Solution

HERON'S FORMULA

Class 09 - Mathematics

Section A

1. **(a)** $\frac{15\sqrt{7}}{4}$ cm

Explanation:

We have, sides of triangle 11 cm, 15 cm and 16 cm.

$\therefore s=\frac{11+15+16}{2}=21$ cm

$\therefore$ Area of triangle $=\sqrt{21(21-11)(21-15)(21-16)}$

$=30\sqrt{7}$ cm^2

Let altitude to the largest side be h cm

$\therefore \frac{1}{2}\times 16\times h=30\sqrt{7}\Rightarrow 8h=30\sqrt{7}$

$\Rightarrow h=\frac{15\sqrt{7}}{4}$ cm

2. **(a)** $8\sqrt{5}\mathrm{cm}^2$

Explanation:

Area of isosceles triangle $=\frac{b}{4}\sqrt{4a^2-b^2}$

Here,

a = 6 cm and b = 8 cm

Thus, we have

$\frac{8}{4}\times\sqrt{4(6)^2-8^2}$

$=\frac{8}{4}\times\sqrt{144-64}$

$=\frac{8}{4}\times\sqrt{80}$

$=\frac{8}{4}\times 4\sqrt{5}$

$=8\sqrt{5}\mathrm{cm}^2$

3.

(d) $25\sqrt{3}$ cm^2

Explanation:

Area of equilateral triangle = $\frac{\sqrt{3}}{4}(\text{Side})^2$

= $\frac{\sqrt{3}}{4}(10)^2$

= $25\sqrt{3}$ sq. cm

4. **(a)** ₹ 2142

Explanation:

We have, $s=\frac{1}{2}(51+37+20)\mathrm{m}=54$ m

$\therefore$ Area $=\sqrt{54(54-51)(54-37)(54-20)}$

$=\sqrt{54\times 3\times 17\times 34}\,\mathrm{m}^2=306$ m^2

$\therefore$ Cost of levelling the field = ₹(306 $\times$ 7) = ₹ 2142

5.

(c) $5\sqrt{3}$cm

Explanation:

Height of equilateral triangle $=\frac{\sqrt{3}}{2}\times$ Side

$=\frac{\sqrt{3}}{2}\times 10$

$=5\sqrt{3}$cm

6.

(d) $1500\sqrt{3}$ m^2

Explanation:

Let the sides of the triangle be
a = 3x, b = 5x, c = 7x
Perimeter of triangle = 300 m
⇒ 3x + 5x + 7x = 300 ⇒ 15x = 300 ⇒ x = 20
∴ a = 3 × 20 = 60 m, b = 5 × 20 = 100 m,
c = 7 × 20 = 140 m
Now, $s=\frac{300}{2}=150\text{ m}$
∴ Area of triangle $\sqrt{150(150-60)(150-100)(150-140)}$
$=\sqrt{150\times90\times50\times10}\,\text{m}^2=1500\sqrt{3}\text{ m}^2$.

7.

(b) 24 cm^2
Explanation:
Perpendicular = $\sqrt{10^2-8^2}=\sqrt{100-64}$ = 6 cm
Area of triangle = $\frac{1}{2}$ × Base × Height
= $\frac{1}{2}$ × 8 × 6 = 24 sq. cm

8.

(c) $16\sqrt{3}$ cm^2
Explanation:
Here , $x\sqrt{3}=\sqrt{48}$
⇒ x = $\sqrt{16}$
Side = 2x
Area of equilateral triangle = $\frac{\sqrt{3}}{4}(\text{Side})^2$
= $\frac{\sqrt{3}}{4}(2x)^2$
= $\sqrt{3}x^2$ sq. cm
= $\sqrt{3}\left(\sqrt{16}\right)^2=16\sqrt{3}$

9.

(c) ₹ 14.95
Explanation:
Area of I $=\frac{\sqrt{3}}{4}\times(10)^2=25\sqrt{3}\text{ cm}^2=43.3\text{ cm}^2$
(II) $s=\frac{10+9+3}{2}=11\text{ cm}$
∴ Area of II $=2\sqrt{11\times1\times2\times8}=8\sqrt{11}\text{ cm}^2=26.56\text{ cm}^2$
(IV) $s=\frac{20+20+4}{2}=22\text{ cm}$
∴ Area of IV $=2\sqrt{22\times2\times2\times18}$
$=24\sqrt{11}\text{ cm}^2=79.68\text{ cm}^2$
∴ Total area of coloured paper used
= (43.3 + 26.56 + 79.68) cm^2 = 149.54 cm^2
Cost of coloured paper used = $\frac{10}{100}\times149.54=₹14.95$

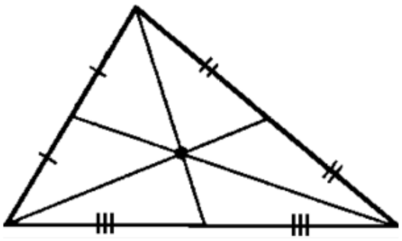
10.

(d) 300 %
Explanation:
Area of triangle with sides a, b, c (A) = $\sqrt{s(s-a)(s-b)(s-c)}$
New sides are 2a, 2b and 2c
Then $s'=\frac{2a+2b+2c}{2}=a+b+c$
⇒ s' = 2s .....(i)
New area = $\sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$
= $\sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= 4\sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)}$
= 4A
Increased area = 4A - A = 3A
% of increased area = $\frac{3\mathrm{A}}{\mathrm{A}} \times 100$ = 300%

11. **(a)** 3k
Explanation:
Semiperimeter of scalene triangle of side k, 2k and 3k = $\frac{k+2k+3k}{2}$ = 3k

12. **(a)** largest
Explanation:
Length of the perpendicular drawn on the smallest side of the scalene triangle is largest.



13.
(c) 12.29 cm^2
Explanation:
Here a = 11 cm, b = 4 cm, c = 8 cm
$\therefore s = \frac{11+4+8}{2} = \frac{23}{2} = 11.5 \text{ cm}$
Area
$= \sqrt{11.5 \times (11.5-11) \times (11.5-4) \times (11.5-8)}$
$= \sqrt{11.5 \times 0.5 \times 7.5 \times 3.5} = \sqrt{150.94}$ = 12.29 cm^2

14.
(d) 20 cm
Explanation:
Given: s - a = 8 cm, s - b = 7 cm and s - c = 5 cm
Adding all equations,
s - a + s - b + s - c = 8 + 7 + 5
$\Rightarrow$ 3s - (a + b + c) = 20 [s = $\frac{a+b+c}{2}$]
$\Rightarrow$ 3s - 2s = 20
$\Rightarrow$ s = 20 cm

15. **(a)** $24\sqrt{5}$ cm
Explanation:
Since longest altitude is drawn opposite to the shortest side in a triangle.
Area of triangle = $\frac{1}{2} \times$ Base $\times$ Height
$\Rightarrow 420\sqrt{5} = \frac{1}{2} \times 35 \times$ Height
$\Rightarrow$ Height = $\frac{420\sqrt{5}\times 2}{35} = 24\sqrt{5}$ cm

16.
(c) 294 cm^2
Explanation:
Let the sides be 3x, 4x and 5x.
Then according to quesiton, 3x + 4x + 5x = 84
$\Rightarrow$ 12x = 84
$\Rightarrow$ x = 7
Therefore, the sides are 3 $\times$ 7 = 21, cm, 4 $\times$ 7 = 28 cm and 5 $\times$ 7 = 35 cm

$s = \frac{21+28+35}{2}$ = 42 cm

Area of triangle = $\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$

= $\sqrt{42\,(42-21)\,(42-28)\,(42-35)}$

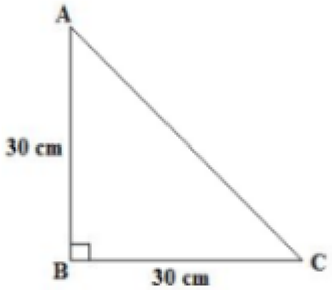
= $\sqrt{42 \times 21 \times 14 \times 7}$

= 21 $\times$ 7 $\times$ 2 = 294 sq. cm

17.

(b) 450 cm^2

Explanation:



Since triangle ABC is an isosceles triangle.

Base = 30

Hence, height = 30 cm

$\Rightarrow$ Area of triangle ABC $= \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 30 \times 30 = 450\text{cm}^2$

18.

(c) $12\sqrt{5}\text{cm}^2$

Explanation:

Let a = 7 cm, b = 9 cm, c = 14 cm

Semi-perimeter = s = $\frac{a+b+c}{2} = \frac{7+9+14}{2}$ = 15 cm

s - a = 15 - 7 = 8cm, s - b = 15 - 9 = 6 cm and s - c = 15 - 14 = 1 cm

Are of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{15 \times 8 \times 6 \times 1}$

= $\sqrt{5 \times 3 \times 4 \times 2 \times 3 \times 2}$

= $12\sqrt{5}\text{cm}^2$

19.

(b) 64 cm

Explanation:

Let the each of the other two equal sides of an isosceles triangle be a cm.

Then, $s = \frac{a+a+24}{2} = (a+12)\text{cm}$

Area of triangle = 192 cm^2

$\Rightarrow$ $\sqrt{(a+12)(a+12-a)(a+12-a)(a+12-24)}$ = 192

$\Rightarrow$ 144 (a^2 - 144) = $(192)^2$

$\Rightarrow$ a^2 = 400 $\Rightarrow$ a = 20 cm

$\therefore$ Perimeter = 20 + 20 + 24 = 64 cm

20. **(a)** 30 cm

Explanation:

Let the sides of $\triangle$ABC be a, b, c

Then, (s - a) = 7 cm, (s - b) = 5 cm and (s - c) = 3 cm

$\Rightarrow$ (s - a) + (s - b) + (s - c) = 15

$\Rightarrow$ 3s - (a + b + c) = 15

$\Rightarrow$ 3s - 2s = 15 $\left[\because s = \frac{a+b+c}{2}\right]$

$\Rightarrow$ s = 15 cm

$\therefore$ Perimeter = 2 $\times$ 15 = 30 cm

21.

(d) A is false but R is true.

Explanation:

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2$, where a is side of triangle

$81\sqrt{3} = \frac{\sqrt{3}}{4}a^2$

81 $\times$ 4 = a^2

324 = a^2

a = 18 cm

s = $\frac{18+18+18}{2}$ = 27 cm

22.

(c) A is true but R is false.

Explanation:

s = $\frac{a+b+c}{2}$

s = $\frac{3+4+5}{2}$ = 6 cm

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{(6)(6-3)(6-4)(6-5)}$

= $\sqrt{(6)(3)(2)(1)}$ = 6 cm^2

23. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion: Area of $\triangle$= $\frac{1}{2}\times$ base $\times$ height

72 = $\frac{1}{2} \times 18 \times b$

b = $\frac{72\times2}{18}$ = 8 cm

24.

(d) A is false but R is true.

Explanation:

The height of the triangle,

h = $\frac{\sqrt{3}}{2}$a

9 = $\frac{\sqrt{3}}{2}$a

a = $\frac{9\times2}{\sqrt{3}} = \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

= $\frac{18\sqrt{3}}{3} = 6\sqrt{3}$ cm

25.

(d) A is false but R is true.

Explanation:

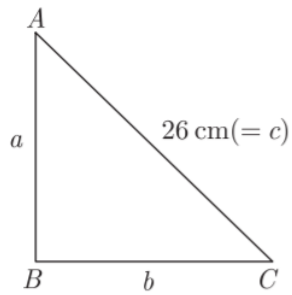
s = $\frac{6+6+6}{2} = \frac{18}{2}$ = 9 cm

Area = $\sqrt{9(9-6)(9-6)(9-6)}$

= $\sqrt{9\times3\times3\times3}$ = $9\sqrt{3}$ cm^2

26. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:



a + b + c = 60

a + b + 26 = 60

a + b = 34 ...(i)

Now, $26^2 = a^2 + b^2$ ...(ii)

Squaring (1) both sides, we get

$(a + b)^2 = (34)^2$

$a^2 + b^2 + 2ab = 34 \times 34$

$(26)^2 + 2ab = 1156$ [From (ii)]

2ab = 1156 - 676

2ab = 480

ab = 240 ...(iii)

Now, $a + \frac{240}{a} = 34$ [From (i) and (iii)]

$a^2 - 24a - 10a + 240 = 0$

a(a - 24) - 10(a - 24) = 0

a = 10, 24

Now, other sides are 10 cm and 24 cm

$s = \frac{26+10+24}{2} = 30$ cm

Area of triangle = $\sqrt{30(30-26)(30-10)(30-24)}$

$= \sqrt{30 \times 4 \times 20 \times 6} = 120$ cm^2

27. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

510 = a + b + c

510 = 25x + 14x + 12x

510 = 51x

x = 10

Three side of the triangle are

$25x = 25 \times 10 = 250$ cm

$14x = 14 \times 10 = 140$ cm and

$12x = 12 \times 10 = 120$ cm

$s = \frac{250+140+120}{2} = 255$ cm

Area = $\sqrt{255 \times 5 \times 115 \times 135}$

= 4449.08 cm^2

Section B

28. We have:

a = 13 cm and b = 20 cm

$\therefore$ Area of an isosceles triangle $= \frac{b}{4}\sqrt{4a^2 - b^2}$

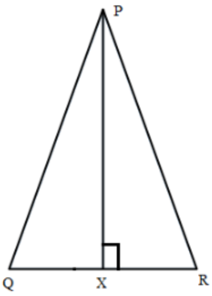
$= \frac{20}{4} \times \sqrt{4(13)^2 - 20^2}$

$= 5 \times \sqrt{676 - 400}$

$= 5 \times \sqrt{276}$

$= 5 \times 16.6$

Area of an isosceles triangle = 83.6 cm^2

29. 

Let $\triangle$PQR be an isosceles triangle and PX $\perp$ QR.

Now,

Area of triangle = 360 cm^2

$\Rightarrow \frac{1}{2} \times QR \times PX = 360$

$\Rightarrow h = \frac{720}{80} = 9\text{cm}$

Now,

$QX = \frac{1}{2} \times 80$ = 40 cm and PX = 9 cm

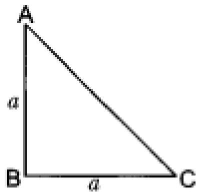
Also, by Pythagoras theorem for $\triangle$PXQ

PX^2 + QX^2 = PQ^2

$\Rightarrow$ a^2 = (40)^2 + 9^2

$a = \sqrt{40^2 + 9^2} = \sqrt{1600 + 81} = \sqrt{1681} = 41\text{cm}$

$\therefore$ Perimeter = 80 + 41 + 41 = 162 cm

30. 

Let the equal sides of an isosceles right triangle are a cm.

And its area = 8 cm^2

So,

Area $= \frac{1}{2}a^2$

$\Rightarrow \frac{1}{2}a^2 = 8$

$\Rightarrow$ a^2 = 16

$\Rightarrow$ a = 4 cm

Hypotenuse = $\sqrt{2}$a = $\sqrt{2}$(4) = 4$\sqrt{2}$cm.

31. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{ Side })^2$

$\Rightarrow \frac{\sqrt{3}}{4} \times (\text{ Side })^2 = 36\sqrt{3}$

$\Rightarrow$ (Side)^2 = 144

$\Rightarrow$ Side = 12 cm

Thus, we have:

Perimeter = 3 $\times$ Side = 3 $\times$ 12 = 36 cm

Hence, Perimeter = 36 cm

32. Let the equal sides of the isosceles triangle be *a* cm each.

$\therefore$ Base of the triangle, $b = \frac{3}{2}a$ cm

Perimeter of triangle = 42 cm

$\Rightarrow$ a + a + $\frac{3}{2}$a = 42

$\Rightarrow \frac{7}{2}$a = 42 $\Rightarrow$ a = 12 cm

and $b = \frac{3}{2}(12)\text{cm}$ = 18 cm

Area of triangle = 71.42 cm^2

$\Rightarrow \frac{1}{2} \times$ Base $\times$ Height = 71.42

$\Rightarrow$ Height $= \frac{71.42 \times 2}{18}$

= 7.94 cm

33. Let $\triangle$ABC be an isosceles triangle and let AL $\perp$ BC

$\therefore \frac{1}{2} \times BC \times AL = 192\text{cm}^2$

$\Rightarrow \frac{1}{2} \times 24\text{cm} \times h = 192\text{cm}^2$

$\Rightarrow h = \left(\frac{192}{12}\right)\text{cm} = 16\text{cm}$

Now, $BL = \frac{1}{2}(BC) = \left(\frac{1}{2} \times 24\right)\text{cm}$ = 12 cm and AL = 16 cm.

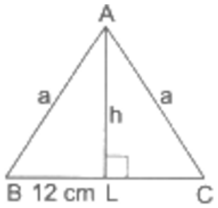
In $\triangle$ABL $AB^2 = BL^2 + AL^2$

$\Rightarrow a^2 = BL^2 + AL^2$

$\therefore a = \sqrt{BL^2 + AL^2} = \sqrt{(12)^2 + (16)^2}\text{cm} = \sqrt{144 + 256}\text{cm}$

$\Rightarrow a = \sqrt{400}\text{cm} = 20\text{cm}$

Hence, perimeter = (20 + 20 + 24) cm = 64 cm.



34. It is given that the sides of a triangle are 8 cm, 15 cm, and 17 cm.

So, a = 8cm, b = 15cm, c = 17cm

$s = \frac{a+b+c}{2} = \frac{8+15+17}{2} = \frac{40}{2} = 20\text{cm}$

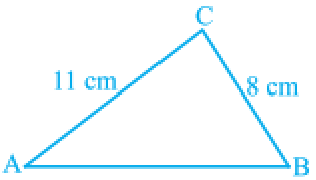
$\therefore Area = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{20(20-8)(20-15)(20-17)}$

$= \sqrt{20 \times 12 \times 5 \times 3} = 60\text{cm}^2$

Hence the area of given triangle is 60 cm^2.

35.



Let a, b, c be the sides of the given triangle and 2s be its perimeter such that

a = 8 cm, b = 11 cm and 2s = 32 cm i.e. s = 16 cm

Now,

a + b + c = 2s

$\Rightarrow$ 8 + 11 + c = 32

$\Rightarrow$ c = 13

$\therefore$ s - a = 16 - 8 = 8, s - b = 16 - 11 = 5 and s - c = 16 - 13 = 3

Hence, Area of given triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{16 \times 8 \times 5 \times 3}$ = $8\sqrt{30}$ cm^2

36. For isosceles triangles two of it's sides are equal so sides are a = 10, b = 10, c = 6

S = $\frac{10+10+6}{2} = \frac{26}{2} = 13cm$

$\therefore$ Area of tringle=$\sqrt{13\ (13-10)\ (13-10)\ (13-6)}$ sq cm

= $\sqrt{13 \times 3 \times 3 \times 7}$ sq cm

= $3\sqrt{91}$ sq cm

37. The perimeter of the given equilateral triangle = 60 cm

As every side of the equilateral triangle is equal.

Length of each of its sides = a = $\frac{60}{3}$cm = 20 cm

Area of the triangle $= \left(\frac{\sqrt{3}}{4} \times a^2\right)$ sq units

$= \left(\frac{\sqrt{3}}{4} \times 20 \times 20\right)\text{cm}^2$

$= (100 \times \sqrt{3})\text{cm}^2$

$= (100 \times 1.732)\text{cm}^2$

= 173.2 cm^2

Hence, the area of the given triangle is 173.2 cm^2.

38. 'a' = a, 'b' = a, 'c' = a

$\therefore s=\frac{'a'+b'+'c'}{2}$

s = $\frac{a+a+a}{2}$ = $\frac{3a}{2}$

$\therefore$ Area of the equilateral triangle

= $\sqrt{s\left(s-'a'\right)\left(s-'b'\right)\left(s-'c'\right)}$

= $\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

= $\sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}$

= $\frac{\sqrt{3}a^2}{4}$ square units.

39. Let the semi-perimeter of the triangle be s.

Let the sides of the triangle be a, b and c.

Given that

s - a = 8, s - b = 7 and s - c = 5 ...(i)

Adding all three equations, we get

3s - (a + b + c) = 8 + 7 + 5

$\Rightarrow$ 3s - (a + b + c) = 20

$\Rightarrow$ 3s - 2s = 20 $\left(\because s=\frac{a+b+c}{2}\right)$

$\Rightarrow$ s = 20 cm ...(ii)

$\therefore$ By Heron's formula

Area of $\triangle=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{20(8)(7)(5)}$ (from (i) and (ii))

area of the triangle$=20\sqrt{14}\mathrm{cm}^2$

Hence, the area of the triangle is $20\sqrt{14}\mathrm{cm}^2$

40. Area of an equilateral triangle $=\frac{\sqrt{3}}{4}\times(\text{ Side })^2$

$\Rightarrow\frac{\sqrt{3}}{4}\times(\text{ Side })^2=81\sqrt{3}$

$\Rightarrow$ (Side)^2 = 324

$\Rightarrow$ Side = 18 cm

Now, we have

For an equilateral triangle, hight $=\frac{\sqrt{3}}{2}\times$ Side

$=\frac{\sqrt{3}}{2}\times 18$

$=9\sqrt{3}\mathrm{cm}$

41. We have Area of ABCD = Area of $\triangle$ABD + Area of $\triangle$BDC...

$=\frac{1}{2}\times BD\times AL+\frac{1}{2}\times BD\times CM$

$=\frac{1}{2}\times BD(AL+CM)$

$=\frac{1}{2}\times 64$ (16.8 + 13.2)

$=32\times 30$

= 960 cm^2

42. Computation of area: Clearly, the park is trianglar with sides

a = BC = 120 m, b = CA = 80 m and c = AB = 50 m

Ifs denotes the semi-perimeter of the park, then

2s = a + b + c $\Rightarrow$ 2s = 120 + 80 + 50 $\Rightarrow$ s = 125

$\therefore$ s - a = 125 - 120 = 5, s - b = 125 - 80 = 45 and s - c = 125 - 50 = 75

Hence, Area of the park = $\sqrt{s(s-a)(s-b)(s-c)}$ = $\sqrt{125\times 5\times 45\times 75}$m^2 = 375$\sqrt{15}$ m^2

Length of the wire needed for fencing = perimeter of the park - width of the gate

= 250m - 3m = 247 m

Cost of fencing = Rs.(20 $\times$ 247) = Rs.4940

43. Since, sides of coloured triangular wall are 15 m, 11 m and 6 m.

$\therefore$ Semi-perimeter of coloured triangular wall

S = $\frac{15+11+6}{2} = \frac{32}{2}$ = 16 m

Now, Using Heron's formula,

Area of coloured triangular wall

= $\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$

= $\sqrt{16\,(16-15)\,(16-11)\,(16-6)}$

= $\sqrt{16 \times 1 \times 5 \times 10}$

= $20\sqrt{2}m^2$

Hence area painted in blue colour = $20\sqrt{2}m^2$

44. i. (c) $15\sqrt{130} \times 50 \times 50 \times 30$

    ii. $32\sqrt{6}$ cm^2

45. i. (b) $2.15 \times 0.35 \times 0.65 \times 1.15$

    ii. No, we don't have enough information to say that the area reserved for animals is double the area reserved for the zoo authorities. The area reserved under zone 1 = area reserved under zone 2 + 3, but we cannot say the area reserved under zone 2 and 3 are equal.

Section C

46. For the triangle having the sides 122 m, 120 m and 22 m:

$s = \frac{122+120+22}{2} = 132$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132(132-122)(132-120)(132-22)}$

$= \sqrt{132 \times 10 \times 12 \times 110}$

= 1320 m^2

For the triangle having the side 22m, 24m and 26m:

$s = \frac{22+24+26}{2} = 36$

Area of the triangle $= \sqrt{36(36-22)(36-24)(36-26)}$

$= \sqrt{36 \times 14 \times 12 \times 10}$

$= 24\sqrt{105}$

= 24 $\times$ 10.25 m^2 (approx.)

= 246 cm^2

Therefore, the area of the shaded portion.

= Area of larger triangle - Area of smaller (shaded) triangle.

= (1320 - 246) m^2

= 1074 m^2

47. The sides of triangular side walls of flyover which have been used for advertisements are 13 m, 14 m, 15 m.

$s = \frac{13+14+15}{2} = \frac{42}{2} = 21m$

$= \sqrt{21(21-13)(21-14)(21-15)}$

$= \sqrt{21 \times 8 \times 7 \times 6}$

$= \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 7 \times 3 \times 2}$

$= 7 \times 3 \times 2 \times 2 = 84m^2$

It is given that the advertisement yield an earning of Rs. 2,000 per m^2 a year.

$\therefore$ Rent for 1 m^2 for 1 year = Rs. 2000

So, rent for 1 m^2 for 6 months or $\frac{1}{2} year = Rs(\frac{1}{2} \times 2000)$ = Rs. 1,000.

$\therefore$ Rent for 84 m^2 for 6 months = Rs. (1000 × 84) = Rs. 84,000.

48. True, Let a = 51m, b = 37m, c = 20m

$s = \frac{a+b+c}{2} = \frac{51+37+20}{2} = \frac{108}{2} = 54\text{m}$

$\therefore$ Area of triangular ground $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-51)(54-37)(54-20)}$

$= \sqrt{54 \times 3 \times 17 \times 34}$

$= \sqrt{9 \times 3 \times 2 \times 3 \times 17 \times 17 \times 2}$

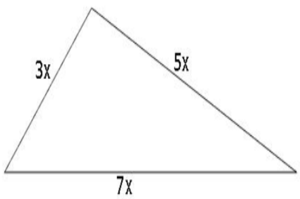
= 3 × 3 × 2 × 17

= 306 m^2

Cost of leveling the ground = Rs.3 $\times$ 306 = Rs.918.
Hence the cost of leveling the ground in the form of a triangle is Rs 918.

49. Given: = 122 m, = 22 m and = 120 m
Semi-perimeter of triangle (s)= $\frac{122+22+120}{2} = \frac{264}{2}$ = 132 m Using Heron's Formula,
Area of triangle =$\sqrt{s\,(s-a)\,(s-b)\,(s-c)}$
=$\sqrt{132\,(132-122)\,(132-22)\,(132-120)}$
$=\sqrt{132\times 10\times 110\times 12}$
$=\sqrt{11\times 12\times 10\times 10\times 11\times 12}$
= 10 $\times$ 11 $\times$ 12
= 1320 m^2
$\because$ Rent for advertisement on wall for 1 year = Rs. 5000 perm^2
$\therefore$ Rent for advertisement on wall for 3 months for 1320 m^2; $\frac{5000}{12}\times 3\times 1320$
= Rs.1650000
Hence rent paid by company = Rs. 16,50,000

50. 'a' = a, 'b' = a and 'c' = a.
$\therefore\ s=\frac{'a'+'b'+'c'}{2}=\frac{a+a+a}{2}=\frac{3a}{2}$
$\therefore$ Area of the signal board
$=\sqrt{s(s-'a')(s-'b')(s-'c')}$
$=\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$
$=\sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}=\sqrt{\frac{3a^4}{16}}=\frac{\sqrt{3}}{4}a^2$
Perimeter = 180 cm
'a' + 'b' + 'c' = 180
$\therefore$ a + a + a = 180
$\therefore$ 3a = 180
$\therefore$ a = 60 cm.
$\therefore$ Area of the signal board = $\frac{\sqrt{3}}{4}a^2$
$=\frac{\sqrt{3}}{4}(60)^2=900\sqrt{3}\,cm^2$
Alternatively,
$s=\frac{3a}{2}=\frac{3}{2}(60)=90\,cm$
Area of the signal board
$=\sqrt{s(s-'a')(s-'b')(s-'c')}$
$=\sqrt{90(90-60)(90-60)(90-60}$
$=\sqrt{90(30)(30(30)}$
$=900\sqrt{3}$ cm^2

51. 

Suppose that the sides in metres are 3x, 5x and 7x.
Then, we know that 3x + 5x + 7x = 300 (Perimeter of the triangle)
Therefore, 15x = 300, which gives x = 20.
So the sides of the triangles are 3 $\times$ 20 m, 5 $\times$ 20 m and 7 $\times$ 20 m
i.e., 60m, 100m and 140m.
We have s = $\frac{60+100+140}{2}$ = 150 m
and area will be = $\sqrt{150(150-60)(150-100)(150-140)}$
= $\sqrt{150\times 90\times 50\times 10}$
= 1500$\sqrt{3}$ m^2

52. $S=\frac{a+a+a}{2}$ units $=\frac{3a}{2} units$

$\therefore$Area of triangle = $\sqrt{\frac{3a}{2}\times(\frac{3a}{2}-a)(\frac{3a}{2}-a)(\frac{3a}{2}-a)}$

= $\sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}}$

$=\frac{a^2}{4}\sqrt{3}$ sq units

Now, perimeter = 180 cm

$\therefore$ each side $=\frac{180}{3}=60cm$

Using above derived formula

$\therefore$ Area of signal board = $\frac{\sqrt{3}}{4}$ $(60)^2$ sq cm

= 900 $\sqrt{3}$ sq cm

53. As the sides of the equal to the base of an isosceles triangle is 3 : 2, so let the sides of an isosceles triangle be 3x, 3x and 2x.

Now, perimeter of triangle = 3x + 3x + 2x = 8x

Given Perimeter of triangle = 32 m

$\therefore 8x=32; x=32\div 8=4$

So, the sides of the isosceles triangle are $(3\times 4)cm, (3\times 4)cm, (2\times 4)cm$ i.e., 12 cm, 12 cm and 8cm

$\therefore\ s=\frac{12+12+8}{2}=\frac{32}{2}=16cm$

$=\sqrt{16(16-12)(16-12)(16-8)}$

$=\sqrt{16\times 4\times 4\times 8}=\sqrt{4\times 4\times 4\times 4\times 4\times 2}$

$=4\times 4\times 2\sqrt{2}=32\sqrt{2}cm^2$

54. Let a = 41m, b = 40m, c = 9m.

$s=\frac{a+b+c}{2}=\frac{41+40+9}{2}=\frac{90}{2}$

s = 45m

Area of triangular field$=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{45(45-41)(45-40)(45-9)}$

$=\sqrt{45\times 4\times 5\times 36}$

$=180\ m^2$

$=1800000\ cm^2$

Number of rose beds$=\frac{\text{Total area}}{\text{Area needed for one rose bed}}=\frac{1800000}{900}=2000$

55. Let the sides of the triangle be x,2x,3x

Perimeter of the triangle = 480 m

$\therefore x+2x+3x=480m$

$6x=480m$

$x=80m$

$\therefore$ The sides are 80m, 160m, 240m

so,

$S=\frac{80+160+240}{2}=\frac{480}{2}$

= 240 m

And,

$\therefore$ Area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}\ sqm$

$=\sqrt{240(240-80)(240-160)(240-240)}$sqm

= 0 sq m

$\therefore$ Triangle doesn't exit with the ratio 1:2:3 whose perimeter is 480 m.

56. a = 8 cm, b = 8 cm, c = 8 cm.

$s=\frac{a+b+c}{2}$

$\therefore\ \frac{8+8+8}{2}\text{m}=12\,\text{cm}$

$\therefore$ Area of the equilateral triangle

$=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{12(12-8)(12-8)(12-8)}$

$=\sqrt{12(4)(4)(4)}$

$=\sqrt{(4)(3)(4)(4)(4)}$

$=16\sqrt{3}\ \text{cm}^2$

Area = $\frac{1}{2}$ × Base × Altitude

$= 16\sqrt{3}$ = $\frac{1}{2}$ × 8 × Altitude

$= 16\sqrt{3}$ = 4 Altitude

Altitude $= \frac{16\sqrt{3}}{4} = 4\sqrt{3}$ cm.

57. Let the Traffic signal board is ΔABC.

According to question, Semi-perimeter of ΔABC (s) $=\frac{a+a+a}{2} = \frac{3a}{2}$

Using Heron's Formula, Area of triangle ABC = $\sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)}$

$=\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$

$=\sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}}$

$=\sqrt{3\left(\frac{a}{2}\right)^4}$

$=\frac{\sqrt{3}a^2}{4}$

Now,If Perimeter of this triangle = 180 cm

$\Rightarrow$ Side of triangle (a) $=\frac{180}{3}$ = 60 cm

Using the above derived formula,

Area of triangle ABC

$=\frac{\sqrt{3}(60^2)}{4}$

$=15\times 60\sqrt{3}$

$=900\sqrt{3}\text{cm}^2$

58. Perimeter = 84 cm.

Ratio of sides = 13 : 14 : 15

Sum of the ratios = 13 + 14 + 15 = 42

∴ One side (a) = $\frac{13}{42}\times 84$ = 26 cm.

Second side (b) = $\frac{14}{42}\times 84$ = 28 cm.

Third side (c) = $\frac{15}{42}\times 84$ = 30 cm

∴ s = $\frac{a+b+c}{2}$

= $\frac{26+28+30}{2}$ = $\frac{84}{2}$ = 42 cm

∴ Area of the triangle

= $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{42(42-26)(42-28)(42-30)}$

= $\sqrt{42(16)(14)(12)}$

= $\sqrt{42(16)(14)(4\times 3)}$

= (42)(4)(2) = 336 cm^2

59. The sides of the triangle field are in the ratio 25:17:12.

Let the sides of triangle be 25x, 17x and 12x.

Perimeter of this triangle = 540 m

25x + 17x + 12x = 540 m

54x = 540 m

x = 10 m

Sides of triangle will be 250 m, 170 m, and 120 m

Semi-perimeter (s) = $\frac{Perimeter}{2}$ = $\frac{540}{2}$ = 270 m

By Heron's formula:

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{270(270-120)(270-170)(270-250)}$

= $\sqrt{270\times 150\times 100\times 20}$

= 9000 m^2

So, area of the triangle is 9000 m^2.

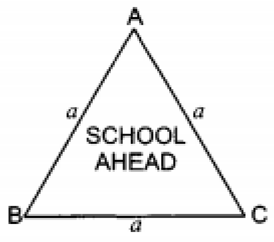
60. In right triangle PSQ,

$PQ^2 = PS^2 + QS^2$ ...[By Pythagoras theorem]

$= (12)^2 + (16)^2$
$= 144 + 256 = 400$
$\Rightarrow$ PQ = $\sqrt{400}$ = 20 cm
Now, for ΔPQR
a = 20 cm, b = 48 cm, c = 52 cm
$\therefore$ s = $\frac{a+b+c}{2}$
$= \frac{20+48+52}{2}$ = 60 cm
$\therefore$ Area of ΔPQR
$= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{60(60-20)(60-48)(60-52)}$
$= \sqrt{60(40)(12)(8)}$
$= \sqrt{(6\times10)(4\times10)(6\times2)(8)}$
$= 6 \times 10 \times 8 = 480$ cm^2
Area of Δ PSQ $=\frac{1}{2} \times$ Base $\times$ Altitude
$= \frac{1}{2} \times 16 \times 12 = 96$ cm^2
$\therefore$ Area of the shaded portion
= Area of ΔPQR - Area of ΔPSQ
$= 480 - 96 = 384$ cm^2

Section D

61. Let a, b, c be the sides of the old triangle and s be its semi-perimeter. Then,
s = $\frac{1}{2}$(a + b + c)
The sides of the new triangle are 2a, 2b and 2c.
Let s' be its semi-perimeter. Then,
s' = $\frac{1}{2}$(2a + 2b + 2c) = a + b + c = 2s
$\Rightarrow$ s' = 2s
Let $\triangle$ and $\triangle$' be the areas of the old and new triangles respectively. Then
$\triangle = \sqrt{s(s-a)(s-b)(s-c)}$ ......(1)
and
$\triangle' = \sqrt{s'\,(s'-2a)\,(s'-2b)\,(s'-2c)}$
$\Rightarrow \triangle' = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$ [$\because$ s' = 2s]
$\Rightarrow \triangle' = \sqrt{16s(s-a)(s-b)(s-c)}$
$\Rightarrow \triangle' = 4\sqrt{s(s-a)(s-b)(s-c)}$ = 4 $\triangle$ [from (1)]
$\therefore$ Increase in the area of the triangle = $\triangle$' - $\triangle$ = 4$\triangle$ - $\triangle$ = 3$\triangle$
Hence, percentage increase in area = $\left(\frac{3\triangle}{\triangle} \times 100\right)$ = 300%

62. 

A traffic signal board is an equilateral triangle with side a.
Perimeter of the signal board,
2s = a + a + a
$\Rightarrow$ s $= \frac{3}{2}a$
Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{\frac{3a}{2}\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$
$= \sqrt{\frac{3a}{2}\times\frac{a}{2}\times\frac{a}{2}\times\frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4}a^2$ sq. units
Now, if perimeter = 180 cm
3a = 180

$\Rightarrow$ a = 60 cm

$\therefore$ Area of signal board $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (60)^2 = 900\sqrt{3}\text{cm}^2$

So, area of the signal board is $900\sqrt{3}\text{cm}^2$.

63. Given that, the difference between the sides at right angles in a right-angled triangle is 14 cm.

Let the sides containing the right angle be x cm and (x -14) cm

Then, the area of the triangle $= \left[\frac{1}{2} \times x \times (x-14)\right] \text{cm}^2$

But, area = 120 cm^2 (given).

$\therefore \frac{1}{2}$ x(x - 14) = 120

$\Rightarrow$ x^2 - 14x - 240 = 0

$\Rightarrow$ x^2 - 24x + 10x - 240

$\Rightarrow$ x(x - 24) + 10(x - 24)

$\Rightarrow$ (x - 24) (x + 10) = 0

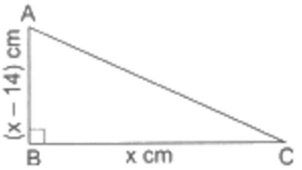
$\Rightarrow$ x = 24 (neglecting x = -10)

$\therefore$ one side = 24 cm, other side = (24 -14) cm = 10 cm

Hypotenuse $= \sqrt{(24)^2 + (10)^2}\text{cm} = \sqrt{576 + 100}\text{cm}$

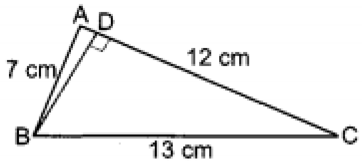
$= \sqrt{676}$ cm = 26 cm

$\therefore$ perimeter of the triangle = (24 +10 + 26) cm = 60 cm.



64. Let, a = 7cm, b = 13cm, c = 12cm

$\therefore \quad s = \frac{a+b+c}{2} = \frac{7+13+12}{2} = \frac{32}{2} = 16\text{cm}$



Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-7)(16-13)(16-12)}$

$= \sqrt{16 \times 9 \times 3 \times 4} = 24\sqrt{3}\text{cm}^2$

Also, Area of $\Delta ABC = \frac{1}{2} AC \cdot BD$

$24\sqrt{3} = \frac{1}{2} \times 12 \times BD$

$\Rightarrow \quad BD = \frac{24\sqrt{3} \times 2}{12} = 4\sqrt{3}\text{cm}$

Hence, the length of perpendicular from the opposite vertex to the given side is $4\sqrt{3}\text{cm}$.

65. Suppose that the sides in metres are 6x, 7x and 8x.

Now, 6x + 7x + 8x = perimeter = 420

$\Rightarrow$ 21x = 420

$\Rightarrow \ x = \frac{420}{21}$

$\Rightarrow$ x = 20

$\therefore$ The sides of the triangular field are $6 \times 20m, 7 \times 20m, 8 \times 20m,$ i.e., 120 m, 140 m and 160 m.

Now, s = Half the perimeter of triangular field.

$= \frac{1}{2} \times 420m = 210m$

Using Heron's formula,

Area of triangular field $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{210(210-120)(210-140)(210-160)}$

$= \sqrt{210 \times 90 \times 70 \times 50}$

$= \sqrt{66150000} = 8133.265m^2$

Hence, the area of the triangular field = 8133.265 m^2.

66. Let x and y be the two lines of the right $\angle$

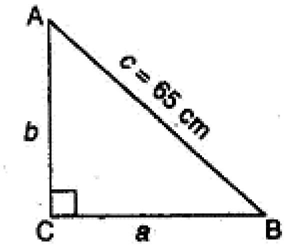
$\therefore$ AB = x cm, BC = y cm and AC = 10 cm

$\therefore$ By the given condition,
Perimeter = 24 cm
x + y +10 = 24 cm
Or x + y = 14 ... (I)
By Pythagoras theorem,
$x^2 + y^2 = (10)^2$ = 100 ... (II)
From (1), $(x + y)^2$ = $(14)^2$
Or $x^2 + y^2$ + 2xy = 196
$\therefore$100 + 2xy =196 [From (II)]
xy =$\frac{96}{2}$= 48 sq cm .... (III)
Area of Δ ABC = $\frac{1}{2}xy$ sq cm
=$\frac{1}{2} \times 48$ sq cm
=24 sq cm.... (IV)
Again, we know that
$\left(x\ -\ y\right)^2 = (x + y)^2 - 4xy$
$=\ (14)^2 - 4 \times 48$ [From (I) & (III)]
Or x - y = $\pm$ 2
(i) When, x-y = 2 and x+y = 14, then 2x = 16
or x = 8, y = 6
(ii) When, X – y = -2 and x + y = 14, then 2x = 12
Or x = 6, y = 8
Verification by using Heron’s formula:
Sides are 6 cm, 8 cm and 10 cm
S =$\frac{24}{2}$ = 12 cm
Area of Δ ABC = $\sqrt{12\ (12-6)\ (12-8)\ (12-10)}$ sq cm
=$\sqrt{12 \times 6 \times 4 \times 2}$ sq cm
= 24 sq cm
Which is same as found in (IV)
Thus, the result is verified.

67. Let a = 18 cm, b = 24 cm and c = 30 cm
$\Rightarrow$ half perimeter = s = $\frac{a+b+c}{2} = \frac{18+24+30}{2} = 36\text{cm}$
By Heron's formula, we have:
Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{36(36-18)(36-24)(36-30)}$
$= \sqrt{36 \times 18 \times 12 \times 6}$
$= \sqrt{12 \times 3 \times 6 \times 3 \times 12 \times 6}$
$= 12 \times 3 \times 6$
= 216 cm^2
We know that the smallest side is 18 cm.
Thus, we can find out the altitude of the triangle corresponding to 18 cm.
We have:
Area of triangle = 216 cm^2
$\Rightarrow \frac{1}{2} \times$ Base $\times$ Height = 216 $\Rightarrow \frac{1}{2} \times$ (18)(Height) = 216
$\Rightarrow \text{Height} = \frac{216 \times 2}{18} = 24\text{cm}$

68. Let:
a = 42 cm, b = 34 cm and c = 20 cm
$\therefore s = \frac{a+b+c}{2} = \frac{42+34+20}{2} = 48\text{cm}$
By Heron's formula, we have:
Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{48(48-42)(48-34)(48-20)}$
$= \sqrt{48 \times 6 \times 14 \times 28}$

$= \sqrt{4 \times 2 \times 6 \times 6 \times 7 \times 2 \times 7 \times 4}$
$= 4 \times 2 \times 6 \times 7$
Area of triangle = 336 cm^2
We know that the longest side is 42 cm.
Thus, we can find out the height of the triangle corresponding to 42 cm.
We have:
Area of triangle = 336 cm^2
$\Rightarrow \frac{1}{2} \times$ Base $\times$ Height = 336
$\Rightarrow \frac{1}{2}$ (42)(height) = 336
$\Rightarrow$ Height $= \frac{336 \times 2}{42}$ = 16 cm

69. 

Let ABC be the right triangle angled at C.
Perimeter = 144 cm.
$\Rightarrow$ a + b + c = 144 . . . (1)
$\Rightarrow$ c = 65 . . .(2)
Subtracting (2) from (1)
a + b = 144 – 65
a + b = 79 . . .(3)
In right triangle ACB,
$a^2 + b^2 = (65)^2$ . . .[By Pythagoras theorem] . . (4)
We know that
$(a + b)^2 = a^2 + b^2 + 2ab$
$\Rightarrow (79)^2 = (65)^2 + 2ab$. . .[Using (3) and (4)]
$\Rightarrow 2ab = (79)^2 – (65)^2$
$\Rightarrow$ 2ab = (79 + 65) (79 – 65)
$\Rightarrow$ 2ab = (144)(14)
$\Rightarrow$ ab = (72)(14)
$\Rightarrow$ ab = 1008
Now, $(a – b)^2 = (a + b)^2 - 4ab$ . . . (5)
$= (79)^2 – 4(1008)$. . .[From (3) and (5)]
= 6241 – 4032 = 2209
a – b = $\sqrt{2209}$
a – b = 47 . . . .(6)
Solving (3) and (6)
a = 63, b = 16 cm.
$\therefore$ Area of the right triangle
= $\frac{1}{2} \times$ base $\times$ height
= $\frac{1}{2} \times 63 \times 16$ = 504 cm^2
Using Heron's Formula
a = 63, b = 16 cm, c = 65 cm.
$\therefore$ s = $\frac{a+b+c}{2}$
= $\frac{63+16+65}{2} = \frac{144}{2}$ = 72 cm.
$\therefore$ Area of the right triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{72(72-63)(72-16)(72-65)}$
= $\sqrt{72(9)(56)(7)}$

$= \sqrt{(8 \times 9)(9)(8 \times 7)(7)}$

$= 8 \times 9 \times 7 = 504\ cm^2$

70. Let the height of the triangle be h meter

$\therefore$ Base = 3h meter [given]

Now,

Area of triangle $= \frac{\text{Total cost}}{\text{Rate}} = \frac{783}{58}$ = 13.5 ha = 135000 m^2

We have:

Area of triangle = 135000 m^2

$\Rightarrow \frac{1}{2} \times$ Base $\times$ Height = 135000

$\Rightarrow \frac{1}{2} \times 3h \times h$ = 135000

$\Rightarrow h^2 = \frac{135000 \times 2}{3}$

$\Rightarrow h^2 = 90000$

$\Rightarrow$ h = 300 m [taking square root both sides]

Thus, we have

Height = h = 300 m

Base = 3h = 900 m.

71. Let:

a = 91 m, b = 98 m, and c = 105 m

$\therefore s = \frac{a+b+c}{2} = \frac{91+98+105}{2}$ = 147 m

$\Rightarrow$ s = 147 m

By Heron's formula, we have:

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{147(147-91)(147-98)(147-105)}$

$= \sqrt{147 \times 56 \times 49 \times 42}$

$= \sqrt{7 \times 3 \times 7 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7 \times 7 \times 3 \times 2}$

$= 7 \times 7 \times 7 \times 2 \times 3 \times 2$

$= 1446\ m^2$

We know that the longest side is 105 m.

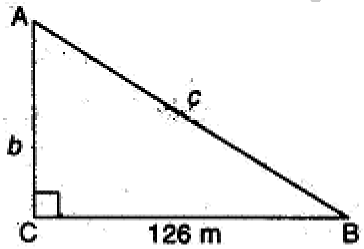
Thus, we can find out the height of the triangle corresponding to 42 cm.

Area of triangle = 4116 m^2

$\Rightarrow \frac{1}{2} \times$ Base $\times$ Height = 4116 $\Rightarrow \frac{1}{2} \times$ (105)(Height) = 4116

$\Rightarrow$ Height $= \frac{4116 \times 2}{105} =$ 78.4 m

72. Let ABC be the right triangle right angles at C.



a = 126 m . . . (1)

In right triangle ACB.

$AB^2 = AC^2 + BC^2$ . . .[By Pythagoras theorem]

$\Rightarrow c^2 = a^2 + b^2$

$\Rightarrow c = \sqrt{a^2 + b^2}$ . . .(2)

$\Rightarrow$ c – b = 42 . . .(3)

$\Rightarrow \sqrt{a^2 + b^2}$ - b = 42 . . .[From (2)]

$\Rightarrow \sqrt{126^2 + b^2}$ - b = 42 . . .[From (1)]

$\Rightarrow \sqrt{126^2 + b^2}$ = (42+b)

$\Rightarrow (126)^2 + b^2 = (42 + b)^2$

$\Rightarrow 15876 + b^2 = 1764 + b^2 + 84b$

$\Rightarrow$ 84b = 15876 – 1764

$\Rightarrow$ 84 b = 14112

$\Rightarrow$ b = $\frac{14112}{84}$

$\Rightarrow$ b = 168 m . . . (4)

From (3) and (4)

c – 168 = 42

$\therefore$ c = 168 + 42 = 210 m . . . (5)

$\therefore$ Area of the right triangle = $\frac{1}{2}$ $\times$ base $\times$ height

=$\frac{1}{2}$ $\times$ 126 $\times$ 168

= 10584 m^2

Using Heron's Formula

a = 126 m, b = 168 m, c = 210 m

$\therefore$ s = $\frac{a+b+c}{2}$

= $\frac{126+168+210}{2}$ = $\frac{504}{2}$ = 252 m

$\therefore$ Area of the right triangle

= $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{252(252-126)(252-168)(252-210)}$

= $\sqrt{252(126)(84)(42)}$

= $\sqrt{(63\times4)(63\times2)(42\times2)(42)}$

= 63 $\times$ 2 $\times$ 2 $\times$ 42 = 10584 m^2

73. Let:

a = 85 m and b = 154 m

Given that perimeter = 324 m

Perimeter= 2s = 324 m

$\Rightarrow$ s =$\frac{324}{2}$m

or, a + b + c = 324

$\Rightarrow$ c = 324 - 85 - 154 = 85 m

By Herons's formula, we have:

Area of triangle $=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{162(162-85)(162-154)(162-85)}$

$=\sqrt{162\times77\times8\times77}$

$=\sqrt{1296\times77\times77}$

$=\sqrt{36\times77\times77\times36}$

$=36\times77$

= 2772 m^2

74. Given that the sides of a triangle are in the ratio 5: 12: 13 and its perimeter is 150 m

Let the sides of the triangle be 5x m, 12x m and 13x m.

We know:

Perimeter = Sum of all sides

or, 150 = 5x + 12x + 13x

or, 30x = 150

or, x = 5

Thus, we obtain the sides of the triangle.

5 $\times$ 5 = 25 m

12 $\times$ 5 = 60 m

13 $\times$ 5 = 65 m

Now,

Let:

a = 25 m, b = 60 m and c = 65 m

$\therefore s=\frac{150}{2}=75\mathrm{m}$

$\Rightarrow$ s = 75 m

By Heron's formula, we have

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{75(75-25)(75-60)(75-65)}$
$= \sqrt{75 \times 50 \times 15 \times 10}$
$= \sqrt{15 \times 5 \times 5 \times 10 \times 15 \times 10}$
$= 15 \times 5 \times 10$
= 750 m^2

75. Let a, b, c be the sides of the given triangle and s be its semi-perimeter.
Then, $s = \frac{a+b+c}{2}$ ...(i)
$\therefore$ Area of the given triangle$= \sqrt{s(s-a)(s-b)(s-c)} = \triangle$ say
As per given condition, the sides of the new triangle will be 2a, 2b, and 2c.
So, the semi-perimeter of the new triangle =
$s' = \frac{2a+2b+2c}{2} = a+b+c$ ...(ii)
From (i) and (ii), we get
s' = 2s
Area of new triangle $= \sqrt{s'\left(s'-2a\right)\left(s'-2b\right)\left(s'-2c\right)}$
$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$
$= \sqrt{16s(s-a)(s-b)(s-c)}$
$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\triangle$
The required ratio = $4\triangle : \triangle$ = 4:1
Therefore the ratio of area of new triangle thus formed and the given triangle is 4 : 1.