Solution

COORDINATE GEOMETRY

Class 10 - Mathematics

Section A

1.

(b) $\sqrt{13}$ units

Explanation:

Distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Diameter = $\sqrt{(1-(-5))^2+(0-4)^2}$

= $\sqrt{36+16}$

= $\sqrt{52}$ = $2\sqrt{13}$

radius = $\frac{\text{diameter}}{2}$

= $\frac{2\sqrt{13}}{2}$

= $\sqrt{13}$

2. **(a)** $\sqrt{a^2+b^2}$

Explanation:

Distance between $(a\cos\theta+b\sin\theta,0)$ and $(0,a\sin\theta-b\cos\theta)$

$=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

$=\sqrt{\sqrt{(0-(a\cos\theta+b\sin\theta))^2+\{(a\sin\theta-b\cos\theta-0)\}^2}}$

$=\sqrt{\{0-(a\cos\theta+b\sin\theta)\}^2+\{(a\sin\theta-b\cos\theta-0)\}^2}$

$=\sqrt{\begin{pmatrix}a^2\cos^2\theta+b^2\sin^2\theta+2ab\sin\theta\cos\theta+\\ a^2\sin^2\theta+b^2\cos^2\theta-2ab\sin\theta\cos\theta\end{pmatrix}}$

$=\sqrt{a^2\times 1+b^2\times 1}=\sqrt{a^2+b^2}$

$\left\{\because \sin^2\theta+\cos^2\theta=1\right\}$

3.

(b) $\sqrt{37}$ units

Explanation:

AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

= $\sqrt{(5-(-1))^2+(-2-(-3))^2}$

= $\sqrt{36+1}$

= $\sqrt{37}$ units

4.

(d) (– 4, 2)

Explanation:

(x, y) = $\{\frac{(-6+(-2))}{2}, \frac{(8+(-4))}{2}\}$

= $(\frac{-8}{2}, \frac{4}{2})$

= (-4,2)

5.

(b) 2 : 1

Explanation:

The point lies on x-axis

Its ordinate is zero

Let this point divides the line segment joining the points (3, 6) and (12, -3) in the ratio m : n

$\therefore 0=\frac{my_2+ny_1}{m+n} \Rightarrow 0=\frac{m(-3)+n\times 6}{m+n}$
$\Rightarrow -3m+6n=0 \Rightarrow 6n=3m$
$\Rightarrow \frac{m}{n}=\frac{6}{3}=\frac{2}{1}$
$\therefore$ Ratio = 2 : 1

6.
(b) $\left(\frac{7}{2}, \frac{9}{4}\right)$
Explanation:
Cordinates of mid points are $\left[\frac{(8-1)}{2}, \left(\frac{3+\left(\frac{3}{2}\right)}{2}\right)\right]$
$=(\frac{7}{2}, \frac{9}{4})$

7.
(b) $AP=\frac{1}{2}AB$
Explanation:
AP = $\sqrt{(2-4)^2+(1-2)^2}$
= $\sqrt{4+1}$ = $\sqrt{5}$ = units
AB = $\sqrt{(8-4)^2+(4-2)^2}$
= $\sqrt{16+4}$ = $\sqrt{20}$ = $2\sqrt{5}$ units
Here AB = 2 $\times$AP
$\therefore$ AP = $\frac{1}{2}$AB

8. **(a)** $\sqrt{41}$
Explanation:
The distance of the point (5, 4) from origin
$=\sqrt{5^2+4^2}$
$=\sqrt{25+16}=\sqrt{41}$ unit

9.
(b) None of these
Explanation:
Let the points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ from an equilateral triangle AB = BC = CA
$\Rightarrow$ AB^2 = BC^2 = CA^2
Now, $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$=(3-0)^2+(\sqrt{3}-0)^2=(3)^2+(\sqrt{3})^2$
= 9 + 3 = 12
$BC^2=(3-3)^2+(\lambda-\sqrt{3})^2$
$=(0)^2+(\lambda-\sqrt{3})^2=(\lambda-3)^2$
and $CA^2=(0-3)^2+(0-\lambda)^2=(-3)^2+(-\lambda)^2$
$=9+\lambda^2$
$AB^2=CA^2 \Rightarrow 12=9+\lambda^2$
$\Rightarrow \lambda^2=12-9=3$
$\therefore\ \lambda=\pm\sqrt{3}$

10.
(d) $\sqrt{52}$
Explanation:
Let us take (3, – 2) and (– 3, 2) as (x_1, y_1) and (x_2, y_2)
Using distance formjula, d = $\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$
d = $\sqrt{(-3-3)^2+(2-(-2))^2}$

d = $\sqrt{(-6)^2+(2+2)^2}$
d = $\sqrt{36+(4)^2}$
d = $\sqrt{36+16}$
d = $\sqrt{52}$

11.
(d) 7
Explanation:
In parallelogram, AB = CD, squaring both sides

$\Rightarrow$ $AB^2 = CD^2$
$\Rightarrow (8-6)^2+(2-1)^2=(p-9)^2+(3-4)^2$
$\Rightarrow 4+1=p^2+81-18p+1$
$\Rightarrow p^2-18p+77=0$
$\Rightarrow (p-7)(p-11)=0$
$\Rightarrow p=7$ and $p=11$

12.
(d) 1 : 2
Explanation:
Let the ratio is k : 1
So the coordinate of the point are
$\left[\left(\frac{5k+2}{k+1}\right),\left(\frac{6k-3}{k+1}\right)\right]$
Since the point lies on x-axis, it's y coordinate will be 0
Comparing the coordinates
$\frac{6k-3}{k+1}$ = 0
6k - 3 = 0
6k = 3
k = $\frac{1}{2}$
Required ratio is
= $\frac{1}{2}$: 1
= 1 : 2

13.
(b) 5 units
Explanation:
According to question,
Coordinates of Chaitanya's house are (6, 5)
Coordinates of Hamida's house are (2, 2)
$\therefore$ Shortest distance between their houses
= $\sqrt{(6-2)^2+(5-2)^2}$ = 5 units.

14. **(a)** 0
Explanation:
Since coordinates of any point on y-axis is (0, y)
Therefore, the abscissa is 0.

15. **(a)** $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
Explanation:

we know that the midpoint formula = $\frac{x_1+x_2}{2}$, $\frac{y_1+y_2}{2}$

The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

16.

(c) -1

Explanation:

A(2, 3) and B(-4, 1) are the given points.

Let C(0.y) be the points are y-axis

AC = $\sqrt{(0-2)^2+(y-3)^2}$

⇒ AC = $\sqrt{4+y^2+9-6y}$

⇒ AC = $\sqrt{y^2-6y+13}$

BC = $\sqrt{(0+4)^2+(y-1)^2}$

⇒ BC = $\sqrt{16+y^2+1-2y}$

⇒ BC = $\sqrt{y^2-2y+17}$

Since AC = BC

$AC^2 = BC^2$

y^2 - 6y + 13 = y^2 - 2y + 17

⇒ -6y + 2y = 17 - 13

⇒ -4y = 4

⇒ y = -1

Therefore, the point on y-axis is (0, -1) and here ordinate is -1.

17.

(d) $2a\sqrt{2}$ units

Explanation:

Let the points be A(a, a) and B$(-\sqrt{3}a, \sqrt{3}a)$

∴AB = $\sqrt{\left(-\sqrt{3}a-a\right)^2+\left(\sqrt{3}a-a\right)^2}$

= $\sqrt{3a^2+a^2+2\sqrt{3}aa+3a^2+a^2-2\sqrt{3}aa}$

= $\sqrt{6a^2+2a^2}$

= $\sqrt{8a^2}$

= $2a\sqrt{2}$ units

18.

(d) (3, 0)

Explanation:

Let the required point be P(x, 0) then,

$AP^2 = BP^2 \Rightarrow (x-7)^2+(0-6)^2-(x+3)^2+(0-4)^2$

$\Rightarrow x^2-14x+85 = x^2+6x+25$

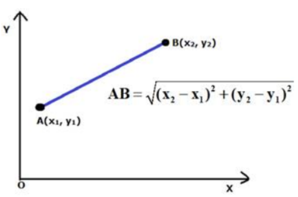
$-20x-60 = x = 3$

19. **(a)** $5\sqrt{2}$

Explanation:

By using the formula:

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



To calculate distance between the points (x_1 , y_1) and (x_2, y_2)

Here we have;

$x_1 = 0, x_2 = -5$

$y_2 = 5, y_2 = 0$

$d^2 = [(-5) - 0]^2 + [0 - 5]^2$

d = $\sqrt{(-5-0)^2+(0-5)^2}$

d = $\sqrt{25+25}$

d = $\sqrt{50}$ = $5\sqrt{2}$

20.

(d) (0, – 10) and (4, 0)

Explanation:

Let the coordinates of P (0, y) and Q (x, 0).

So, the mid - point of P (0, y) and Q (x, 0) = M

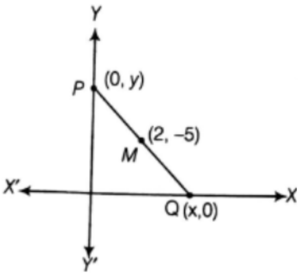
Coordinates of M = $\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$

$\therefore$ Mid - point of a line segment having points (x_1, y_1) and (x_2, y_2)

= $\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$

Given,

Mid - point of PQ is (2, - 5)



$\therefore 2 = \frac{x+0}{2}$ = 4 = x + 0

x = 4

$-5 = \frac{y+0}{2}$ = - 10 = y + 0

- 10 = y

So,

x = 4 and y = - 10

Thus, the coordinates of P and Q are (0, - 10) and (4, 0)

21. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Image of points of type (0, k) is (0, –k) only.

22. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

The x coordinate of the point (0, 4) is zero.

Point (0, 4) lies on y-axis.

23. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Distance of point (5, 12) from 8 origin is given, d = $\sqrt{(5-0)^2+(12-0)^2}$

= $\sqrt{25+144} = \sqrt{169}$ = 13

24. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

We know that the coordinates of the point P(x, y) which divides the line segment $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m_1 : m_2$

is $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ So, Reason is correct.

Here, x-coordinate = $\frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{(1\times-1)+(2\times1)}{1+2}$ = $\frac{1}{3}$

and y-coordinate = $\frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{(1\times1)+(2\times2)}{1+2}$ = $\frac{1+4}{3}$ = $\frac{5}{3}$

So, Assertion is correct.

25.

(d) A is false but R is true.

Explanation:

Let joining of (1, 1) and (5, 5) meet x-axis in k : 1. Now, x = $\frac{5k+1}{k+1}$ and y = $\frac{5k+1}{k+1}$

But for x-axis y = 0. So, $\frac{5k+1}{k+1}$ = 0 $\Rightarrow$ k = $-\frac{1}{5}$

$\Rightarrow$ 1 : 5 externally

26. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Image of points of type (h, 0) is (–h, 0) only.

27.

(c) A is true but R is false.

Explanation:

Let, A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are all rational coordinates,

$\operatorname{ar}(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

= $\frac{\sqrt{3}}{4}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]$

LHS = rational

RHS = irrational

Hence, (x_1, y_1) (x_2, y_2) and (x_3, y_3) cannot be all rational.

28.

(d) A is false but R is true.

Explanation:

PQ = 10

PQ^2 = 100

$(10 - 2)^2 + (y + 3)^2 = 100$

$(y + 3)^2 = 100 - 64 = 36$

y + 3 = $\pm$6

y = -3 $\pm$6

y = 3, -9

29.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Distance of point (h, k) from its image under x-axis is 2k units and distance of point (h, k) under y-axis is 2h units.

30.

(d) A is false but R is true.

Explanation:

A is false but R is true.

Section B

31. 1

Explanation:

It is given that A(0, 2) is equidistant from the points B(3, p) and C(p, 5).

$\therefore$ AB = AC

$\Rightarrow \sqrt{(3-0)^2+(p-2)^2} = \sqrt{(p-0)^2+(5-2)^2}$ (Distance formula)

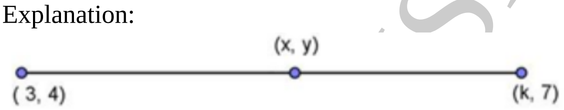
Squaring on both sides, we get

$9 + p^2 - 4p + 4 = p^2 + 9$
$\Rightarrow -4p + 4 = 0$
$\Rightarrow p = 1$
Thus, the values of p is 1.

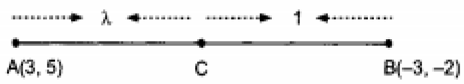
32. 10
Explanation:
The given points are A(9, 3) and B(15, 11)
Then, $x_1 = 9$, $y_1 = 3$, $x_2 = 15$, $y_2 = 11$
By distance formula,
$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(15-9)^2 + (11-3)^2} = \sqrt{6^2 + 8^2}$
$= \sqrt{36+64} = \sqrt{100}$ = 10 units

33. 13
Explanation:
The given point is A(5, -12) and let O(0, 0) be the origin
Then, $AO = \sqrt{(5-0)^2 + (-12-0)^2}$
$= \sqrt{5^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169}$
= 13 units

34. -1
Explanation:
At mid-point of AB = $\left(\frac{\frac{x}{2}+x+1}{2}\right) = 5$
or, x = 6
$\left(\frac{\frac{y+1}{2}+y-3}{2}\right) = -2$
or, y + 1 + 2y - 6 = -8
y = -1

35. -15
Explanation:


Since, (x, y) is the mid-point
$x = \frac{3+k}{2}, y = \frac{4+7}{2} = \frac{11}{2}$
Again,
2x + 2y + 1 = 0
$\Rightarrow 2 \times \frac{(3+k)}{2} + 2 \times \frac{11}{2} + 1 = 0$
$\Rightarrow$ 3 + k + 11 + 1 = 0
$\Rightarrow$ 3 + k + 12 = 0
$\Rightarrow$ k + 15 = 0
$\Rightarrow$ k = -15

36. According to the question, A(3, 5) and B (-3, -2)
Let the point C divide AB in the ratio λ : 1.


By using section formula, $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$
The coordinates of C are
$\left(\frac{-3\lambda+3}{\lambda+1}, \frac{-2\lambda+5}{\lambda+1}\right)$
But, the coordinates of C are given as $\left(\frac{3}{5}, \frac{11}{5}\right)$
$\therefore \quad \frac{-3\lambda+3}{\lambda+1} = \frac{3}{5}$ and $\frac{-2\lambda+5}{\lambda+1} = \frac{11}{5}$
$\Rightarrow \quad -15\lambda + 15 = 3\lambda + 3$ and $-10\lambda + 25 = 11\lambda + 11$

$\Rightarrow \quad 18\lambda = 12$ and $21\lambda = 14$

$\Rightarrow \quad \lambda = \frac{2}{3}$

Hence, the point C divides AB in the ratio 2 : 3.

37. The point P on x-axis will have its ordinate = 0

Let the coordinates of point P be (x, 0)

Let the given points be A(–2, 5) and B(2, –3)

Then PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x + 2)^2 + (0 – 5)^2 = (x – 2)^2 + (0 + 3)^2$

$\Rightarrow x^2 + 4 + 4x + 5^2 = x^2 + 4 – 4x + 3^2$

$\Rightarrow x^2 + 4x + 4 + 25 = x^2 + 4 – 4x + 9$

$\Rightarrow$ 4x + 4x = 9 – 25

$\Rightarrow$ 8x = – 16

$\Rightarrow x = \frac{-16}{8} = -2$

$\therefore$ The point equidistant from given points is (–2, 0)

38. Let the point P(x, 2) divides the line segment joining the points A(12, 5) and B(4, -3) in the ratio k:1.

Then, by section formula, $\frac{(mx_2+nx_1)}{m+n}, \frac{(my_2+ny_1)}{m+n}$

Coordinates of P are $\left(\frac{k\times4+1\times12}{k+1}, \frac{k\times(-3)+1\times5}{k+1}\right)$

$= \left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right)$

Given, coordinates of P are (x, 2)

$\therefore \frac{-3k+5}{k+1} = 2$

$\Rightarrow$ -3k + 5 = 2k + 2

$\Rightarrow$5k = 3

$\Rightarrow k = \frac{3}{5}$

Therefore ,the ratio in which the point P(x, 2) divides the join of A(12, 5) and B(4, -3) is equal to 3:5

39. Let P and Q divide the line segment joining A(5, -3) and B(-4, 3) in three equal parts such that AP : PB = 1 : 2

$P\left(\frac{1\times-4+2\times5}{1+2}, \frac{1\times3+2\times-3}{1+2}\right)$

i.e., P(2, -1)

AQ : QB = 2 : 1

$Q\left(\frac{2\times-4+1\times5}{2+1}, \frac{2\times3+1\times-3}{2+1}\right)$

i.e., Q(-1, 1)

40. Let coordinates of R be (x, y).

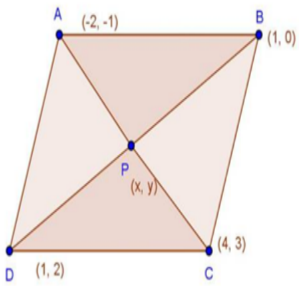
PR : RQ = 2 : 1

$x = \frac{2(3)+1(-2)}{2+1} = \frac{4}{3}$

$y = \frac{2(2)+1(5)}{2+1} = 3$

$\therefore$ Cordinates of the point R $\left(\frac{4}{3}, 3\right)$

41.



Let P(x, y) be the given points.

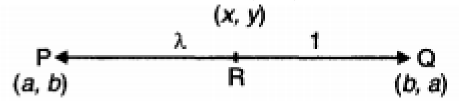
We know that diagonals of a parallelogram bisect each other.

$x = \frac{-2+4}{2}$

$\Rightarrow \quad x = \frac{2}{2} = 1$

$y = \frac{-1+3}{2} = \frac{2}{2} = 1$

$\therefore$ Coordinates of P are (1, 1)

42. 

According to the question, R(x, y) is a point on the line segment joining the points P(a, b) and Q(b, a)

Let point R(x + y) divides the line joining P(a,b) and Q(b,a) in the ratio λ : 1.

$\therefore x = \frac{\lambda b + a}{\lambda + 1}$

$y = \frac{\lambda a + b}{\lambda + 1}$

Adding, $x + y = \frac{\lambda b + a + \lambda a + b}{\lambda + 1}$

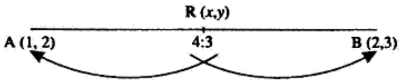
$= \frac{\lambda(a+b) + 1 \times (a+b)}{\lambda + 1}$

$= \frac{(\lambda+1) \times (a+b)}{\lambda + 1} = a + b$

$\Rightarrow x + y = a + b$

Hence Proved.

43. Given that $\frac{AR}{RB} = \frac{4}{3}$ or AR : RB = 4 : 3



Let the coordinates of R be (x, y).

Then $x = \frac{4 \times 2 + 3 \times 1}{4+3} = \frac{8+3}{7} = \frac{11}{7}$

$y = \frac{4 \times 3 + 3 \times 2}{4+3} = \frac{12+6}{7} = \frac{18}{7}$

$\therefore$ The coordinates of R are $\left(\frac{11}{7}, \frac{18}{7}\right)$

44. 

Let C divides AB in the ratio 1 : 2

$\therefore$ C $\left(\frac{1 \times 4 + 2 \times 5}{1+2}, \frac{1 \times 5 + 2 \times 3}{1+2}\right)$, i.e., C $\left(\frac{14}{3}, \frac{11}{3}\right)$

Let D divides AB in the ratio 2 : 1

$\therefore$ D $\left(\frac{2 \times 4 + 1 \times 5}{2+1}, \frac{2 \times 5 + 1 \times 3}{2+1}\right)$, i.e., D $\left(\frac{13}{3}, \frac{13}{3}\right)$

45. AD is the median of triangle ABC

$\therefore$ D is the mid-point of BC.

$\therefore$ Its coordinates are $\left(\frac{7}{2}, \frac{9}{2}\right)$

$P \rightarrow \left(\frac{(2)\left(\frac{7}{2}\right)+(1)(4)}{2+1}, \frac{(2)\left(\frac{9}{2}\right)+(1)(2)}{2+1}\right)$ [Using section formula]

$\Rightarrow P \rightarrow \left(\frac{11}{3}, \frac{11}{3}\right)$

$E \rightarrow \left(\frac{4+1}{2}, \frac{2+4}{2}\right)$

$\Rightarrow E \rightarrow \left(\frac{5}{2}, 3\right)$

Section C

46. Let Home represented by point H(4, 5), Library by point L(-1, 3), Skate Park by point P(3, 0) and School by S(4, 2).

i. Distance between Home and School, HS = $\sqrt{(4-4)^2 + (2-5)^2} = 3\ metres$

ii. Now, HL = $\sqrt{(-1-4)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29}$

LS = $\sqrt{[4-(-1)]^2 + (2-3)^2} = \sqrt{25+1} = \sqrt{26}$

Thus, HL + LS = $\sqrt{29} + \sqrt{26} = 10.48\ metres$

So, extra distance covered by Ramesh is = HL + LS - HS = 10.48 - 3 = 7.48 metres

iii. Now, HP = $\sqrt{(3-4)^2 + (0-5)^2} = \sqrt{1+25} = \sqrt{26}$

PS = $\sqrt{[4-3]^2 + (2-0)^2} = \sqrt{1+4} = \sqrt{5}$

Thus, HP + PS = $\sqrt{26} + \sqrt{5} = 7.33\ metres$

So, extra distance covered by Ramesh is = HP + PS - HS = 7.33 - 3 = 4.33 metres

47. Let ABCD be a parallelogram in which the co-ordinates of the vertices are A (3,-4); B (-1,-3) and C(-6,2). We have to find the co-ordinates of the forth vertex.

Let the forth vertex be D(x, y)

Since ABCD is a parallelogram, the diagonals bisect each other. Therefore the midpoint of the diagonals of the parallelogram will coincide.

Now to find the mid-point P(x, y) of two points A(x_1, y_1) and B(x_2, y_2).

we use section formula as,

$P(x, y)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The mid-point of the diagonals of the parallelogram will coincide.

So, coordinate of mid-point AC = Coordinate of mid-point of BD

Therefore,

$\left(\frac{x-1}{2}, \frac{y-3}{2}\right)=\left(\frac{3-6}{2}, \frac{2-4}{2}\right)$

$\left(\frac{x-1}{2}, \frac{y-3}{2}\right)=\left(-\frac{3}{2},-1\right)$

Now equate the individual terms to get the unknown value. So,

x = -2

y = 1

So the forth vertex is D(-2, 1)

48. Distance between P(2, -3) and Q(x, 5) = 10

$\Rightarrow \sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

$\Rightarrow \sqrt{(x-2)^2+(5+3)^2}=10$

squaring,

$\Rightarrow$ $(x - 2)^2 + (8)^2 = (10)^2$

$\Rightarrow$ $x^2 - 4x + 4 + 64 = 100$

$\Rightarrow$ $x^2 - 4x + 68 - 100 = 0$

$\Rightarrow$ $x^2 - 4x - 32 = 0$

$\Rightarrow$ $x^2 - 8x + 4x - 32 = 0$

$\Rightarrow$ x(x - 8) + 4(x - 8) = 0

$\Rightarrow$ (x - 8) (x + 4) = 0

Either x - 8 = 0, then x = 8

or x + 4 = 0, then x = -6

$\therefore$ x = 8, -4

49. Coordinates of points on a circle are A(2, 1), B(5, -8) and C(2, -9).

Let the coordinates of the centre of the circle be O(x, y).

OA = OB

$\sqrt{(x-2)^2+(y-1)^2}=\sqrt{(x-5)^2+(y+8)^2}$

$(x - 2)^2 + (y - 1)^2 = (x - 5)^2 + (y + 8)^2$

$x^2 + 4 - 4x + y^2 + 1 - 2y = x^2 + 25 - 10x + y^2 + 64 + 16y$

6x - 18y - 84 = 0

x - 3y - 14 = 0 .... (i)

Similarly, OC = OB

$\sqrt{(x-2)^2+(y+9)^2}=\sqrt{(x-5)^2+(y+8)^2}$

$(x - 2)^2 + (y + 9)^2 = (x - 5)^2 + (y + 8)^2$

$x^2 + 4 - 4x + y^2 + 81 + 18y = x^2 + 25 - 10x + y^2 + 64 + 16y$

6x + 2y - 4 = 0

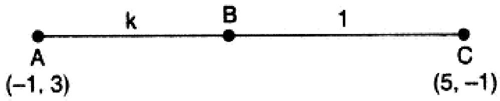
3x + y - 2 = 0 .... (ii)

By solving (i) and (ii), we get,

x = 2 and y = -4

So, the coordinates of the centre of circle are (2, -4).

50. Let A → (–1, 3) B → (2, p) and C → (5, –1)



If the points A, B and C are collinear then let B divide AC in the ratio K : 1 internally.

Then, $B \rightarrow \left\{\frac{(\mathrm{K})(5)+(1)(-1)}{\mathrm{K}+1}, \frac{(\mathrm{K})(-1)+(1)(3)}{\mathrm{K}+1}\right\}$

$\Rightarrow \mathrm{B} \rightarrow \left(\frac{5\mathrm{K}-1}{\mathrm{K}+1}, \frac{-\mathrm{K}+3}{\mathrm{K}+1}\right)$

But, B is given to be (2,p)

$\therefore \frac{5\mathrm{K}-1}{\mathrm{K}+1} = 2$

⇒ 5K - 1 = 2(K + 1)

⇒ 5K - 1 = 2K + 2

⇒5K - 2K = 2 + 1

⇒3K = 3

$\Rightarrow \mathrm{K} = \frac{3}{3} = 1$ and $\frac{-\mathrm{K}+3}{\mathrm{K}+1} = \mathrm{p}$

$\Rightarrow \frac{-1+3}{1+1} = p$

⇒ p = 1

Hence, the required value of p is 1.

51. Given: P(2, 2) is equidistant from the points A(-2, k) and B(-2k, -3),

We have, AP = BP

$AP^2 = BP^2$

$(2 + 2)^2 + (2 - k)^2 = (2 + 2k)^2 + (2 + 3)^2$

$16 + 4 + k^2 - 4k = 4 + 4k^2 + 8k + 25$

$20 + k^2 - 4k = 29 + 4k^2 + 8k$

$3k^2 + 12k + 9 = 0$

$k^2 + 4k + 3 = 0$

$k^2 + 3k + k + 3 = 0$

k(k + 3) + 1(k + 3) = 0

(k + 1) (k + 3) = 0

k = -1, -3

For k = -1, we have,

AP = $\sqrt{(2+2)^2+(2-k)^2} = \sqrt{16+(2+1)^2} = \sqrt{25} = 5$

For k = -3, we have,

AP = $\sqrt{(2+2)^2+(2+3)^2} = \sqrt{16+25} = \sqrt{41}$

52. Given coordinates of point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ and coordinates of the line segment joining the point $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, -5).

Let the required ratio be k:1

Then, by section formula,$\left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+nx_1}{m+n}\right]$

Coordinates of $P = \left(\frac{k\times 2+1\times\frac{1}{2}}{k+1}, \frac{k\times(-5)+1\times\frac{3}{2}}{k+1}\right)$

$= \left(\frac{2k+\frac{1}{2}}{k+1}, \frac{-5k+\frac{3}{2}}{k+1}\right)$

$= \left(\frac{4k+1}{2(k+1)}, \frac{-10k+3}{2(k+1)}\right)$

Given, coordinates of $P = \left(\frac{3}{4}, \frac{5}{12}\right)$

$\therefore \frac{4k+1}{2(k+1)} = \frac{3}{4}$

⇒ 16k + 4 = 6k + 6

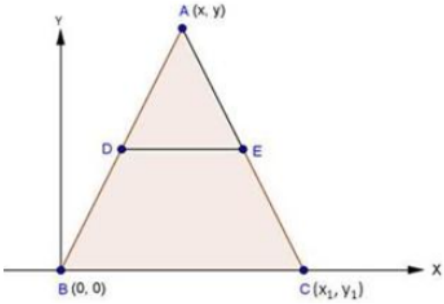
$\implies$ 16k-6k=6-4

⇒ 10k = 2

$\Rightarrow k = \frac{1}{5}$

So, the required ratio is 1:5.

53. Let ABC be a triangle such that BC is along x-axis.



Coordinates of A, B and C are (x, y), (0, 0) and (x_1, y_1)

D and E are the mid-points of AB and AC respectively.

Coordinates of D are $\left(\frac{x+0}{2}, \frac{y+0}{2}\right)$

$= \left(\frac{x}{2}, \frac{y}{2}\right)$

Coordinates of E are $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

Length of BC $= \sqrt{x_1^2 + y_1^2}$

Length of $DE = \sqrt{\left(\frac{x+x_1}{2} - \frac{x}{2}\right)^2 + \left(\frac{y+y_1}{2} - \frac{y}{2}\right)^2}$

$= \sqrt{\left(\frac{x_1}{2}\right)^2 + \left(\frac{y_1}{2}\right)^2}$

$= \sqrt{\frac{x_1^2}{4} + \frac{y_1^2}{4}}$

$= \sqrt{\frac{1}{4}\left(x_1^2 + y_1^2\right)}$

$= \frac{1}{2}\sqrt{x_1^2 + y_1^2}$

$= \frac{1}{2}BC$

Hence proved that length of $DE = \frac{1}{2}$ of BC

54. Let A(1, 7), B(4, 2), C(-1, -1) and D(-4, 4) be the given points. One way of showing that ABCD is a square is to use the property that all its sides should be equal both its diagonals should also be equal. Now,

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$

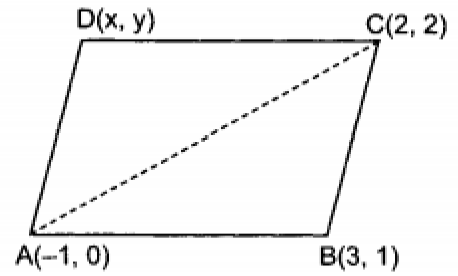
$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$

$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$

$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

55. 

Area of $\triangle$ABC

$= \frac{1}{2}$ $[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}$ [-1(1 - 2) + 3(2 - 0) + 2(0 - 1)]

$= \frac{1}{2}$ [1 + 6 - 2] = $\frac{5}{2}$ sq. units

Area of $\|$gm = 2 $\times$ area of $\triangle$ABC

$\Rightarrow$ Area of $\|$gm = 2 $\times$ $\frac{5}{2}$ = 5 sq. units

Let coordinates of D are (x, y)

Mid point of AC $= \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$

Mid-point of BD $= \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$

$\therefore$ Diagonals of a $\|\mathrm{gm}$ bisect each other

$\therefore$ Mid-point of BD = Mid-point of AC

$\Rightarrow \left(\frac{3+x}{2}, \frac{1+y}{2}\right) = \left(\frac{1}{2}, 1\right)$

$\Rightarrow \frac{3+x}{2} = \frac{1}{2}$ and $\frac{1+y}{2} = 1$

$\Rightarrow$ x = - 2

$\Rightarrow$ y = 1

Now AD = $\sqrt{(-1+2)^2 + (0+1)^2} = \sqrt{2}$

Also area of $\|\mathrm{gm}$ = base $\times$ height

$\Rightarrow$ AD $\times$ height = 5

$\Rightarrow \sqrt{2} \times$ height = 5

$\Rightarrow$ height $= \frac{5}{\sqrt{2}} = \frac{5}{2}\sqrt{2}$ units.

Section D

56. i. (4, 8) and (-3, 7)
 ii. 8 units
 iii. 1280 cubic feet
 OR
 7 or -1

57. i. Distance travelled by second bus = 7.2 km
 $\therefore$ Total fare = 7.2 $\times$ 15 = ₹08
 ii. Required distance = $\sqrt{(2+2)^2 + (3+3)^2}$
 $= \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = 2\sqrt{13}$ km $\approx$ 7.2 km
 iii. Required distance = $\sqrt{(3+2)^2 + (2+3)^2}$
 = $\sqrt{5^2 + 5^2} = 5\sqrt{2}$ km
 OR
 Distance between B and C
 = $\sqrt{(3-2)^2 + (2-3)^2}$ = $\sqrt{1+1} = \sqrt{2}$ km
 Thus, distance travelled by first bus to reach to B
 = AC + CB = $5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$ km $\approx$ 8.48 km
 and distance travelled by second bus to reach to B
 = AB = $2\sqrt{13}$ km = 7.2 km
 $\therefore$ Distance of first bus is greater than distance of the cond bus, therefore second bus should be chosen.

58. i. Position of Neena = (3, 6)
 Position of Karan = (6, 5)
 Distance between Neena and Karan = $\sqrt{(6-3)^2 + (5-6)^2}$
 = $\sqrt{9 + (-1)^2}$
 = $\sqrt{10}$
 ii. Co-ordinate of seat of Akash = 2, 3
 iii. 

 Co-ordinate of middle point = $\left(\frac{2+5}{2}, \frac{3+2}{2}\right)$
 = 3.5, 2.5
 OR
 Binu = (5, 5); Karan = (6, 5)
 Distance = $\sqrt{(6-5)^2 + (5-2)^2}$
 $= \sqrt{1+9}$
 $= \sqrt{10}$

59. i. Since, PQRS is a square
 $\therefore$ PQ = QR = RS = PS
 Length of PQ = 200 - (-200) = 400

$\therefore$ The coordinates of R = (200, 400)
and coordinates of S = (-200, 400)

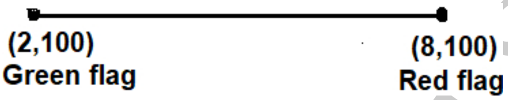
ii. Area of square PQRS = $(\text{side})^2$
= $(PQ)^2$
= $(400)^2$
= 1,60,000 sq. units

iii. By Pythagoras theorem
$(PR)^2 = (PQ)^2 + (QR)^2$
= 1,60,000 + 1,60,000
= 3,20,000
$\Rightarrow$ PR = $\sqrt{3,20,000}$
= 400 $\times$ $\sqrt{2}$ units

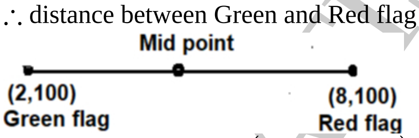
OR

Since, point S divides CA in the ratio K : 1
$\therefore \left(\frac{Kx_2+x_1}{K+1}, \frac{Ky_2+y_1}{K+1}\right)$ = (-200, 400)
$\Rightarrow \left(\frac{K(200)+(-600)}{K+1}, \frac{K(800)+0}{K+1}\right)$ = (-200, 400)
$\Rightarrow \left(\frac{200K-600}{K+1}, \frac{800K}{K+1}\right)$ = (-200, 400)
$\therefore \frac{800K}{K+1}$ = 400
$\Rightarrow$ 800K = 400K + 400
$\Rightarrow$ 400K = 400
$\Rightarrow$ K = 1

60. i. Co-ordinate of green flag = (2,100)

ii. 

distance between Red flag and Green flag
d $= \sqrt{(8-2)^2 + (100-100)^2}$
= $\sqrt{6^2 + 0^2}$
d = 6
$\therefore$ distance between Green and Red flag is 6 m.

iii. 

Position of blue flag = $\left(\frac{2+8}{2}, \frac{100+100}{2}\right)$
= (5,100)

OR

Distance = $\sqrt{(5-2)^2 + (100-100)^2}$
$= \sqrt{9+0}$
= 3 m

61. i. Mid point of FG is $\left(\frac{-3+1}{2}, \frac{0+4}{2}\right)$ = (-1, 2)

ii. a. $AC = \sqrt{(-1-3)^2 + (-2-4)^2}$
$= \sqrt{52}$ or $2\sqrt{13}$

OR

b. The coordinates of required point are $\left(\frac{1\times3+3\times3}{1+3}, \frac{1\times2+3\times4}{1+3}\right)$ i.e. $\left(3, \frac{7}{2}\right)$

iii. D(-2, -5)

62. i. Point of intersection of diagonals is their midpoint
So, $\left[\frac{(1+7)}{2}, \frac{(1+5)}{2}\right]$
= (4, 3)

ii. Length of diagonal AC
AC = $\sqrt{(7-1)(7-1) + (5-1)(5-1)}$

$= \sqrt{52}$ units

iii. Area of campaign board

$= 6 \times 4$

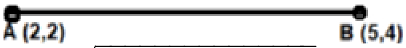
= 24 units square

OR

Ratio of lengths = $\frac{AB}{AC}$

$= \frac{6}{\sqrt{52}}$

$= 6 : \sqrt{52}$

63. i. 

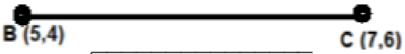
AB = $\sqrt{(5-2)^2 + (4-2)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

ii. Middle point AB = $\left(\frac{2+5}{2}, \frac{2+4}{2}\right)$

= (3.5, 3)

iii. 

BC = $\sqrt{(7-5)^2 + (6-4)^2}$

$= \sqrt{4+4}$

$= 2\sqrt{2}$

OR

Middle point of BC = $\left(\frac{5+7}{2}, \frac{4+6}{2}\right)$

= (6, 5)

64. i. We have, P(-3, 4), Q(3, 4) and R(-2, -1).

$\therefore$ Coordinates of centroid of $\triangle$PQR

$= \left(\frac{-3+3-2}{3}, \frac{4+4-1}{3}\right) = \left(\frac{-2}{3}, \frac{7}{3}\right)$

ii. Coordinates of T = $\left(\frac{-2+3}{2}, \frac{-1+4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

iii. Coordinates of U = $\left(\frac{-2-3}{2}, \frac{-1+4}{2}\right) = \left(\frac{-5}{2}, \frac{3}{2}\right)$

OR

The centroid of the triangle formed by joining the mid-points of sides of a given triangle is the same as that of the given triangle.

So, centroid of $\triangle$STU = $\left(\frac{-2}{3}, \frac{7}{3}\right)$

65. i. A(1, 9) and B(5, 13)

ii. C(9, 13) and D(13, 9)

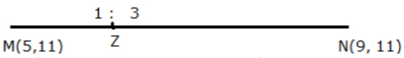
Mid-point of CD is (11, 11)

iii. a. M(5, 11) and Q(9, 3)

MQ = $\sqrt{(9-5)^2 + (3-11)^2} = \sqrt{80}$ or $4\sqrt{5}$

OR

b. M(5, 11) and N(9, 11)

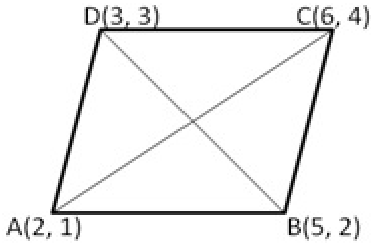


$Z\left(\frac{1\times9+3\times5}{1+3}, \frac{1\times11+3\times11}{1+3}\right)$

$Z(6, 11)$

Section E

66. Let A(2, 1), B(5, 2), C(6, 4) and D(3, 3) are the angular points of a parallelogram ABCD. Then



Now, distance between,

$AB = \sqrt{(5-2)^2 + (2-1)^2}$

$= \sqrt{(3)^2 + (1)^2}$

$= \sqrt{10} = \sqrt{10}\, units$

Distance between,

$BC = \sqrt{(6-5)^2 + (4-2)^2}$

$= \sqrt{(1)^2 + (2)^2}$

$= \sqrt{1+4} = \sqrt{5}\, units$

Distance between,

$DC = \sqrt{(6-3)^2 + (4-3)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}\, units$

And distance between,

$AD = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}\, units$

Therefore, AB = DC and AD = BC

Now,

D$iagonal\ AC = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5\, units$

And,

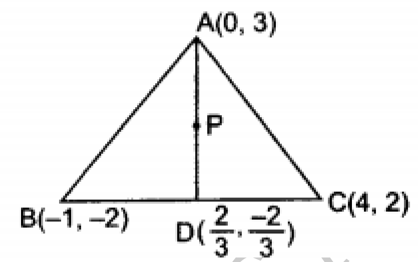
$Diagonal\ BD = \sqrt{(3-5)^2 + (3-2)^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{5}\, unit.$

Since,

Diagonal AC $\neq$ Diagonal BD

Thus ABCD is not a rectangle but it is a parallelogram because its opposite sides are equal and diagonals are not equal.

67.



$\because$ BD : CD = 1 : 2

$\therefore$ Coordinate of D are

$\left(\frac{1\times4+2\times-1}{1+2}, \frac{1\times2+2\times-2}{1+2}\right)$, i.e, $\left(\frac{2}{3}, \frac{-2}{3}\right)$

AD = $\sqrt{\left(\frac{2}{3}-0\right)^2 + \left(\frac{-2}{3}-3\right)^2}$

$= \sqrt{\frac{4}{9} + \frac{121}{9}} = \sqrt{\frac{125}{9}} = \frac{5\sqrt{5}}{3}$ units

DP = AD - AP = $\frac{5\sqrt{5}}{3} - \frac{2\sqrt{5}}{3} = \frac{3\sqrt{5}}{3} = \sqrt{5}$ units

$\therefore \frac{\text{AP}}{\text{DP}} = \frac{\frac{2\sqrt{5}}{3}}{\sqrt{5}} = \frac{2}{3}$

$\Rightarrow$ P divides AD in the ratio 2 : 3.

$\therefore$ x-coordinates of P is

x = $\frac{2\times\frac{2}{3}+3\times0}{2+3} = \frac{4}{15}$

Similarly, y-coordinates of P is

y = $\frac{2\times\frac{-2}{3}+3\times3}{2+3} = \frac{23}{15}$

$\therefore$ Coordinates of P are $\left(\frac{4}{15}, \frac{23}{15}\right)$.

68. Let the vertex of triangles are A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3)

and L(3,4),M(4,6) and N(5,7) are the midpoints of side AB, BC and AC respective.

Since, L, M and N are mid-point than they divide the sides AB, BC and AC in the ratio of 1:1

Midpoint formula

$x = \frac{x_1+x_2}{2}$

$y = \frac{y_1+y_2}{2}$

So,

$\frac{x_1+x_2}{2} = 3$

$x_1 + x_2 = 6$ ..... (1)

$\frac{y_1+y_2}{2} = 4$

$y_1 + y_2 = 8$ ..... (2)

by the same way

$x_2 + x_3 = 8$ .... (3)

$y_2 + y_3 = 12$ ... (4)

$x_1 + x_3 = 10$ .... (5)

$y_1 + y_3 = 14$ ... (6)

Subtract eq (3) and (5)

$x_2 - x_1 = 2$ .... (7)

subtract eq. (4) and (6)

$y_2 - y_1 = -2$ ... (8)

on adding eq. (1) and (7)

$x_1 + x_2 = 6$

$-x_1 + x_2 = -2$

$2x_2 = 4$

$x_2 = 2$

Therefore $x_1 = 4$ and $x_3 = 6$

on adding eq. (2) and (8), we get

$y_1 + y_2 = 8$

$-y_1 + y_2 = -2$

$2y_2 = 6$

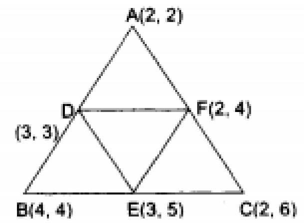
$y_2 = 3$

Therefore, $y_1 = 5$ and $y_3 = 9$

So A(4, 5), B(2, 3) and C(6, 9).

69. Let A(2, 2), B(4, 4) and C(2, 6) be the vertices of the given ΔABC. Let D, E and F be the midpoints of AB, BC and CA respectively.

Then, the coordinates of D, E and F are



$D\left(\frac{2+4}{2}, \frac{2+4}{2}\right), E\left(\frac{4+2}{2}, \frac{4+6}{2}\right)$ and $F\left(\frac{2+2}{2}, \frac{2+6}{2}\right)$

i.e., D(3, 3), E(3, 5) and F(2, 4).

For ΔDEF, we have

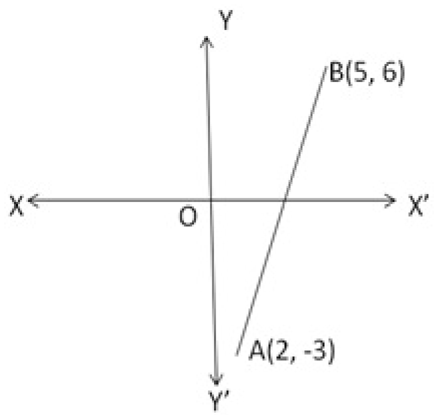
($x_1 = 3$, $y_1 = 3$), ($x_2 = 3$, $y_2 = 5$) and ($x_3 = 2$, $y_3 = 4$)

$\therefore \quad \operatorname{art}(\Delta DEF) = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|3 \cdot (5-4) + 3 \cdot (4-3) + 2 \cdot (3-5)|$

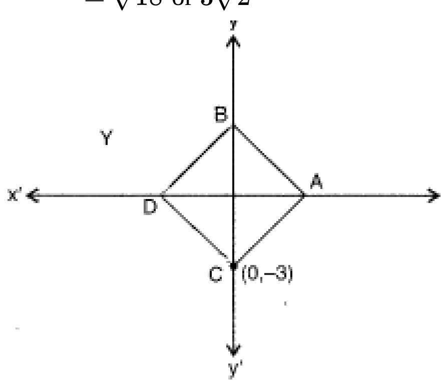
$= \frac{1}{2}|(3 \times 1) + (3 \times 1) + 2 \times (-2)|$
$= \frac{1}{2}|3 + 3 - 4| = \left(\frac{1}{2} \times 2\right) = 1 sq. unit.$
Hence, the area of ΔDEF is 1 sq unit.

70. Let the x-axis cut the join of A(2, -3) and B(5, 6) in the ratio k:1 at the point P
Then, by the section formula, the coordinates of P are $\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$



But P lies on the x-axis so, its ordinate must be 0
$\therefore \frac{6k-3}{k+1} = 0$
$\Rightarrow 6k - 3 = 0, k = \frac{1}{2}$
So the required ratio is 1:2
Thus the x-axis divides AB in the ratio 1:2
Putting $k = \frac{1}{2}$ in $\frac{5k+2}{k+1}$, we get the point P as
$P\left(\frac{5 \times \frac{1}{2} + 2}{\frac{1}{2} + 1}, 0\right)$ or $P(3, 0)$
Thus, P is (3, 0) and k= 1:2

71. i. Position of the pole C(5, 4)
ii. B(6, 6)
$\therefore BO = \sqrt{(6-0)^2 + (6-0)^2} = \sqrt{36+36} = \sqrt{72}$ or $6\sqrt{2}$
iii. a. A(2, 7), B(6, 6), C(5, 4)
Let D(x, y)
ABCD forms a parallelogram
$\therefore$ Mid-point of AC = Mid-point of BD
$\left(\frac{2+5}{2}, \frac{7+4}{2}\right) = \left(\frac{6+x}{2}, \frac{6+y}{2}\right)$
$\frac{7}{2} = \frac{6+x}{2} \Rightarrow \mathrm{x} = 1$
$\frac{11}{2} = \frac{6^2+y}{2} \Rightarrow y = 5$
Position of the fourth pole D is (1, 5)
OR
b. AC $= \sqrt{(5-2)^2 + (4-7)^2} = \sqrt{9+9}$
$= \sqrt{18}$ or $3\sqrt{2}$

72.


Co-ordinates of point B are (0,3)
$\therefore$BC = 6 unit

Let the co-ordinates of point A be (x, 0)

or, AB = $\sqrt{x^2+9}$

$\because$ AB = BC

$\therefore$ $x^2 + 9 = 36$

or, $x^2 = 27$ or, x = $\pm 3\sqrt{3}$

Co-ordinates of point A = ($3\sqrt{3}$,0)

Since ABCD is a rhombus

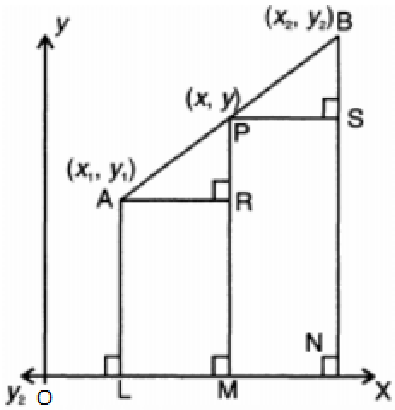
or, AB = AC = CD = DB

$\therefore$ Co-ordinate of point D = ($-3\sqrt{3}$,0)

73. i. Derivation of Section Formula:

Let A(x_1,y_1) and B(x_2, y_2) be two points.

Let P(x, y) be a point on line AB, such that P divides it in the ratio $m_1 : m_2$



Let AB be a line segment joining the points A(x_1, y_1), B(x_2, y_2).

Let P have coordinates (x, y).

Draw AL, PM, BN $\perp$ to x-axis.

It is clear from fig., that

AR = LM = (distance between origin and point M) - (distance between origin and point L).

$\therefore$ AR = LM = OM - OL = $x - x_1$

Similarly, PR = PM - RM = $y - y_1$

And, PS = (Distance between origin and point N) - (Distance between point M and origin) = ON - OM = $x_2 - x$

Similarly, BS = BN - SN = $y_2 - y$

$\Delta APR \sim \Delta PBS$ [AAA]

$\frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$

Now, $\frac{AR}{PS} = \frac{AP}{PB}$

$\Rightarrow \frac{x-x_1}{x_2-x} = \frac{m_1}{m_2}$

$\Rightarrow m_2(x-x_1) = m_1(x_2-x)$

$\Rightarrow m_2x - m_2x_1 = m_1x_2 - m_1x$

$\therefore x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

Similarly, $\frac{PR}{BS} = \frac{AP}{PB}$

$\Rightarrow \frac{y-y_1}{y_2-y} = \frac{m_1}{m_2}$

$\therefore y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

$\therefore$ Coordinates of P are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

ii. Let (- 4, 6) divides the line segment joining the point. A(-6, 10) and B(3, - 8) in k : 1

So, $x_1 = -6$, $y_1 = 10$, $x_2 = 3$, $y_2 = -8$, x = -4, y = 6, $m_1 = k$, $m_2 = 1$

Using section formula,

$-4 = \frac{k(3)+1(-6)}{k+1}$

$\Rightarrow -4k - 3k = -6 + 4$

$\Rightarrow -7k = -2$

$\Rightarrow k=\frac{2}{7}$

Therefore, the ratio = 2 : 7

74. (-1, -2), (1, 0), (-1, 2), (-3, 0)

Let A $\rightarrow$ (-1, -2), B $\rightarrow$ (1, 0)

C $\rightarrow$ (-1, 2) and D $\rightarrow$ (-3, 0)

Then,

$AB=\sqrt{[1-(-1)]^2+[0-(-2)]^2}$

$=\sqrt{(2)^2+(2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

$BC=\sqrt{(-1-1)^2+(2-0)^2}$

$=\sqrt{(-2)^2+(2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

$CD=\sqrt{[(-3)-(-1)]^2+(0-2)^2}$

$=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

$DA=\sqrt{[(-1)-(-3)]^2+(-2-0)^2}$

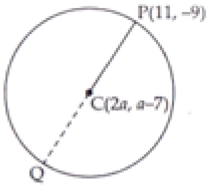
$=\sqrt{(2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$

$AC=\sqrt{[(-1)-(-1)]^2+[(2)-(-2)]^2}=4$

$BD=\sqrt{[(-3)-(1)]^2+(0-0)^2}=4$

Since AB = BC = CD = DA (i.e., all the four sides of the quadrilateral ABCD are equal) and AC = BD (i.e. diagonals of the quadrilateral ABCD are equal). Therefore, ABCD is a square.

75. Let C(2a, a – 7) be the centre of the circle and it passes through the point P(11, –9).



$\therefore$ PQ = $10\sqrt{2}$

⇒ CP = $5\sqrt{2}$

⇒ $CP^2 = (5\sqrt{2})^2 = 50$

⇒ $(2a – 11)^2 + (a – 7 + 9)^2 = 50$

⇒ $(2a)^2 + (11)^2 – 2(2a)(11) + (a + 2)^2 = 50$

⇒ $4a^2 + 121 – 44a + (a)^2 + (2)^2 + 2(a)(2) = 50$

⇒ $5a^2 – 40a + 125 = 50$

⇒ $a^2 – 8a + 25 = 10$

⇒ $a^2 – 8a + 25 – 10 = 0$

⇒ $a^2 – 8a + 15 = 0$

⇒ $a^2 – 5a – 3a + 15 = 0$

⇒ a(a – 5) –3(a – 5) = 0

⇒ (a – 5) (a – 3) = 0

⇒ a – 5 = 0 or a – 3 = 0

⇒ a = 5 or a = 3

Hence, the required values of a are 5 and 3.