Solution

CIRCLES

Class 09 - Mathematics

Section A

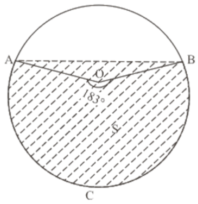
1.

(d) in the interior of S

Explanation:

Given: m$\widehat{AB}$ = 183^o and C is mid-point of arc ABO is the centre.

With the given information the corresponding figure will look like the following,

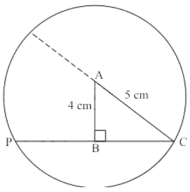


From the figure, so the centre of the circle O lies inside the shaded region S.

2.

(d) 6 cm

Explanation:



In the circle produce CB to P. Here PC is the required chord.

We know that perpendicular drawn from the centre to the chord divide the chord into two equal parts.

So, PC = 2BC

Now in $\triangle$ABC apply Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

$\Rightarrow BC^2 = 5^2 - 4^2$

$\Rightarrow BC^2 = 25 - 16$

$\Rightarrow BC^2 = 9$

==>BC = 3 cm

So, PC = 2 $\times$ BC

= 2 $\times$ 3

PC= 6 cm

3.

(b) 8 cm

Explanation:

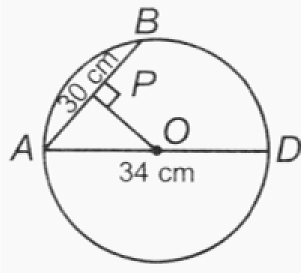
Given, AD = 34 cm, AB = 30 cm

$\therefore \quad AO = \frac{1}{2}AD = \frac{1}{2}(34) = 17$ cm

Draw OP $\perp$ AB.

Since, perpendicular drawn from centre to the chord bisects the chord.

$\therefore \quad AP = \frac{1}{2}AB = 15$ cm



Now, in right angled △APO

$(OP)^2 = (17)^2 - (15)^2 = 289 - 225 = 64$

⇒ OP = 8 cm

∴ Distance of AB from the centre of the circle is 8 cm.

4.

(d) 89°, 37°

Explanation:

In △AEB, ∠ABE + ∠BEA + ∠BAE = 180°

⇒ 35° + ∠BEA + 54° = 180°

⇒ ∠BEA = 91°

Now, ∠AFD + ∠DEA = 180° (opposite angles of cyclic quadrilateral)

⇒ x + 91° = 180° => x = 89°

In △AFC, ∠AFC + ∠FCA + ∠CAF = 180°

⇒ 89° + y + 54° = 180° ⇒ y = 37°

5.

(d) 58°

Explanation:

Since, angle subtended by an arc at centre is double the angle subtended by it on the remaining part of the circle.

∴ ∠AOB = 2∠ACB = 2 x 29° = 58° ⇒ x = 58°

6.

(b) 150^o

Explanation:

Radius of circle = 12 cm

Arc length = 10 π cm

And

Arc length = $\frac{\theta}{360} \times 2\pi r$

$\Rightarrow 10\pi = \frac{\theta}{360} \times 2 \times \pi \times 12$

$\Rightarrow \theta = 150^\circ$

7.

(b) 110°

Explanation:

Given, ∠BEC = 130° and ∠ECD = 20°

Now, ∠ABD = ∠ACD (Angles in the same segment)

∴ ∠ABD = 20° Now, in △AEB

∠EBA + ∠BAE = ∠BEC (exterior angle property)

⇒ 20° + ZBAC = 130° ⇒ ZBAC = 110°

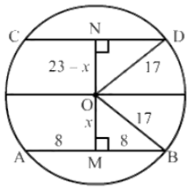
8.

(d) 90°

Explanation:

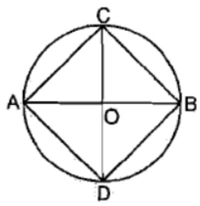
∠CDB = ∠CAB (Angles in same segment are equal)
⇒ ∠CDB = x
Given, ∠CPB = 90°
In △DPB, ∠PDB + ∠PBD = ∠CPB (exterior angle property)
⇒ x + y = 90°

9. **(a)** 30 cm
Explanation:
Given that: Radius of the circle is 17 cm, distance between two parallel chords AB and CD is 23 cm, where AB = 16 cm. We have to find the length of CD.



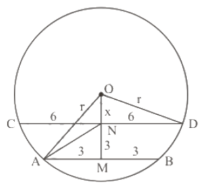
We know that the perpendicular drawn from the centre of the circle to any chord divides it into two equal parts.
AM = MB = 8 cm
Let OM = x cm ==> ON = 23 - x
In right angled triangle OMB,
x = $\sqrt{17^2 - 8^2 = 15}$
Now, in triangle OND, ON = (23 - x) cm = (23 - 15) cm = 8 cm
ND = $\sqrt{OD^2 - ON^2}$
⇒ ND = $\sqrt{17^2 - 8^2 = 15}$
Therefore, the length of the other chord is
CD = 2 × 15 = 30 cm

10.
(c) square
Explanation:



Let AB and CD be the diagonals of a circle such that AB ⊥ CD.
Joining points A, B, C, D in the order we see that AB and CD are the equal diagonals of quad. ACBD which intersect at a right angle. every angle is equal to 90°
∴ ACBD is a square.

11.
(d) $3\sqrt{5}$ cm
Explanation:
Let the distance between the center and the chord CD be x cm and the radius of the circle is r cm.
We have to find the radius of the following circle:



In right angled triangle, OND,
$x^2 + 36 = r^2$ .....(i)
Now, in right angled triangle AOM,
$r^2 = 9 + (x + 3)^2$ ......(ii)

From (i) and (ii), we have,

$r^2 = 9 + ((\sqrt{r})^2 - 36 + 3)^2$

$\Rightarrow r^2 = 9 + r^2 - 36 + 9 + 6\sqrt{r^2 - 36}$

$\Rightarrow 3 = \sqrt{r^2 - 36}$

$\Rightarrow 9 = r^2 - 36$ [squaring both the sides]

$\Rightarrow r^2 = 45 \Rightarrow r = 3\sqrt{5}$ cm

12.

(d) 90^o

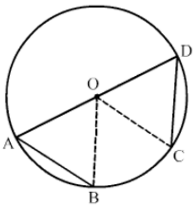
Explanation:

The angle in a semicircle measures 90^o.

13.

(b) 60^o

Explanation:



As we know that equal chords make an equal angle at the center.
Therefore,

$\angle AOB = \angle BOC = \angle COD$

$\angle AOB + \angle BOC + \angle COD = 180^o$ [Sum of Linear pair of angle is 180^o]

$\Rightarrow 3\angle AOB = 180^o$

$\Rightarrow \angle AOB = 60^o$

14. **(a)** 30^o

Explanation:

Since the chord is equal to the radius therefore, it will form an equilateral triangle inside the circle with the third vertex being the centre of the circle.

So the chord will make an angle of 60^o at the centre. As the angle made by the chord at any other point of the circumference would be half.

So, we have that angle made at the major segment would be 30^o.

15.

(d) 60°

Explanation:

Given, $\angle PQR = 150°$

$\therefore$ Reflex $\angle POR = 2\angle PQR = 2(150°) = 300°$

Now, $\angle POR = 360°$ - Reflex $\angle POR = 360° - 300° = 60°$ ...(i)

Also, $OP = OR \Rightarrow \angle OPR = \angle ORP$ ...(ii) (Angles opposite to equal sides of a triangle are equal)

In $\triangle OPR$, $\angle OPR + \angle ORP + \angle POR = 180°$

$\Rightarrow 2\angle OPR + 60° = 180°$ [From (i) & (ii)]

$\Rightarrow 2\angle OPR = 120° \Rightarrow \angle OPR = 60°$

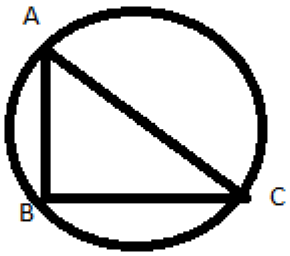
16.

(c) 8 cm

Explanation:

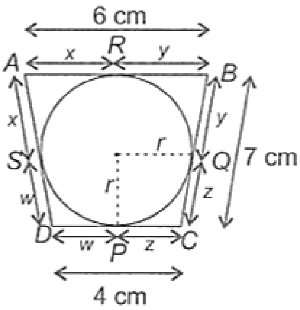
Given, AB = 8 cm and OM = ON = 4 cm
Since equal chords are equidistant from the centre.
∴ CD = 8 cm

17. **(a)** 10 cm
Explanation:



Since AB is perpendicular to BC, therefore ABC is a right-angled triangle right angled at B. As clear from the figure, AC would act as the diameter
$AB^2 + BC^2 = AC^2$
$12^2 + 16^2 = AC^2$
$AC = 20$
Since AC is diameter so radius = 10 cm.

18.
(b) 3 cm
Explanation:



x + y = 6 cm ...(i)
y + z = 7 cm ...(ii)
z + w = 4 cm ...(iii)
Using (ii) and (iii),
y - w = 3 cm
Now using (i);
x + (3 + w) = 6 cm
⇒ x + w = 3 cm
⇒ AD = 3 cm

19.
(c) 3 : 8
Explanation:
The length of an arc subtending an angle θ in a circle of radius r is given by the formula,
Length of the arc = $\frac{\theta}{360^\circ} 2\pi r$
Here, it is given that the are subtends an angle of 135^o with its centre. So the length of the given arc in a circle with radius r is given as
Length of the arc = $\frac{135^\circ}{360^\circ} 2\pi r$ ......(1)
The circumference of the same circle with radius r = 2πr. ....(2)
The ratio between the lengths of the arc and the circumference of the circle will be
$\frac{\text{Lenght of the arc}}{\text{Cirrumference of the circle}}$ = $\frac{135^\circ(2\pi r)}{360^\circ(2\pi r)}$ = $\frac{135^\circ}{360^\circ}$ = $\frac{3}{8}$ [FROM (1) and (2)]
RATIO = 3 : 8

20. **(a)** 75°

Explanation:

Given, $\angle AOC = 55°$, $\angle BOC = 155°$

reflex $\angle AOB = \angle AOC + \angle BOC = 55° + 155° = 210°$

$\therefore \angle AOB = 360° - 210° = 150°$

Since angle subtended by an arc at centre is double the angle subtended by it on the remaining part of the circle.

$\therefore \angle ACB = \frac{1}{2} \angle AOB = 75°$

21.

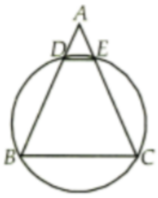
(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Both A and R are true but R is not the correct explanation of A.

22. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:



To prove DE || BC

i.e., $\angle B$ $\angle ADE$

In $\triangle ABC$, we have

AB = AC

$\angle B = \angle C$ ...(i)

In the cyclic quadrilateral CBDE, side BD is produced to A. We know that an exterior angle of cyclic quadrilateral is equal to interior opposite angle of cyclic quadrilateral.

$\angle ADE = \angle C$ ...(ii)

From (i) and (ii), we get

$\angle B = \angle ADE$

Hence, DE || BC

23.

(c) A is true but R is false.

Explanation:

Given, one angle is 40^o and opposite angle 140^o.

We know, Opposite angles in a cyclic quadrilateral are supplementary, i.e. they sum up to 180^o.

Then, $40^o + 140^o = 180^o$.

Therefore, the given assertion is correct but reason is incorrect.

24. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Since ABCD is a cyclic quadrilateral, so, its opposite angles are supplementary

$\angle A + \angle C = 180^o$ ...(i)

Also, $\angle A - \angle C = 60^o$ ...(i)

On solving (i) and (ii), we get

$\angle A = 120^o$, $\angle C = 60^o$

25.

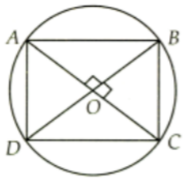
(c) A is true but R is false.

Explanation:

Two or more circles are called concentric circles if and only if they have same centre but different radii.

26. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:



Let AB and CD be two perpendicular diameters of a circle with centre O.

Now, ∠ABC = 90^o [Angle in semicircle is a right angle]

Similarly ∠ACD = ∠ADC

= ∠BAD = 90^o ...(i)

In △AOB and △AOD, we have

AO = AO (Common)

∠AOB = ∠AOD (Each 90^o, given)

BO = OD (Radii of circle)

$\triangle AOB \cong \triangle AOD$ (By SAS congruence)

AB = AD (By C.P.C.T.)

Similarly, we have, AD = DC, DC = BC; BC = AB

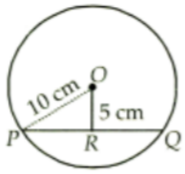
Hence, AB = BC = CD = DA ...(ii)

Also, it is given that diagonals of ABCD intersect at 90^o ...(iii)

By (i), (ii) and (iii) ABCD is a square.

27. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:



Let, PQ be a chord of a circle with centre O and radius 10cm. Draw OR ⊥ PQ.

Now, OP = 10cm and OR = 5cm

In right triangle ORP, we get

$OP^2 = PR^2 + OR^2$

$PR^2 = OP^2 - OR^2$

$PR^2 = 10^2 - 5^2 = 75$

PR = $\sqrt{75}$ = 8.66

Since, the perpendicular from the centre to a chord bisects the chord.

Therefore, PQ = 2 × PR

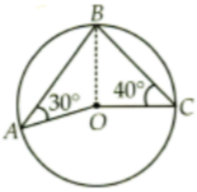
= 2 × 8.66 = 17.32 cm

28.

(d) A is false but R is true.

Explanation:

Join BO.



In ΔAOB, we have

OA = OB [radius]

∠OBA = ∠OAB [Angle opposite to equal sides of a triangle are equal]

$\angle OBA = 30^o$ ...(i)

Similarly, in ΔBOC, we get OB = OC

$\angle OCB = \angle OBC$

$\angle OBC = 40^o$ ...(ii)

$\angle ABC = \angle OBA + \angle OBC$

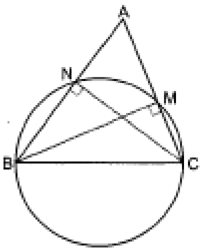
$= 30^o + 40^o = 70^o$ [using (i) and (ii)]

Since angle subtended by an arc of a circle at the centre of the circle is double the angle subtended by the arc on the circumference.

$\angle AOC = 2 \times \angle ABC$

$= 2 \times 70^o = 140^o$

Section B

29. 

Consider BC as a diameter of the circle.

Angles subtended by the diameter in a semicircle is 90°.

Given, $\angle BNC = \angle BMC = 90°$

So the points M and N should be on the same circle.

Hence, BCMN is a cyclic quadrilateral. Therefore, the points B, C, M, and N are cyclic.

Hence proved.

30. We have, $\angle OAB = 35^\circ$

Then, $\angle OBA = \angle OAB = 35^\circ$ ... (Angle opposite to equal sides are equal)

In triangle AOB, by angle sum property

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$

$\angle AOB + 35^\circ + 35^\circ = 180^\circ$

$\angle AOB = 110^\circ$

Therefore, $\angle AOB$ + Reflex $\angle AOB = 360^\circ$ ... (Complete angle)

110° + Reflex $\angle AOB = 360^\circ$

Reflex $\angle AOB = 250^\circ$

By degree measure theorem,

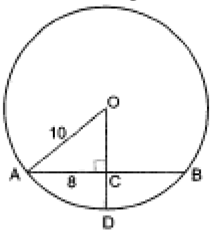
Reflex $\angle AOB = 2 \angle ACB$

$250^\circ = 2x$

$\Rightarrow x = 125^\circ$.

31. Given : In given figure OA=10cm and Ab=16 cm

To find : Length of CD



Solution : As OD $\perp$ AB

$\Rightarrow$ AC = CB

($\perp$ from the centre to the chord bisects the chord)

$\therefore AC = \frac{AB}{2} = 8$

In right $\triangle OCA$,

$OA^2 = AC^2 + OC^2$

$(10)^2 = 8^2 + OC^2$

$OC^2 = 100 - 64$

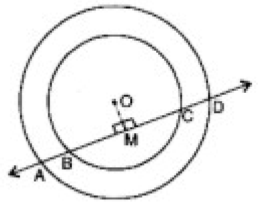
$OC^2 = 36$

$\therefore OC = \sqrt{36}$

OC = 6 cm

CD = OD - OC = 10 - 6 = 4 cm.

32. Given: In figure, AB and CD are equal chords of a circle whose centre is O. $OM \perp AB$ and $ON \perp CD$
To prove: $\angle \mathrm{OMN} = \angle \mathrm{ONM}$
Proof : Chord AB = Chord CD
$\therefore$ OM = ON .... (1)
|$\because$ Equal chords of a circle are equidistant from the centre of the circle
In $\triangle OMN$
OM = ON |From (1)
$\therefore \angle \mathrm{OMN} = \angle \mathrm{ONM}$ |Angles opp. to equal sides
33. We know that the sum of all angles at a point is 360°
$\therefore 90^\circ + 120^\circ + \angle BOC = 360^\circ \Rightarrow \angle BOC = 150^\circ$
$\angle BOC = 2\angle BAC$
$\Rightarrow 2x^o = 150^o$
=> $x^o = 75^o$
34. Given: A line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D.
To prove : AB = CD
Construction : Draw $OM \perp BC$



Proof: $\because$ The perpendicular drawn from the centre of a circle to a chord bisects the chord.
$\therefore$ AM = DM
and BM = CM
Subtracting (2) from (1), we get
AM - BM = DM - CM
$\Rightarrow$ AB = CD
35. $\angle APB = 90^\circ$ [angle in a semicircle].
now in $\triangle$ABP
$\angle BAP + \angle APB + \angle ABP = 180^\circ$
$\Rightarrow \angle BAP + 90^\circ + 35^\circ = 180^\circ$
$\Rightarrow \quad \angle BAP = (180^\circ - 125^\circ) = 55^\circ$
Now, $x^\circ = \angle BQP = \angle BAP = 55^\circ$ [$\angle$ in the same segment].
Hence, $x^o = 55^o$.
36. In $\triangle$ADB,
$\angle$ADB = 90^o (Angle in semi-circle) ...(i)
Now, by using angle sum properly in $\triangle$ABD, we have
$\angle$ABD + $\angle$ADB + $\angle$DAB = 180^o
$\Rightarrow$ $\angle$ABD + 90^o + 55^o = 180^o ...[From equation (i) and given $\angle$DAB = 55^o]
$\Rightarrow$ $\angle$ABD + 180o - 145^o
$\Rightarrow$ $\angle$ABD = 35o ...(ii)
Now, $\angle$ABD = 90^o ...(Angle between tangent and radius)
or, $\angle$ABD + $\angle$DBC = 90^o
or, $\angle$DBC = 90^o - $\angle$ABD

or, $\angle DBC = 90^o - 35^o$ ...[From equation (ii)]

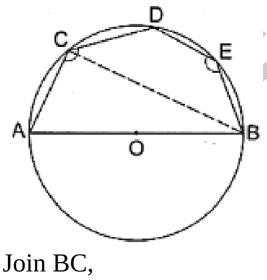
or, $\angle DBC = 55^o$

37. In the given diagram join AB. Also $\angle ABD = 90°$ (because angle in a semicircle is always 90°)
Similarly, we have $\angle ABC = 90°$
So, $\angle ABD + \angle ABC = 90° + 90° = 180°$
Therefore, DBC is a line i.e., B lies on the line segment DC.

38. Radius of circle (OA) = 6 cm
Distance (OC) = 4 cm
In $\triangle OCA$, by using Pythagoras theorem
$AC^2 + OC^2 = OA^2$
$AC^2 + 4^2 = 6^2$
$AC^2 = 36 – 16$
$AC^2 = 20$
AC = 4.47 cm
We know that, The perpendicular distance from centre to chord bisects the chord
AC = BC = 4.47 cm
Then, AB = 4.47 + 4.47
= 8.94 cm.

39. From the given figure, in $\triangle ABC$, we can write
$\angle ABC + \angle ACB + \angle BAC = 180^o$ (by angle sum property)
$69^o + 31^o + \angle BAC = 180^o$
$\Rightarrow \angle BAC = 180° -100° = 80°$
$\angle BDC = \angle BAC$ (Angles in the same segment)
$\therefore \angle BDC = 80°$

40. Given: Perimeter of $\triangle PQR = 20$
i.e., PQ + PR + PR = 20
$\Rightarrow$ PQ + (QC + CR) + PR = 20
$\Rightarrow$ (PQ + QA) + (RB + PR) = 20 ...[\because Q A=Q C \text { and } R C=R B \text { as tangents from external points Q and R, respectively]
$\Rightarrow$ PA + PB = 20
$\Rightarrow$ PA + PA = 20 ...[$\because$ PA = PB as tangents from external point P]
$\Rightarrow$ 2PA = 20
$\Rightarrow$ PA = 10 cm

41. 

Join BC,
Then, $\angle ACB = 90°$ (Angle in the semicircle)
Since DCBE is a cyclic quadrilateral.
$\angle BCD + \angle BED = 180°$
Adding $\angle ACB$ both the sides, we get
$\angle BCD + \angle BED + \angle ACB = \angle ACB + 180°$
$(\angle BCD + \angle ACB) + \angle BED = 90° + 180°$
$\angle ACD + \angle BED = 270°$

42. We have, $\angle ABD = 40^\circ$
$\angle ACD = \angle ABD = 40^\circ$ ... (Angle on same segment)
In triangle PCD, by angle sum property
$\angle PCD + \angle CPD + \angle PDC = 180^\circ$

$40^\circ + 110^\circ + x = 180^\circ$

$x = 30^\circ$.

43. Given that, PQ is a diameter of circle which bisects chord AB to C

To prove: PQ bisects ∠AOB

Proof: In △AOC and △BOC,

OA = OB (Radius of circle)

OC = OC (Common)

AC = BC (Given)

Then, △ADC ≅ △BOC (By SSS congruence rule)

∠AOC = ∠BOC (By c.p.c.t)

Hence, PQ bisects ∠AOB.

Section C

44. We observe that the arc BC makes ∠BOC = z at the centre and ∠BAC = x at a point on the circumference.

∴ z = 2x

In △OBC,we have

∠OBC + ∠OCB + ∠BOC = 180°

⇒ y + z + t = 180°

⇒ t = 180° - 2y

Now, z = 360° - t

z = 360° - (180° - 2y)

⇒ 2x = 180° + 2y... [∵ z = 2x]

⇒ 2x - 2y = 180°

⇒ x - y = 90°

⇒ ∠BAC - ∠OBC = 90°

45. i. Minor arc AB subtends ∠AOB at the centre and ∠APB at a point on the remaining part of the circle.

$\therefore \angle AOB = 2\angle APB = 2 \times 70^\circ = 140^\circ$

ii. AOBC is a cyclic quadrilateral

$\therefore \angle AOB + \angle ACB = 180^\circ$

$\Rightarrow 140^\circ + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - 140^\circ = 40^\circ$

iii. $\angle ADB = \angle ACB = 40^\circ$ [$\angle s$ in the same segment].

46. ∠CED + ∠BEC = 180^o [Linear Pair Axiom]

⇒ ∠CED + 130^o = 180^o

⇒ ∠CED = 180^o - 130^o = 50^o .... (1)

∠ECD = 20^o .... (2)

In △CED,

∠CED + ∠ECD + ∠CDE = 180^o [Sum of all the angles of a triangle is 180^o]

⇒ 50^o + 20^o + ∠CDE = 180^o [Using (1) and (2)]

⇒ 70^o + ∠CDE = 180o

⇒ ∠CDE = 180^o - 70^o

⇒ ∠CDE = 110^o .... (3)

Now, ∠BAC = ∠CDE [Angles in the same segment of a circle are equal

= 110^o] Using (3)

47. From the given figure we can say that, ∠RMS = 90° (Angle in the semicircle as RS is diameter)

therefore, ∠RSM = 180°- (30°+90°) = 60°

∠RPM + ∠RSM = 180° (because opposite angles of cyclic quadrilateral are always supplementary)

∠RPM + ∠60° = 180°

⇒ ∠RPM = 120°

48. Given: ∠BEC = 130° and ∠ECD = 20°

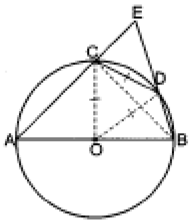
∠DEC = 180° - ∠BEC = 180° - 130° = 50° [Linear pair]

Now in △DEC,

$\angle$DEC + $\angle$DCE + $\angle$EDC = 180° [Angle sum property]
$\Rightarrow$ 50° + 20° + $\angle$EDC = 180° $\Rightarrow$ $\angle$EDC = 110°
$\Rightarrow$ $\angle$BAC = $\angle$EDC = 110° [Angles in same segment]

49. We know that the opposite angles of a cyclic quadrilateral are supplementary.
$\therefore \angle A+\angle C=180^{\circ}$ and $\angle B+\angle D=180^{\circ}$
$\Rightarrow$ (2x + 4) + (4y - 4) = 180 and (x +10) + (5y + 5) = 180
$\Rightarrow$ 2x + 4y = 180 and x + 5y = 165
$\Rightarrow$ x + 2y = 90 ...(i) and x + 5y = 165 ...(ii).
On solving (i) and (ii), we get x = 40 and y = 25.
$\therefore \angle A=(2\times 40+4)^{\circ}=84^{\circ}, \angle B=(40^{\circ}+10^{\circ})=50^{\circ}$
$\angle C=(4\times 25-4)^{\circ}=96^{\circ}$ and $\angle D=(5\times 25+5)^{\circ}=130^{\circ}$

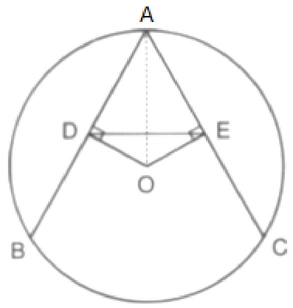
50. Construction: Join OC, OD, BD and BC



Proof:In $\triangle$OCD, we have
OC = OD (Each equal to radius)
and CD = r (given)
So OC = OD = CD
$\therefore$ $\angle$ODC is an equilateral triangle.
$\Rightarrow$ $\angle$COD = 60°
Also, $\angle$COD = 2 $\angle$CBD
$\Rightarrow$ 60° = 2 $\angle$CBD $\Rightarrow$ $\angle$CBD = 30°
Now since $\angle$ACB is angle in a semi-circle.
$\angle$ACB = 90°
$\Rightarrow$ $\angle$BCE = 180° - $\angle$ACB = 180° - 90° = 90°
Thus, in $\triangle$ BCE, we have
$\angle \mathrm{CEB}+\angle \mathrm{ECB}+\angle \mathrm{CBE}=180^{\circ}$
$\angle \mathrm{CEB}+90^{\circ}+30^{\circ}=180^{\circ}$
$\angle \mathrm{CEB}=180^{\circ}-120^{\circ}=60^{\circ}$
Hence $\angle \mathrm{CEB}=60^{\circ}$

51. $\angle$APB = 50° (Given)
By degree measure theorem,we have,
$\angle$AOB =2 $\angle$APB
$\angle$AOB = 2 $\times$ 50 = 100°
Since, OA = OB (Radii)
Therefore, $\angle$OAB = $\angle$OBA ... (Angle opposite to equal sides are equal)
Let, $\angle$OAB = x
In Triangle OAB,
$\angle$OAB + $\angle$OBA + $\angle$AOB = 180°
x + x + 100° = 180°
2x = 80°
x = 40°
$\angle$OAB = $\angle$OBA = 40°

52. Given: In figure, AB and AC are two equal chords of a circle whose centre is O. $OD \perp AB$ and $OE \perp AC$
To prove: ADE is an isosceles triangle



Proof: AB = AC

OD = OE [∵ Equal chords are equidistant from the centre]

∴ In $\triangle ODE$

$\angle ODE = \angle OED$ [Angle opposite to equal sides]

$\Rightarrow 90^\circ - \angle ODE = 90^\circ - \angle OED$

$\Rightarrow \angle ODA - \angle ODE = \angle OEA - \angle OED$

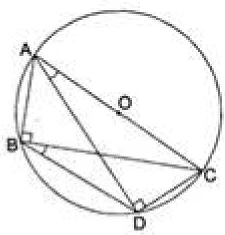
$\Rightarrow \angle ADE = \angle AED$

$\therefore AD = AE$ [Sides opposite to equal angles]

$\therefore \triangle ADE$ is an isosceles triangle.

53. Given: ABC and ADC are two right triangles with common hypotenuse AC.

To prove: $\angle CAD = \angle CBD$



Proof : AC is the common hypotenuse and ABC and ADC are two right triangles.

$\therefore \angle ABC = 90^\circ = \angle ADC$

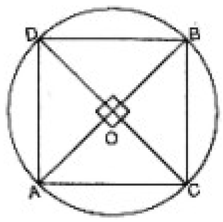
⇒ Both the triangles are in the same semi-circle

∴ Points A, B, D and C are concyclic

∴DC is a chord

$\therefore \angle CAD = \angle CBD$ |∵ Angles in the same segment are equal.

54. Given: Two diameters AB and CD of a circle intersect each other at right angles.



To prove: The quadrilateral ACBD formed by joining their end points is a square.

Proof: A diameter essentially passes through the centre of the circle.

∴ Diameters AB and CD intersect each other at O, the centre of the circle.

$\angle A = \angle B = \angle C = \angle D = 90^\circ (\text{ each })$ [∵ Angle in a semi-circle is 90^o]

Quadrilateral ACBD is a rectangle .... (1)

In $\triangle OAC$ and ΔOAD

$\angle AOC = \angle AOD$ |Each = 90^o

OA = OA [common]

OC = OD [Radii of the same circle]

$\therefore \Delta OAC \cong \Delta OAD$ [SAS]

$\therefore AC = AD$ .... (2) [c.p.c.t]

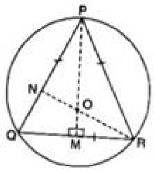
In view of (1) and (2)

Quadrilateral ABCD is a square

55. Construction: Draw PM ⊥ QR and RN ⊥ PQ

Determination : PQ = QR = RP

$\therefore \triangle$PQR is equilateral. We know that in an equilateral triangle, the medians and the altitudes are the same. So, PM and RN are median. They intersect at O where O is the centre of the circle.



Also, PO = 2 OM = 20 (medians intersect each other in the ratio 2: 1)

$\Rightarrow$ OM = 10 m $\Rightarrow$ PM = OP + OM = 20 + 10 = 30 m

Let QM = x

Then, QM = MR = x [$\because$ PM bisects QR]

$\therefore$ QM = $\frac{1}{2}$QR $\Rightarrow$ x = $\frac{1}{2}$QR $\Rightarrow$ QR = 2x

Similarly, PQ = 2x

In right triangle PMQ,

$PQ^2 = PM^2 + QM^2$ |By Pythagoras Theorem

$\Rightarrow (2x)^2 = (30)^2 + x^2$

$\Rightarrow 4x^2 = 900 + x^2$

$\Rightarrow 4x^2 - x^2 = 900$

$\Rightarrow 3x^2 = 900$

$\Rightarrow x^2 = \frac{900}{3} = 300$

$\Rightarrow x = \sqrt{300} = 10\sqrt{3}$

$\Rightarrow$ PQ = 2x = $2(10\sqrt{3})$

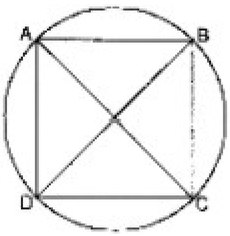
Hence, the length of the string of each phone is $20\sqrt{3}$ m.

56. Given: ABCD is a cyclic quadrilateral in which AB || DC.

To prove :

i. AD = BC
ii. AC = BD

Proof:



i. $\because$ AB || DC and transversal AC intersects them [Alt. Int. $\angle$s]

$\therefore \angle$DCA = $\angle$BAC

$\therefore$ arc AD $\cong$ arc BC

$\therefore$ chord AD = chord BC

$\Rightarrow$ AD = BC

ii. $\overline{AD} \cong \overline{BC}$ [Proved above]

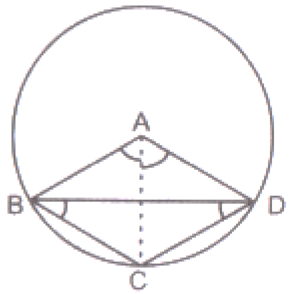
$\Rightarrow \overline{AD} + \overline{DC} = \overline{BC} + \overline{DC}$ [Adding $\overline{DC}$ to both sides]

$\Rightarrow \overline{AC} \cong \overline{BD}$

$\Rightarrow$chord AC = chord BD

$\Rightarrow$ AC = BD

57. To prove: $\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$



Construction: Join AC.

Since angle subtended by an arc at the centre is double of the angle subtended by it at point on the remaining part of the circle.

Therefore, $\angle CAD = 2\angle CBD$ ...(1)

And $\angle BAC = 2\angle CDB$ ...(2)

Adding (1) and (2), we get

$\angle CAD + \angle BAC = 2(\angle CBD + \angle CDB)$

$\Rightarrow \angle BAD = 2(\angle CBD + \angle CDB)$

Hence, $\angle CBD + \angle CBD = \frac{1}{2}\angle BAD$

58. Join BC.

Then, $\angle ACB = 90^\circ$ [angle in a semicircle].

Now, DCBE is a cyclic quadrilateral.

$\Rightarrow \angle ACB + \angle BCD + \angle DEB = 90^\circ + 180^\circ \quad [\because \angle ACB = 90^\circ]$

$\Rightarrow \angle ACD + \angle DEB = 270^\circ \quad [\because \angle ACB + \angle BCD = \angle ACD]$

$\therefore$ the numerical value of $(\angle ACD + \angle DEB)$ is 270°.

Section D

59. i. In ΔAOP and ΔBOP

$\angle$APO = $\angle$BPO (Given)

OP = OP (Common)

AO = OB (radius of circle)

ΔAOP $\cong$ ΔBOP

AP = BP (CPCT)

ii. In right ΔCOQ

$CO^2 = OQ^2 + CQ^2$

$\Rightarrow 10^2 = 8^2 + CQ^2$

$\Rightarrow CQ^2 = 100 - 64 = 36$

$\Rightarrow$ CQ = 6

CD = 2CQ

$\Rightarrow$ CD = 12 cm

iii. In right ΔAOB

$AO^2 = OP^2 + AP^2$

$\Rightarrow 10^2 = 6^2 + AP^2$

$\Rightarrow AP^2 = 100 - 36 = 64$

$\Rightarrow$ AP = 8

AB = 2AP

$\Rightarrow$ AB = 16 cm

OR

There is one and only one circle passing through three given non-collinear points.

60. i. (c) 180^o

ii. Show that in a right triangle the sum of legs is longest for an isosceles right triangle when hypotenuse remains same.

Take for example the length of diameter (hypotenuse) = 5 units.

Road D and Road B are equal hence (Road D = 3.53 units).

Let Road E be = 1, Road F = 4.89 units.

Therefore, length of Road B + Road D is greater than Road E + Road F.

iii. (c) Road G divides Road F into two equal.

iv. Yes, Priya is correct because arc corresponding to two equal roads (chords) are congruent.

61. i. ABCD is cyclic quadrilateral.
A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.
Here all four vertices A, B, C and D lie on a circle.

ii. We know that the sum of both pair of opposite angles of a cyclic quadrilateral is 180º.
$\angle$C + $\angle$A = 1800
$\angle$C = 1800 - 1000 = 800

iii. We know that
The sum of both pair of opposite angles of a cyclic quadrilateral is 180º.
$\angle$B + $\angle$D = 1800
$\angle$B = 1800 - 800 = 1000

OR

I. In a cyclic quadrilateral, all the four vertices of the quadrilateral lie on the circumference of the circle.
II. The four sides of the inscribed quadrilateral are the four chords of the circle.
III. The sum of a pair of opposite angles is 180° (supplementary). Let $\angle$A, $\angle$B, $\angle$C, and $\angle$D be the four angles of an inscribed quadrilateral. Then, $\angle$A + $\angle$C = 180° and $\angle$B + $\angle$D = 180°.

Section E

62. Let O be the centre of the circle.
Join OA and OC.
Since perpendicular from the centre of the circle to the chord bisects the chord.
$\therefore AE = EB = \frac{1}{2}AB = \frac{1}{2} \times 5 = \frac{5}{2}cm$
And CF = FD = $\frac{1}{2}CD = \frac{1}{2} \times 11 = \frac{11}{2}cm$
Let OE = x
$\therefore OF = 6 - x$ $\therefore$OF = 6 - x
Let radius of the circle be r.
In right angled triangle AEO,
$AO^2 = AE^2 + OE^2$ $AO^2 = AE^2 + OE^2$
[Using Pythagoras theorem]
$\Rightarrow r^2 = \left(\frac{5}{2}\right)^2 + x^2$ ...(i)
Again In right angled triangle CFO,
$OC^2 = CF^2 + OF^2$
[Using Pythagoras theorem]
$\Rightarrow r^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$ ...(ii)
Equating eq. (i) and (ii),
$\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$
$\Rightarrow \frac{25}{4} + x^2 = \frac{121}{4} + 36 + x^2 - 12x$
$\Rightarrow 12x = \frac{121}{4} - \frac{25}{4} + 36$
$\Rightarrow 12x = \frac{96}{4} + 36$
$\Rightarrow$ 12x = 24 + 36
$\Rightarrow$ 12x = 63
$\Rightarrow$ x = 5
Putting the value of x in eq. (i)
$r^2 = \left(\frac{5}{2}\right)^2 + 5^2$ $r^2 = \left(\frac{5}{2}\right)^2 + 5^2$
$\Rightarrow r^2 = 31.25$
$\Rightarrow$ r = 5.6 cm (approx.)

63. i. $\angle QPR$
$\because$ PR is a diameter
$\therefore \angle PRQ = 90^\circ$ |Angle in a semi-circle is 90º
In $\triangle$PQR
$\angle QPR + \angle PRQ + \angle PQR = 180^\circ$ |Angle sum property of a triangle

$\Rightarrow \angle QPR + 90^\circ + 65^\circ = 180^\circ$
$\Rightarrow \angle QPR + 155^\circ = 180^\circ$
$\Rightarrow \angle QPR = 180^\circ - 155^\circ$
$\Rightarrow \angle QPR = 25^\circ$

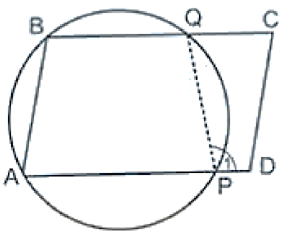
ii. $\angle$PRS

$\because$ PQRS is a cyclic quadrilateral
$\therefore \angle PSR + \angle PQR = 180^\circ$
|$\because$ Opposite angles of a cyclic quadrilateral are supplementary
$\Rightarrow \angle PSR + 65^\circ = 180^\circ$
$\Rightarrow \angle PSR = 180^\circ - 65^\circ$
$\Rightarrow \angle \text{PSR} = 115^\circ$
In $\triangle PSR$
$\angle PSR + \angle SPR + \angle PRS = 180^\circ$ |Angles sum property of a triangle
$\Rightarrow 115^\circ + 40^\circ + \angle PRS = 180^\circ$
$\Rightarrow 155^\circ + \angle PRS = 180^\circ$
$\Rightarrow \angle PRS = 180^\circ - 155^\circ$
$\Rightarrow \angle PRS = 25^\circ$

iii. $\angle QPM$

$\therefore$ PQ is a diameter
$\therefore \angle \text{PMQ} = 90^\circ$ |$\because$ Angle is a semi-circle is 90^o
In $\triangle PMQ$
$\angle PMQ + \angle PQM + \angle QPM = 180^\circ$ |Angle sum property of a triangle
$\Rightarrow 90^\circ + 50^\circ + \angle QPM = 180^\circ$
$\Rightarrow 140^\circ + \angle QPM = 180^\circ$
$\Rightarrow \angle QPM = 180^\circ - 140^\circ$
$\Rightarrow \angle QPM = 40^\circ$

64. ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. We have to prove that P, Q, C and D are concyclic. Join PQ.



Now, side AP of the cyclic quadrilateral APQB is produced to D.
$\therefore$ Ext. $\angle$ 1 = int. opp. $\angle$B
$\because$ BA||CD and BC cuts them
$\therefore \angle \text{B} + \angle \text{C} = 180^\circ$ [$\because$ Sum of int. $\angle$S on the same side of the transversal is 180°]
or $\angle$1 + $\angle$C = 180° [$\because$ $\angle$1 = $\angle$B (proved)]
$\therefore$ PDCQ is cyclic quadrilateral.
Hence, the points P, Q, C and D are concyclic.

65. Given, AB is a diameter of the circle C(O, r) and radius OD is perpendicular to AB. C is any point on DB.
Required: To find $\angle BAD$ and $\angle ACD$
Determination: In right triangle OAD,
$AD^2 = OA^2 + OD^2$ .... (1) |Pythagoras Theorem
In right triangle OBD,
$BD^2 = OB^2 + OD^2$ |Pythagoras Theorem
$= OA^2 + OD^2$ .... (2) |$\because$ OA = OB (radii of the same circle)
From (1) and (2),
$AD_2 = BD^2$
$\Rightarrow$ AD = BD

$\therefore \angle ABD = \angle BAD$ |Angle opposite to equal sides of a triangle are equal

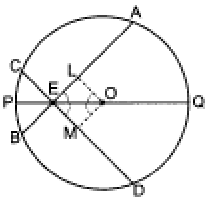
But $\angle ABD + \angle BAD = 90^\circ$.

|$\because$ In $\triangle ABD, \angle ADB = 90^\circ$ and the sum of the three angles of a Δ is 180^o

$\therefore \angle ABD = \angle BAD = 45^\circ$

Thus, $\angle BAD = 45^\circ$

Now, $\angle ACD = \angle ABD$ |Angles in the same segment = 45^o

66. 

Given: AB and CD are two chords of a circle with centre O, intersecting at point E. PQ is a diameter through E, such that ∠AEQ = ∠DEQ.

To prove: AB = CD

Construction: Draw OL ⊥ AB and OM ⊥ CD

Proof: ∠LOE + ∠LEO + ∠OLE = 180° (Angle sum property of a triangle)

⇒ ∠LOE + ∠LEO + 90° = 180°

∠LOE + ∠LEO =90° ....(i)

Similarly, ∠MOE + ∠MEO + ∠OME = 180°

⇒∠MOE + ∠MEO + 90° = 180°

∠MOE + ∠MEO = 90° . ...(ii)

From (i) and (ii) we get

∠LOE + ∠LEO = ∠MOE + ∠MEO ....(iii)

Also, ∠LEO = ∠MEO (Given) ...(iv)

From (iii) and (iv) we obtain

∠LOE = ∠MOE

Now in triangles OLE and OME

∠LEO = ∠MEO (Given)

∴ ∠LOE = ∠MOE (Proved above)

EO = EO (Common)

∴ by ASA congruence criterion we have:

△OLE ≅ △ OME

∴ OL = OM (by CPCT)

Thus, chords AB and CD are equidistant from the centre O of the circle. Since, chords of a circle which are equidistant from the centre are equal.

∴ AB = CD

67. i. Arc BC subtends ∠BOC at the centre and ∠BAC at a point on the remaining part of the circumference. ∠BOC and ∠BAC both are on the same arcBC

$\therefore \angle BOC = 2 \times \angle ABC = 2 \times 20^\circ = 40^\circ$

ii. CD || BA and OC cuts them.

$\therefore \angle OCD = \angle BOC = 40^\circ$ [alt. int. $\angle s$].

Now, in △OCD, we have

$\angle COD + \angle OCD + \angle ODC = 180^\circ$

$\Rightarrow \angle COD + 40^\circ + 40^\circ = 180^\circ \quad [\because \angle OCD = \angle ODC = 40^\circ]$

$\Rightarrow \angle COD = 180^\circ - 80^\circ = 100^\circ$

iii. Since the angle at the centre is double the angle at a point on the remaining part of the circumference, ∠CAD and ∠COD both are on the same arcCD

$\angle CAD = \frac{1}{2}\angle COD = \left(\frac{1}{2} \times 100^\circ\right) = 50^\circ$

iv. Since BA || CD and AC cuts them, we have

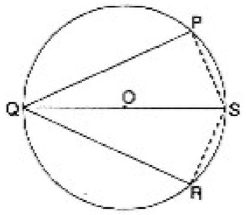
$\angle ACD = \angle CAB = 20^\circ$ [alt. int $\angle s$]

In △ACD, we have

$\angle CAD + \angle ACD + \angle ADC = 180^\circ$

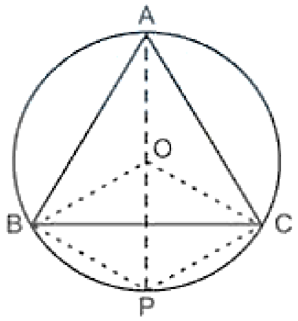
$\Rightarrow 50^\circ + 20^\circ + \angle ADC = 180^\circ$
$\Rightarrow \angle ADC = 180^\circ - 70^\circ = 110^\circ$

68. Given: PQ and RQ are two chords of a circle equidistant from the centre.
To prove: The diameter QS passing through Q bisects $\angle \mathrm{PQR}$ and $\angle \mathrm{PQR}$
Construction : Join PS and RS.

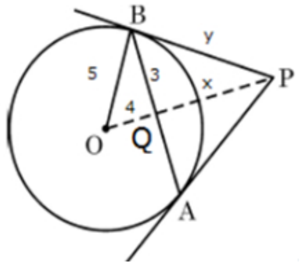


Proof : Chords PQ and RQ are equidistant from the centre.
$\therefore PQ = RQ$ | $\because$ chords of a circle equidistant from the centre are equal
Also $\angle QPS = \angle QRS = 90^\circ$ | $\because$ An angle in a semi-circle is a right angle
$\therefore \triangle$s PQS and RQS are right $\triangle s$
Now in right $\triangle s$ PQS and RQS
PQ = RQ |Proved above
Hyp. QS = Hyp.QS |Common
$\therefore \triangle PQS \cong \triangle RQS$ |R.H.S. Axiom
$\therefore \angle PQS = \angle RQS$ |c.p.c.t
and $\angle PSQ = \angle RSQ$ |c.p.c.t
i.e. The diameter QS passing through Q bisects $\angle \mathrm{PQR}$ and $\angle \mathrm{PSR}$ Proved.

69. Since equal chords of a circle subtends equal angles at the centre, so we have chord AB = chord AC [Given]



So $\angle \mathrm{AOB} = \angle \mathrm{AOC}$ ...(i)
since the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle,
$\therefore \quad \angle \mathrm{APC} = \frac{1}{2} \angle \mathrm{AOC}$ ...(ii)
and $\angle \mathrm{APB} = \frac{1}{2} \angle \mathrm{AOB}$ ...(iii)
$\therefore \quad \angle \mathrm{APC} = \angle \mathrm{APB}$ [from (i), (ii) and (iii)]
Hence, PA is the bisector of $\angle \mathrm{BPC}$.

70. 

Since OP $\perp$ AB and bisects it
$\therefore$ BQ = QA = 3 cm
Using Pythagoras Theorem in $\triangle$OQB, OQ = 4 cm
Taking PQ = x cm and PB = y cm,
Using Pythagoras Theorem in $\triangle$OBP and $\triangle$PQB
$x^2 + 9 = y^2$ and $(x+4)^2 = y^2 + 25$
Solving equations to get x = $\frac{9}{4}$ and y = $\frac{15}{4}$

71. Join OA, OC and OB.

Clearly, ∠OCA is the angle in a semi-circle.

∴ ∠OCA = 90^o

In right triangles OCA and OCB, we have

OA = OB = r

∠OCA = ∠OCB = 90^o and, OC = OC

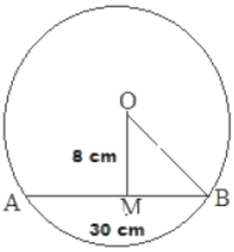
So, by RHS criterion of congruence, we get

△OCA ≅ △OCB

∴ AC = CB

72. Let AB be the chord of the given circle with centre O. The perpendicular distance from the centre of the circle to the chord is 8 cm.

Join OB.



Then OM = 8 cm and AB = 30 cm

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore MB = \left(\frac{AB}{2}\right) = \left(\frac{30}{2}\right) \text{cm} = 15\text{cm}$

From the right angled ΔOMB, we have:

$OB^2 = OM^2 + MB^2$ {paythgoras theorem}

⇒ $OB^2 = 8^2 + 15^2$

⇒ $OB^2 = 64 + 225$

⇒ $OB^2 = 289$

⇒ $OB = \sqrt{289}\text{cm}$ =17 cm

Hence, the required length of the radius is 17 cm.

73. Let position of three boys Ankur, Syed and David are denoted by the points A, B and C respectively.

AB = BC = CA = a [say]

Since equal sides of equilateral triangle are as equal chords and perpendicular distances of equal chords of a circle are equidistant from the centre.

∴OD = OE = OF = x cm [say]

Join OA, OB and OC.

⇒ Area of △AOB = Area of △BOC = Area of △AOC

And Area of △ABC

= Area of △AOB + Area of △BOC + Area of △AOC

⇒ And Area of △ ABC = $3 \times Area\ of BOC$

⇒ $\frac{\sqrt{3}}{4}a^2 = 3\ \left(\frac{1}{2}BC \times OE\right)$

⇒ $\frac{\sqrt{3}}{4}a^2$ = 3 ($\frac{1}{2}$ $\times a \times x$)

⇒ $\frac{a^2}{a} = 3 \times \frac{1}{2} \times \frac{4}{\sqrt{3}} \times x$

⇒ $a = 2\sqrt{3}x$

Now, CE ⊥ BC

∴ BE = EC = $\frac{1}{2}$ BC [∵ Perpendicular drawn from the centre bisects the chord]

⇒ BE = EC = $\frac{1}{2}a$

⇒ BE = EC = $\frac{1}{2}\left(2\sqrt{3}x\right)$ [Using eq. (i)]

⇒ BE = EC = $\sqrt{3}x$

Now in right angled triangle BEO,

$OE^2 + BE^2 = OB^2$ [Using Pythagoras theorem]

⇒ $x^2 + (\sqrt{3}x)^2 = (20)^2$

$\Rightarrow x^2 + 3x^2 = 400$

$\Rightarrow 4x^2 = 400$

$\Rightarrow x^2 = 100$

$\Rightarrow$ = 10 m

And $a = 2\sqrt{3}x$ = $2\sqrt{3} \times 10$ = $20\sqrt{3}$ m

Thus distance between any two boys is $20\sqrt{3}$ m.

74. i. We know that the opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \angle BAD + \angle BCD = 180^\circ$

$\Rightarrow 100^\circ + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 100^\circ = 80^\circ$

ii. AB || DC and DA is the transversal.

$\therefore \angle ADC + \angle BAD = 180^\circ$ [sum of internally opposite angles]

$\Rightarrow \angle ADC + 100^\circ = 180^\circ$

$\Rightarrow \angle ADC = 180^\circ - 100^\circ = 80^\circ$

iii. Using the fact that the opposite angles of a cyclic quadrilateral are supplementary, we have

$\angle ABC + \angle ADC = 180^\circ$

$\Rightarrow \angle ABC + 80^\circ = 180^\circ \quad [\because \angle ADC = 80^\circ]$

$\Rightarrow \angle ABC = 180^\circ - 80^\circ = 100^\circ$

==> ∠ABC =100°

$\therefore \angle BCD = 80^\circ, \angle ADC = 80^\circ$ and $\angle ABC = 100^\circ$

75. Let Reshma, Salma and Mandip takes the position C, A and B on the circle.

Since AB = AC = 6 cm

The center lies on the bisector of ∠BAC.

Let M be the point of intersection of BC and OA.

Again, since AB = AC and AM bisects ∠CAB.

∴ AM ⊥CB and M is the mid-point of CB.

Let OM = x, then MA = 5 - x

From right angled triangle OMB, $OB^2 = OM^2 + MB^2$

$\Rightarrow 5^2 = x^2 + MB^2$ ...(i)

Again, in right angled triangle AMB, $AB^2 = AM^2 + MB^2$

$\Rightarrow 6^2 = (5-x)^2 + MB^2$ ...(ii)

Equating the value of MB^2 from eq. (i) and (ii),

$5^2 - x^2 = 6^2 - (5 - x)^2$

$\Rightarrow (5-x)^2 - x^2 = 6^2 - 5^2$

$\Rightarrow 25-10x + x^2 - x^2 = 36 - 25$

$\Rightarrow 10x = 25-11$

$\Rightarrow 10x = 14 \Rightarrow x=\frac{14}{10}$

Hence, from eq. (i),

$MB^2 = 5^2 - x^2 = 5^2 - (\frac{14}{10})^2$

$= \left(5 + \frac{14}{10}\right)\left(5 - \frac{14}{10}\right) = \frac{64}{10} \times \frac{36}{10}$

$\Rightarrow MB = \frac{8\times6}{10}$ = 4.8 cm

$\therefore BC = 2MB = 2 \times 4.8 = 9.6$ cm