac{\sqrt{3}}{2}$

Now, $\dfrac{1}{\tan A} + \dfrac{\sin A}{1 + \cos A}$

$= \dfrac{1}{\frac{1}{\sqrt{3}}} + \dfrac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}$

$= \dfrac{\sqrt{3}}{1} + \dfrac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$

$= \dfrac{\sqrt{3}}{1} + \dfrac{1}{2} \times \dfrac{2}{2+\sqrt{3}}$

$= \dfrac{\sqrt{3}}{1} + \dfrac{1}{2+\sqrt{3}}$

$= \dfrac{2\sqrt{3} + 3 + 1}{2+\sqrt{3}}$

$= \frac{2\sqrt{3}+4}{2+\sqrt{3}}$

$= \frac{2\left(\sqrt{3}+2\right)}{2+\sqrt{3}} = 2$

75. Given, 2 cosθ - sinθ = x and

cosθ - 3 sinθ = y

Put the values of x and y in $2x^2 + y^2 - 2xy$ (LHS) , we get

$= 2(2\cos\theta - \sin\theta)^2 + (\cos\theta - 3\sin\theta)^2 - 2(2\cos\theta - \sin\theta)(\cos\theta - 3\sin\theta)$

$= 2(4\cos^2\theta - 4\cos\theta\sin\theta + \sin^2\theta) + (\cos^2\theta - 6\cos\theta\sin\theta + 9\sin^2\theta) - 2(2\cos^2\theta - 7\cos\theta\sin\theta + 3\sin^2\theta)$

$= 8\cos^2\theta - 8\cos\theta\sin\theta + 2\sin^2\theta + \cos^2\theta - 6\cos\theta\sin\theta + 9\sin^2\theta - 2(2\cos^2\theta - 7\cos\theta\sin\theta + 3\sin^2\theta)$

$= 8\cos^2\theta - 8\cos\theta\sin\theta + 2\sin^2\theta + \cos^2\theta - 6\cos\theta\sin\theta + 9\sin^2\theta - 4\cos^2\theta + 14\cos\theta\sin\theta - 6\sin^2\theta$

$= 5\cos^2\theta + 5\sin^2\theta$

$= 5(\cos^2\theta + \sin^2\theta)$

$= 5$

= RHS