## Solution

## ARITHMETIC PROGRESSIONS

## Class 10 - Mathematics

### Section A

1.

**(b)** 7

**Explanation:**

$a_n = 7n + 4$

d = ?

$a_2 = 7 \times 2 + 4$

= 18

$a_1 = 7 \times 1 + 4$

= 11

$d = a_2 - a_1$

= 18 - 11

d = 7

2.

**(d)** p + q - n

**Explanation:**

We have given that $a_p = q$ and $a_q = p$

$\Rightarrow$ q = a + (p - 1)d and ...(i)

p = a + (q - 1)d ...(ii)

Subtracting (ii) from (i), we get

q - p = d (p - q) $\Rightarrow$ d = -1

Now, g = a + 1 - p [From (i)]

$\Rightarrow$ a = q + p - 1

$\therefore a_n = a + (n - 1)d = q + p - 1 + (n - 1)(-1)$

= q + p - 1 + 1 - n = q + p - n

3.

**(b)** 31

**Explanation:**

Reversing the given A.P.,

we have, 49, 46, 43, ..., -11

Here, a = 49, d = 46 - 49 = -3 and n = 7

$\therefore a_n = a + (n - 1)d$

$\Rightarrow a_7 = 49 + (7 - 1) \times (-3)$

$= 49 + 6 \times (-3)$

$\Rightarrow a_7 = 49 - 18 = 31$

4. **(a)** 22

**Explanation:**

Given, $a_1 = 14$, $t_n = 119$

$d = a_2 - a_1 = 19 - 14 = 5$

$t_n = a_1 + (n - 1)d$

119 = 14 + (n - 1)5

119 - 14 = 5n - 5

105 + 5 = 5n
110 = 5n
n = 22

5.

**(c)** (a), (b) and (c)

**Explanation:**

a. Given A.P. is $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$,....
$S_n$ = $\frac{n}{2}$[2($\sqrt{2}$) + (n - 1)($\sqrt{2}$)
= $\frac{n}{2} \times \sqrt{2}$ (2 + n - 1) = $\frac{n}{\sqrt{2}}$(n + 1)

b. Since, $a_n$ = 3n + 2
Here, $a_1$ = 3(1) + 2 = 5
$a_2$ = 3(2) + 2 = 8
∴ Common difference = $a_2$ - $a_1$ = 3

c. Given A.P. is (-5), (-8), (-11),... (-230)
Now, $a_n$ = a + (n - 1) ⇒ -230 = -5 + (n - 1) (-3)
⇒ $\frac{-225}{-3}$ = (n - 1) ⇒ n = 75 + 1 = 76
∴ $S_n$ = $\frac{76}{2}$((-5) + (-230)) = -8930

6.

**(d)** 37

**Explanation:**

a = 3
d = 3
$a_n = 111$
n = ?
$a_n = a + (n-1)d$
$111 = 3 + (n-1)3$
$108 = (n-1)3$
$\frac{108}{3} = (n-1)$
$36 = n - 1$
n = 37

7.

**(c)** 8

**Explanation:**

Given: $a_{18}$ - $a_{14}$ = 32
⇒ a + (18 - 1)d - [a + (14 - 1)d] = 32
⇒ a + 17d - a - 13d = 32
⇒ 4d = 32
⇒ d = 8

8.

**(d)** 6

**Explanation:**

$S_n$=Sum of n terms of an A.P. and $S_{2n} = 3S_n$
$S_n = \frac{n}{2}[2a + (n-1)d], \;\; S_{2n} = \frac{2n}{2}[2a + (2n-1)d]$
and $S_{3n} = \frac{3n}{2}[2a + (3n-1)d]$
We know that $S_{3n} = 3\left(S_{2n} - S_n\right)$ and $S_{2n} = 3S_n$
$\frac{S_{3n}}{S_n} = \frac{3(S_{2n} - S_n)}{S_n} = \frac{3(3S_n - S_n)}{S_n}$

$= \frac{3\times 2\text{Sn}}{\text{Sn}} = \frac{6}{1}$

$\therefore\ \text{S}_{3n} : \text{S}_n = 6 : 1$

9.

**(c)** 508th term

**Explanation:**

$a_9$ = a + 8d

$a_9$ = 499 (given)

∴ a + 8d = 499 .... (i)

$a_{499}$ = a + 498d

$a_{499}$ = 9

∴ a + 498d = 9 ...... (ii)

Subtract (i) from (ii)

⇒ 490d = - 490

⇒ d = -1

Substitute the value of d

⇒ a + 8(-1) = 499

⇒ a = 507

∴ For $a_n$ = 0

⇒ a + (n - 1)d = 0

⇒ 507 + (n - 1)(-1) = 0

⇒ n = 508

So, 508th term is equal to zero.

10. **(a)** 15

**Explanation:**

15

11. **(a)** 4n + 3

**Explanation:**

Let a be the first term and d be the common difference of an A.P. and $S_n = 2n^2 + 5n$

$\therefore \text{S}_1 = 2(1)^2 + 5 \times 1 = 2 + 5 = 7$

$S_2 = 2(2)^2 + 5 \times 2 = 8 + 10 = 18$

So, first term, $a_1$ = 7

and the second term, $a_2 = \text{S}_2 - \text{S}_1 = 18 - 7 = 11$

$\therefore\ d = a_2 - d_1 = 11 - 7 = 4$

Now $a_n = a + (n-1)d$

$= 7 + (n-1)4 = 7 + 4n - 4$

$= 4n + 3$

12.

**(d)** 23

**Explanation:**

a = 7, $a_n$ = 84, $S_n = \frac{2093}{2}$

$a_n$ = a + (n - 1)d

84 = 7 + (n - 1)d

77 = (n - 1)d ...(i)

Now,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\frac{2093}{2} = \frac{n}{2}[2 \times 7 + 77]$

2093 = n[14 + 77]

$n = \frac{2093}{91}$

n = 23

13.

**(c)** 6

**Explanation:**

We have 2x, (x + 10), (3x + 2) are in A.P.

We know that if $a_1$, $a_2$ and $a_3$ are in AP then $a_2 - a_1 = a_3 - a_2$

x + 10 − 2x = 3x + 2 − x − 10

⇒x = 6

14.

**(d)** 6

**Explanation:**

Given: $a = 1, l = 11$ and $S_n = 36$

$\therefore S_n = \frac{n}{2}(a + l)$

$\Rightarrow 36 = \frac{n}{2}(1 + 11)$

$\Rightarrow 72 = n \times 12$

$\Rightarrow n = 6$

15.

**(b)** 31500

**Explanation:**

Number of TV produced in $6^{th}$ year = 8000

⇒ $a_6$ = 8000 ⇒ a + 5d = 8000 ...(i)

Also, number of TV produced in $9^{th}$ year = 11300

⇒ $a_9$ = 11300 ⇒ a + 8d = 11300 ...(ii)

Subtracting (i) from (ii), we get

3d = 3300 ⇒ d = 1100

From (i), a = 2500

∴ Production in 6 years, i.e.,

$S_6 = \frac{6}{2}[2(2500) + (6 - 1)(1100)] = 31500$

16.

**(b)** 1

**Explanation:**

If a, b and c are in A.P.,

b - a = c - b

-(a - b) = -(b - c)

a - b = b - c

dividing both sides by b - c

$\frac{a-b}{b-c} = \frac{b-c}{b-c}$

$\frac{a-b}{b-c} = 1$

17.

**(c)** 158

**Explanation:**

20th term from the end = l - 19 d

= (253 -19 × 5)

$= (253 - 95)$
$= 158.$

18.

**(d)** $16^{th}$
**Explanation:**
$a = -29$
$d = 3$
$a_n = 16$
$16 = a + (n - 1)d$
$16 = -29 + (n - 1)3$
$\frac{45}{3} = n - 1$
$15 + 1 = n$
$n = 16$
$a_{16} = 16$

19. **(a)** 5k + 4 and 6k + 5
**Explanation:**
Given: k, 2k + 1, 3k + 2, 4k + 3, ...
Here, d = 2k + 1 - k = k + 1
Therefore, the next two terms are
4k + 3 + k + 1 = 5k + 4 and 5k + 4 + k + 1 = 6k + 5

20. **(a)** 8
**Explanation:**
$a_{23} - a_{19} = 32$
$a + 22d - a - 18d = 32$
$4d = 32$
$d = 8$

21.

**(b)** Both A and R are true but R is not the correct explanation of A.
**Explanation:**
Both A and R are true but R is not the correct explanation of A.

22. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:**
Both A and R are true and R is the correct explanation of A.

23.

**(c)** A is true but R is false.
**Explanation:**
$a_{10} = a + 9d$
$= 5 + 9(3) = 5 + 27 = 32$

24. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:**
Both are correct. Reason is the correct explanation.
Assertion,
$a_n = 7 - 4n$
$d = a_{n - 1} - a_n$
$= 7 - 4(n + 1) - (7 - 4n)$
$= 7 - 4n - 4 - 7 + 4n = -4$

25. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:**
Both are correct. Reason is the correct reasoning for Assertion.
Assertion, $S_{10} = \frac{10}{2}[2(-0.5) + (10 - 1)(-0.5)]$
= 5[-1 - 4.5]
= 5(-5.5) = -27.5

26. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:**
nth term of an AP be $a_n = S_n - S_{n-1}$
$a_n = 3n^2 - 4n - 3(n - 1)^2 + 4(n - 1)$
$a_n = 6n - 7$
So, both A and R are true and R is the correct explanation of A.

27.
**(d)** A is false but R is true.
**Explanation:**
**Assertion:** Even natural numbers divisible by 5 are 10, 20, 30, 40, ...
They form an A.P. with,
a = 10, d = 10
$S_{100} = \frac{100}{2}[2(10) + 99(10)]$ = 50500
So, reason is correct.

28.
**(d)** A is false but R is true.
**Explanation:**
We have,
$a_n = a + (n - 1)d$
$a_{21} - a_7 = \{a + (21 - 1)d\} - \{a + (7 - 1)d\} = 84$
a + 20d - a - 6d = 84
14d = 84
$d = \frac{18}{14} = 6$
d = 6
So, A is false but R is true.

29. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:**
Both A and R are true and R is the correct explanation of A.

30. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:**
Both A and R are true and R is the correct explanation of A.

**Section B**

31. 20400
Explanation:
Let the production during first year be a and let d be the increase in production every year. Then,
$T_6 = 16000 \Rightarrow a + 5d = 16000$ ... (i)
and $T_9 = 22600 \Rightarrow a + 8d = 22600$. ... (ii)
On subtracting (i) from (ii), we get $3d = 6600 \Rightarrow d = 2200$
Putting d = 2200 in (i), we get
$a + 5 \times 2200 = 16000 \Rightarrow a + 11000 = 16000 \Rightarrow a = 16000 - 11000 = 5000$
Thus, a = 5000 and d = 2200

Production during $8^{th}$ year is given by

$T_8 = (a + 7d) = (5000 + 7 \times 2200) = (5000 + 15400) = 20400$

32. 15

Explanation:

Two digits numbers are 10, 11, 12, 13, 14.... 97, 98, 99 in which only 12, 18, 24,...96 are divisible by 6.

So, this forms an AP 12, 18,......,96

whose first term (a) = 12, common difference(d) = 18 - 12 = 6.

Let there are n terms in the above sequence, then last term ($a_n$) = 96

$\Rightarrow$ a + (n - 1)d = 96

$\Rightarrow$ 12 + (n - 1)(6) = 96

$\Rightarrow$ 12 + 6n - 6 = 96

$\Rightarrow$ 6n = 90

$\Rightarrow$ n = 15

Hence, 15 numbers of two digits are divisible by 6.

33. 54

Explanation:

$T_n = (4_n - 10)$ [Given]

$T_1 = (4\times 1 - 10) = -6$

$T_2 = (4\times 2 - 10) = -2$

$T_3 = (4\times 3 - 10) = 2$

$T_4 = (4\times 4 - 10) = 6$

Clearly, [-2-(-6)] = [2-(-2)] = [6 - 2] = 4

So, the terms -6, -2, 2, 6,... forms an AP.

Thus we have

$T_{16} = a + (n - 1)d = a + 15d = -6 + 15 \times 4 = 54$

34. 163

Explanation:

A.P. 7, 10, 13,....., 184

Last term (l) = 184

Common difference(d) = 10 - 7 = 3

$\therefore$ $8^{th}$ term from end

= l - (n - 1)d

= 184 - (8 - 1)$\times$ 3

= 184 - 21

= 163

35. 19

Explanation:

Here it is given an AP

where a = 72 and d = -4

Suppose the $n^{th}$ term = 0

$T_n = a + (n - 1)d = 0$

So 72 + (n - 1)(-4) = 0

72 - 4n + 4 = 0

-4n = -72 - 4 = -76

$n = \frac{-76}{-4} = 19$

Hence, the $19^{th}$ term of the given AP is 0.

36. 98450

Explanation:

Multiple of 5 lying between 101 and 999 are 105, 110, 115, ..., 995 which are in AP.

Here a = 105 and d = 5. where a is first term and d is common difference

$a_n = a + (n - 1)d$

$\Rightarrow 995 = 105 + (n - 1)5$

$\Rightarrow 890 = 5n - 5$

$\Rightarrow 895 = 5n$

$\therefore n = 179$

$\therefore S_n = \frac{n}{2}[a + l] = \frac{179}{2}[105 + 995] = \frac{179}{2} \times 1100 = 98450.$

37. From the given numbers, we can have

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_4 - a_3 = 27 - 9 = 18$

since $a_{k+1} - a_k$ i.e. the common difference is not same for all values of k

Hence, it is not an AP. So, we can not find next three terms.

38. $a_1 = \sqrt{27}$

$= 3\sqrt{3}$

$a_2 = \sqrt{48}$

$= 4\sqrt{3}$

$d = a_2 - a_1$

$= 4\sqrt{3} - 3\sqrt{3}$

$= \sqrt{3}$

4th of A.P

$a_4 = a_1 + 3d$

$= \sqrt{27} + 3\sqrt{3}$

$= \sqrt{27} + \sqrt{27}$

$= 2\sqrt{27}$

$= \sqrt{108}$

$a_5 = a_1 + 4d$

$= \sqrt{27} + 9\sqrt{3}$

$= 3\sqrt{3} + 4\sqrt{3}$

$= \sqrt{147}$

hence, next two term of AP are $\sqrt{108}$ and $\sqrt{147}$

39. Here, we are given two A.P. sequences whose $n^{th}$ terms are equal. We need to find n.

So let us first find the $n^{th}$ term for both the A.P.

First A.P. is 9, 7, 5...

Here

First term (a) = 9

Common difference of the A.P. (d) = 7 - 9

= -2

Now as we know

$a_n = a + (n - 1)d$

So for nth term

$a_n = a + (n - 1) d$

So for nth term

an = 9 + (n - 1)(-2)

= 9 - 2n + 2

= 11 - 2n ...(i)

Second A.P. is 15, 12, 9...

Here,

First term (a) = 15

Common difference of the A.P. (d) = 12 - 15 = -3

Now as we know

$a_n = a + (n - 1)d$

So for nth term

$a_n = 15 + (n - 1)(-3)$

$= 15 - 3n + 3$

$= 18 - 3n$ ...(ii)

Now, we are given that the nth terms for both the A.P. sequences are equal, we equate (i) and (ii),

$11 - 2n = 18 - 3n$

$3n - 2n = 18 - 11$

$n = 7$

Therefore n = 7

40. The given terms can be written as follows:

1, 9, 25, 49,....

Here, $a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

$a_4 - a_3 = 49 - 25 = 24$

Since $a_{k+1} - a_k$ is not same for all values of k

Hence, it is not an AP. So we can not find next three terms.

41. $a_n = 0$

$54 + (n - 1)(-3) = 0$

$n = 19$

$S_{19} = \frac{19}{2}(54 + 0)$

$= 19 \times 27 = 513$

42. Let the first term, common difference be a and d respectively.

As we know

$a_n = a + (n - 1)d$

and Given

$a_5 + a_7 = 52$

$a + 4d + a + 6d = 52$

$2a + 10d = 52$

$a + 5d = 26$

$a = 26 - 5d$ ...(1)

also we have given

$a_{10} = 46$

$a + 9d = 46$

$26 - 5d + 9d = 46$ [ from eqn 1]

$4d = 46 - 26$

$4d = 20$

$d = 5$

using this value in eqn 1 , we get

$a = 26 - 5d$

$a = 26 - 5(5)$

$a = 1$

as a = 1 and d = 5

So, required AP is a, a + d, a + 2d, a + 3d, …. i.e., 1, 1 + 5, 1 + 2(5), 1 + 3(5), …. i.e., 1, 6, 11, 16, …

43. Given that a man receives Rs. 60 for the first week and Rs. 3 more each week than the preceding week.

Money earned by the 20th week

= 60 + 63 + 66 + ... up to 20 terms

$= \frac{20}{2}[2 \times 60 + 19 \times 3] = 10(177) = 1770.$

44. a = 10, d = 10

First term a = 10

Second term = 10 + d = 10 + 10 = 20

Third term = 20 + d = 20 + 10 = 30

Fourth term = 30 + d = 30 + 10 = 40

Hence, first four terms of the given AP are 10, 20, 30, 40

45. According to the question,we have,

$b_n = 5 + 2n$

Put n = 1,

$b_1 = 5 + 2\times 1 = 5 + 2 = 7$

$b_2 = 5 + 2\times 2 = 5 + 4 = 9$

Common difference(d)= 9 - 7 = 2

$S_n = \frac{n}{2}[a + (n-1)d]$ .

Sum of 15 terms, $S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1) \times 2]$

$= \frac{15}{2}[14 + 28]$

$= \frac{15}{2} \times 42$

= 315

Therefore, the sum of the first 15 terms of sequences having $n^{th}$ term as $b_n = 5 + 2n$ is 315.

**Section C**

46. Given that, nth term of the series is $a_n = 3 - 4n$

For $a_1$,

Put n = 1 so $a_1 = 3 - 4(1) = -1$

For $a_2$,

Put n = 2, so $a_2 = 3 - 4(2) = -5$

For $a_3$,

Put n = 3 so $a_3 = 3 - 4(3) = -9$

For $a_4$,

Put n = 4 so $a_4 = 3 - 4(4) = -13$

So AP is - 1, - 5, - 9, - 13, …

$a_2 - a_1 = -5 - (-1) = -4$

$a_3 - a_2 = -9 - (-5) = -4$

$a_4 - a_3 = -13 - (-9) = -4$

Since, the each successive term of the series has the same difference. So, it forms an AP with common difference, d = - 4

We know that, sum of n terms of an AP is

$S_n = \frac{n}{2}(2a + (n-1)d)$

Where a = first term

d = common difference

and n = no of terms

$S_{20} = \frac{20}{2}[2(-1) + (20-1)(-4)]$

= 10[ - 2 - 76]

= - 780

So Sum of first 20 terms of this AP is - 780.

47. The given AP is 9, 17, 25,...

Here, a = 9

d = 17 - 9 = 8

Let n terms of the AP must be taken

Then, $S_n = 636$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 636$

$\Rightarrow \frac{n}{2}[2(9) + (n-1)8] = 636$

$\Rightarrow$ n[9 + (n - 1)4] = 636

$\Rightarrow$ n[9 + 4n - 4] = 636

$\Rightarrow$ n[(4n + 5)] = 636

$\Rightarrow 4n^2 + 5n - 636 = 0$

$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow$ n(4n + 53) - 12(4n + 53) = 0

$\Rightarrow$ (4n + 53) (n - 12) = 0

$\Rightarrow$ 4n + 53 = 0 or n - 12 = 0

$\Rightarrow n = -\frac{53}{4}$ or n = 12

$n = -\frac{53}{4}$ is in admissible as n, being the number of terms, is a natural number

$\therefore$ n = 12

Hence, 12 terms of the AP must be taken.

48. All the numbers between 100 and 500 which are divisible by 8 are

104, 112, 120, 128, ..., 496

Here, $a_1 = 104$

$a_2 = 112$

$a_3 = 120$

$a_4 = 128$

$\therefore a_2 - a_1 = 112 - 104 = 8$

$a_3 - a_2 = 120 - 112 = 8$

$a_4 - a_3 = 128 - 120 = 8$

$\because a_2 - a_1 = a_3 - a_2 = a_4 - a_3 =$ .... (8 each)

$\therefore$ This sequence is an arithmetic progression whose difference is 8.

Here, a = 104

d = 8

l = 496

Let the number of term s be n . Then,

l = a + (n - 1)d

$\Rightarrow$ 496 = 104 + (n - 1)8

$\Rightarrow$ 392 = (n - 1)8

$\Rightarrow$ (n - 1)8 = 392

$\Rightarrow$ n - 1 = 49

$\Rightarrow$ n = 49 + 1

$\Rightarrow$ n = 50

$\therefore S_n = \frac{n}{2}(a + l)$

$= \left(\frac{50}{2}\right)(104 + 496)$

= (25) (600)

= 15000

49. Let the first term of given AP =a and common difference =d

Now, we know that in general the $n^{th}$ term of an A.P is given by $T_n$=a+(n-1)d

$\Rightarrow T_4 = a + (4 - 1)d$, $T_{25} = a + (25 - 1)d$, and $T_{11} = a + (11 - 1)d$

$\Rightarrow T_4=a+3d$, $T_{25}=a+24d$, and $T_{11}=a+10d$

Now, $T_4=0$ [given]

$\Rightarrow$a+3d=0

$\Rightarrow$a=-3d .....................................(1)

$\therefore T_{25}=a+24d=(-3d+24d)=21d$ [using (1)]

And $T_{11}=a+10d=-3d+10d=7d$ ( substituting a from equation 1)

$\therefore T_{25}=21d=3\times 7d=3\times T_{11}$

50. $a_3 = 16$

$\Rightarrow$ a + 2d = 16 ..... (i)

$a_7 = a_5 + 12$

⇒ a + 6d = a + 4d + 12

⇒ 2d = 12

⇒ d = 6

Put the value of d in eq. (i)

$a + 2 \times 6 = 16$

⇒ a = 16 - 12

⇒ a = 4

4, 10, 16....

51. The first 40 positive integers divisible by 6 are 6, 12, 18, 24, .....

Here, $a_2 - a_1 = 12 - 6 = 6$

$a_3 - a_2 = 18 - 12 = 6$

$a_4 - a_3 = 24 - 18 = 6$

i.e.$a_{k+1} - a_k$ is the same every time.

So, the above list of numbers form an AP.

Here, a = 6

d = 6

n = 40

∴ Sum of the first 40 positive integers = $S_{40}$

$= \frac{40}{2}[2a + (40 - 1)d]$ .........$\{\because S_n = \frac{n}{2}[2a + (n-1)d]\}$

= 20[2a + 39d]

$= (20)[2 \times 6 + 39 \times 6]$

= (20) (246)

= 4920

52. Initial monthly salary of Tanvy in 2015 = Rs 40000

Annual increment = Rs 2500

Therefore an AP is formed

and here a = 40000 and d = 2500

Let the monthly salary of Tanvy becomes Rs 65000 in $n^{th}$ year

So, $a_n$ = a + (n-1 )d

⇒ 65000=40000+(n-1)$\times$ 2500

⇒ 40000 + (n -1) $\times$ 2500 = 65000

⇒ (n -1) $\times$ 2500 = 65000 - 40000 = 25000

⇒ (n-1)=$\frac{25000}{2500}$ = 10

Hence n = 11

Thus, the 11th annual salary received by Tanvy will be Rs.65000. Thus, after 10 years, i.e., in the year 2025, her annual salary will be Rs.65000.

53. Given, a = 10, and $S_{14}$ = 1050

Let the common difference of the A.P. be d

we know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{14} = \frac{14}{2}[2 \times 10 + (14 - 1)d]$

1050 = 7(20 + 13d)

or 20 + 13d = $\frac{1050}{7}$

20 + 13d = 150

13d = 150 - 20

13d = 130

d = $\frac{130}{13}$

d = 10

Now, $a_{21}$ = a + (n − 1)d

= 10 + (21 - 1) 10

= 10 + 20 $\times$ 10

= 10 + 190

= 210

Hence, $a_{20}$ = 210

54. Let the first price be ₹ a.

Since each prize after the first is ₹200 less than the preceding prize,therefore,the prizes are ₹ a, ₹(a-200), ₹(a-400), ₹(a-600).

common difference d =(a-200)-a=-200. Then,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 2800 = \frac{4}{2}[2a+3(-200)]$

$\Rightarrow$ 1400 = 2a-600

$\Rightarrow$ 2a = 2000

$\Rightarrow$ a = 1000

So, the prizes are ₹ 1000, ₹ 800, ₹ 600 and ₹400.

55. Let a be the first term and d be the common difference of the given AP.

According to the question,we are given that,

$\frac{T_{11}}{T_{18}} = \frac{2}{3} \Rightarrow \frac{a+(11-1)d}{a+(18-1)d} = \frac{2}{3}$

$\Rightarrow \frac{a+10d}{a+17d} = \frac{2}{3} \Rightarrow$ 3a+30d= 2a +34d

$\Rightarrow$a=4d. .....(i)

Ratio of 5th term to 21st term$= \frac{T_5}{T_{21}} = \frac{a+(5-1)d}{a+(21-1)d}$

$= \frac{a+4d}{a+20d} = \frac{4d+4d}{4d+20d}$ { from (i)}

$= \frac{8d}{24d} = \frac{1}{3} = 1 : 3$

Ratio of sum of first 5 terms to sum of first 21 terms

$= \frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a+(5-1)d]}{\frac{21}{2}[2a+(21-1)d]} = \frac{5(2a+4d)}{21(2a+20d)}$

$= \frac{10(a+2d)}{42(a+10d)} = \frac{10(4d+2d)}{42(4d+10d)}$ [from(i)]

$= \frac{60d}{588d} = \frac{60}{588} = \frac{5}{49} = 5 : 49$ .

**Section D**

56. i. Number of pots in the $10^{th}$ row

= $a_{10}$ = a + 9d = 29

ii. $a_5$ - $a_2$ = (a + 4d) - (a + d) = 3d = 9

iii. $S_n$ = 100 $\Rightarrow \frac{n}{2}[2(2) + (n-1)3$ = 100

$3n^2$ + n - 200 = 0 $\Rightarrow$ (3n + 25)(n - 8) = 0

$\therefore$ n = 8 (n = $-\frac{25}{3}$ rejected).

**OR**

$S_{12} = \frac{12}{2}[2(2) + 11(3)]$

= 222

57. i. Money saved on 1st day = Rs. 27.5

$\because$ Sehaj increases his saving by a fixed amount of Rs. 2.5

$\therefore$ His saving form an AP with a = 27.5 and d = 2.5

$\therefore$ Money saved on 10th day,

$a_{10}$ = a + 9d = 27.5 + 9(2.5)

= 27.5 + 22.5 = Rs. 50

ii. $a_{25}$ = a = 24d

= 27.5 + 24(2.5)

= 27.5 + 60 = Rs. 87.5

iii. Total amount saved by Sehaj in 30 days.

= $\frac{30}{2}[2 \times 27.5 + (30 - 1) \times 2.5]$

= 15(55 + 29(2.5)

= Rs. 1912.5

58. i. a = 1000

d = 100

$S_n$ = 1,18,000

$t_{30}$ = a + 29 d

= 1000 + 29 $\times$ 100

= 1000 + 2900

$t_{30}$ = 3900

i.e., he will pay ₹ 3900 in $30^{th}$ installment.

ii. $S_n = \frac{n}{2}$ {2a + (n - 1)d}

$S_{30} = \frac{30}{2}$ {2 $\times$ 1000 + (30 - 1) $\times$ 100}

$S_{30}$ = 15 {2000 + 2900}

$S_{30}$ = 15 $\times$ 4900

$S_{30}$ = 73,500

i.e., he will pay ₹ 73500 in 30 installments.

iii. $S_n = \frac{n}{2}$ {a + l}

1,18,000 = $\frac{40}{2}$ {1000 + l}

1,18,000 = 20,000 + 20 l

98,000 = 20 l

l = 4900

i.e., the last installment will be of ₹ 4900.

**OR**

$t_{10}$ = a + 9d

= 2000 + 9 $\times$ 100

$t_{10}$ = 2000 + 900

$t_{10}$ = ₹ 2900

59. i. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

i.e. $S_n = 360$

$\Rightarrow \frac{n}{2}[2 \times 30 + (n-1)(-1)] = 360$ $\{S_n = \frac{n}{2}(2a + (n-1)d)\}$

$\Rightarrow 720 = n(60 - n + 1)$

$\Rightarrow$ 720 = 60n - $n^2 + n$

$\Rightarrow n^2 - 61n + 720 = 0$

$\Rightarrow n^2 - 16n - 45n + 720 = 0$ [by factorization]

$\Rightarrow n(n-16) - 45(n-16) = 0$

$\Rightarrow (n-16)(n-45) = 0$

$\Rightarrow (n-16) = 0$ or $(n-45) = 0$

$\Rightarrow n = 16$ or $n = 45$

Hence, number of rows is either 45 or 16.

n = 45 not possible so n = 16

$a_{45} = 30 + (45-1)(-1)$ $\{a_n = a + (n-1)d\}$

$= 30 - 44 = -14$ [$\because$ The number of logs cannot be negative]

Hence the number of rows is 16.

ii. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

Number of bricks on top row are $n = 16$ ,

$a_{16}$ = 30 + (16 - 1) (-1) {$a_n$ = a + (n - 1)d}

= 30 - 15 = 15

Hence, and number of bricks in the top row is 15.

iii. Number of bricks in the bottom row = 30. in the next row = 29, and so on.
therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.
Suppose number of rows is n, then sum of number of bricks in n rows should be 360
Number of bricks in 10th row a = 30, d = -1, n = 10
$a_n = a + (n - 1)d$
$\Rightarrow a_{10} = 30 + 9 \times -1$
$\Rightarrow a_{10} = 30 - 9 = 21$
Therefore, number of bricks in 10th row are 21.
**OR**
Number of bricks in the bottom row = 30. in the next row = 29, and so on.
Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.
Suppose number of rows is n, then sum of number of bricks in n rows should be 360.
$a_n = 26$, a = 30, d = -1
$a_n = a + (n - 1)d$
$\Rightarrow 26 = 30 + (n - 1) \times -1$
$\Rightarrow 26 - 30 = -n + 1$
$\Rightarrow n = 5$
Hence 26 bricks are in 5th row.

60. i. Let production in a 1st year be a unit and increase in production (every year) be d units.
$\because$ Increase in production is constant, therefore unit produced every year forms an AP.
Now, $a_3 = 6000$
$a + 2d = 6000 \Rightarrow a = 6000 - 2d$ ..(1)
and $a_7 = 7000 \Rightarrow a + 6d = 7000$
$\Rightarrow (6000 - 2d) + 6d = 7000 \Rightarrow 4d = 1000$ [using eq. (1)]
$\Rightarrow d = 250$
When d = 250, eq. (1) becomes
a = 6000 - 2(250) = 5500
$\therefore$ Production in 1st year = 5500
ii. Production in fifth year
$a_5 = a + 4d = 5500 + 4(250) = 6500$
iii. Total production in 7 years = $\frac{7}{2}(5500 + 7000) = 43750$.

61. i. 1st installment = ₹ 3425
2nd installment = ₹ 3225
3rd installment = ₹ 3025
and so on
Now, 3425, 3225, 3025, ... are in AP, with
a = 3425, d = 3225 - 3425 = -200
Now 6th installment = $a_n = a + 5d = 3425 + 5(-200)$ = ₹ 2425
ii. Total amount paid = $\frac{15}{2}(2a + 14d)$
$= \frac{15}{2}[2(3425) + 14(-200)] = \frac{15}{2}(6850 - 2800)$
$= \frac{15}{2}(4050)$ = ₹ 30375
iii. $a_n = a + (n - 1)d$
$\Rightarrow a_{10} = 3425 + 9 \times (-200) = 1625$
$\Rightarrow a_{11} = 3425 + 10 \times (-200) = 1425$
$a_{10} + a_{11} = 1625 + 1425 = 3050$
**OR**
$a_n = a + (n - 1)d$ given $a_n = 2625$
$2625 = 3425 + (n - 1) \times - 200$

$\Rightarrow$ - 800 = (n - 1) $\times$ - 200
$\Rightarrow$ 4 = n - 1
$\Rightarrow$ n = 5
So, in 5th installment, she pays ₹ 2625.

62. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
We have,$a_3$ = 600 and $a_7$ = 700 $\Rightarrow$ a + 2d = 600 and a + 6d = 700. Solving these equations, we get;a = 550 and d = 25.

1. We have, a = 550
$\therefore$ Production in the first year is of 550 TV sets.
2. The production in the 10th term is given by $a_{10}$.
Therefore, production in the 10th year = $a_{10}$ = a + 9d = 550 + 9 $\times$ 25 = 775. So, production in 10th year is of 775 TV sets.
3. Total production in 7 years
= Sum of 7 terms of the A.P. with first term a (= 550) and common difference d (= 25).
$= \frac{7}{2}\{2 \times 550 + (7 - 1) \times 25\}$
$= \frac{7}{2}(1100 + 150) = 4375.$

63. i. Let $1^{st}$ year production of TV = x
Production in $6^{th}$ year = 16000
$t_6$ = 16000
$t_9$ = 22,600
$t_6$ = a + 5d
$t_9$ = a + 8d
**16000 = x + 5d ...(i)**
**22600 = x + 8d ...(ii)**
**- - -**
**-6600 = - 3d**
d = 2200
Putting d = 2200 in equation ...(i)
16000 = x + 5 $\times$ (2200)
16000 = x + 11000
x = 16000 - 11000
x = 5000
$\therefore$ Production during $1^{st}$ year = 5000

ii. Production during 8th year is (a + 7d) = 5000 + 7(2200) = 20400

iii. Production during first 3 year = Production in ($1^{st}$ + $2^{nd}$ + $3^{rd}$) year
Production in $1^{st}$ year = 5000
Production in $2^{nd}$ year = 5000 + 2200
= 7200
Production in $3^{rd}$ year = 7200 + 2200
= 9400
$\therefore$ Production in first 3 year = 5000 + 7200 + 9400
= 21,600
**OR**
Let in $n^{th}$ year production was = 29,200
$t_n$ = a + (n - 1)d
29,200 = 5000 + (n - 1) 2200
29,200 = 5000 + 2200n - 2200
29200 - 2800 = 2200n
26,400 = 2200n

$\therefore n=\frac{26400}{2200}$

n = 12

i.e., in $12^{th}$ year, the production is 29,200

64. i. First Term = 1 and Common difference = 4

ii. First Term = 4 and Common difference = 12

iii. a. Required number of squares = 1 + (9) $\times$ 4 = 37

Required number of sticks = 4 + 9 $\times$ 12 = 112

b. 88 = 4 + (m - 1) $\times$ 12

$\Rightarrow$ m = 8

Number of squares formed in $8^{th}$ fig. = 1 + 7 $\times$ 4 = 29

65. Number of bricks in the bottom row=30. in the next row=29, and so on.

therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,... , which is an AP with first term , $a$=30 and common difference, $d$= 29 - 30 = -1.

Suppose number of rows is $n$, then sum of number of bricks in n rows should be 360

i.e. $S_n=360$

$\Rightarrow \frac{n}{2}[2\times 30+(n-1)(-1)]=360\ \{S_n=\frac{n}{2}(2a+(n-1)d)\}$

$\Rightarrow 720=n(60-n+1)$

$\Rightarrow$ 720 = 60n - $n^2+n$

$\Rightarrow n^2-61n+720=0$

$\Rightarrow n^2-16n-45n+720=0$ [by factorisation]

$\Rightarrow n(n-16)-45(n-16)=0$

$\Rightarrow (n-16)(n-45)=0$

$\Rightarrow (n-16)=0$ or $(n-45)=0$

$\Rightarrow n=16$ or $n=45$

Hence, number of rows is either 45 or 16.

When, $n=16$ ,

$a_{16}=30+(16-1)(-1)\ \{a_n=a+(n-1)d\}$

$=30-15=15$

When, $n=45$

$a_{45}=30+(45-1)(-1)\ \{a_n=a+(n-1)d\}$

$=30-44=-14$ [$\because$ The number of logs cannot be neagtive]

Hence, the number of rows is 16 and number of logs in the top row is 15.

**Section E**

66. We have,

2 + 6 + 10 + ...... x = 1800

Here ; 2, 6, 10 , .....x are in arithematic progression where, a = 2 is first term and d = 4 is common difference.

Using formula to find the number of terms in AP.

x = a + (n-1)d

x = 2 + (n - 1) .4

x = 2 + 4n - 4

x = 4n - 2

x + 2 = 4n

n = $\frac{(x+2)}{4}$

Now, using formula, $S_n=\frac{n}{2}(a+T_n)$

Here, $S_n$ = 1800, n = $\frac{x+2}{4}$, a = 2, $T_n$ = x

1800 = $\frac{\left(\frac{x+2}{4}\right)}{2}$ [2 + x ]

1800 = $\frac{x+2}{8}\times$ (x + 2)

1800 $\times$ 8 = $(x+2)^2$

14400 = $(x+2)^2$

$(120)^2=(x+2)^2$

x + 2 = 120 ⇒ x = 118

Hence, value of x = 118

67. Given A.P is: a, a + d, a + 2d .....

Here, we first need to write the expression for $a_n - a_k$

Now, as we know,

$a_n = a + (n - 1)d$

So for the nth term

$a_n = a + (n - 1)d$

Similarly for $k^{th}$ term

$a_k = a + (k - 1)d$

So,

$a_n - a_k = (a + nd - d) - (a + kd - d)$

= a + nd - d - a - kd + d

= nd - kd

= (n - k)d

So $a_n - a_k = (n - k)d$

We are given $a_{10} - a_5 = 200$

Here

Let us take the first term as 'a' and the common difference as 'd'

Now, as we know,

$a_n = a + (n - 1)d$

Here we find $a_{30}$ and $a_{20}$

So, for 10th term,

$a_{10} = a + (10 - 1)d$

= a + 9d

Also for 5th term

$a_5 = a + (5 - 1)d$

= a + 4c

So, $a_{10} - a_5 = (a + 9d) - (a + 4d)$

200 = a = 9d - a - 4d

200 = 5d

$d = \frac{200}{5}$

d = 40

Therefore the common difference for the A.P is d = 40

68. Here,

P = 1000

R = 10% per annum

Amount at the end of $1^{st}$ year,

$\mathrm{A}_1 = 1000\left(1 + \frac{10}{100}\right)^1$

$A_1 = 1100$

For $2^{nd}$ year, the amount is

$A_2 = 1100\left(1 + \frac{10}{100}\right)^1$

$A_2 = 1210$

For $3^{rd}$ year compound interest,

$A_3 = 1210\left(1 + \frac{10}{100}\right)^1$

$A_3 = 1331$

For $1^{st}$ year, $2^{nd}$ year and $3^{rd}$ year, the respective amounts are

1100, 1210 and 1331

If any sequence is in A.P. then common difference between any two consecutive terms is constant.

So, 1210 - 1100 = 110

1331 - 1210 = 121

Since it is not constant, so it is not in A.P.

69. Natural numbers between 1 and 100, which are divisible by 3

= 3, 6, 9, .... 99. As it forms Arithmetic progression, we can find its first term "a" and common difference "d"

Now first term (a) = 3

Common difference (d) = 6 - 3 = 3

Last term ($a_n$) = 99

As $T_n = a + (n-1)d$

$\Rightarrow$ a + (n - 1)d = 99

$\Rightarrow$ 3 + (n - 1) $\times$ 3 = 99

$\Rightarrow$ 3 + 3n - 3 = 99

$\Rightarrow$ 3n = 99

$\Rightarrow n = \frac{99}{3} = 33$

Since, $S_n = \frac{n}{2}[a + a_n]$

Therefore, Sum of 33 terms $S_{33} = \frac{n}{2}[a + a_n]$

$= \frac{33}{2}[3 + 99]$

$= \frac{33}{2} \times 102$

= 33 $\times$ 51

= 1683

Hence, the sum of all natural numbers between 1 and 100 which are divisible by 3 is equal to 1683.

70. Given

a = 22 , $a_n$= -6 ,$S_n$ = 64

$a_n$ = -6

a + (n - 1) = -6

22 + (n - 1)d = -6

(n - 1)d = -28 ...(i)

$S_n$ = 64

$\frac{n}{2}(a + a_n)$ = 64

$\frac{n}{2}(22 - 6)$ = 64

n = $\frac{64 \times 2}{16}$ = 8

$\therefore$ Number of terms is 8.

From equation (i)

(n - 1)d = -28

7d = -28

$\therefore$ d = -4

Common difference = -4.

71. Since, the difference between the savings of two consecutive months is ₹20, therefore the series is an A.P.

Here, the savings of the first month is ₹50

First term, a = 50, Common difference, d = 20

No. of terms = no. of months

No. of terms, n = 12

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{12}{2}[2 \times 50 + (12-1)20]$

= 6[100 + 220]

= 6(320)

= 1920

After a year, Ramakali will save ₹1920.

Yes, Ramakali will be able to fulfill her dream of sending her daughter to school.

72. Here, (-4) + (-1) + 2 + 5 + ... + x = 437.

Now,

-1 - (-4) = -1 + 4 = 3
2 - (-1) = 2 + 1 = 3
5 - 2 = 3
Thus, this forms an A.P. with a = -4, d = 3, l = x
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow 437 = \frac{n}{2}[2 \times (-4) + (n - 1) \times 3]$
$\Rightarrow 874 = n[-8 + 3n - 3]$
$\Rightarrow 874 = n[3n - 11]$
$\Rightarrow 874 = 3n^2 - 11n$
$\Rightarrow 3n^2 - 11n - 874 = 0$
$\Rightarrow 3n^2 - 57n + 46n - 874 = 0$
$\Rightarrow 3n(n - 19) + 46(n - 19) = 0$
$\Rightarrow 3n + 46 = 0$ or n = 19
$\Rightarrow n = -\frac{46}{3}$ or n = 19
Numbers of terms cannot be negative or fraction.
$\Rightarrow$ n = 19
Now, $S_n = \frac{n}{2}[a + l]$
$\Rightarrow 437 = \frac{19}{2}[-4 + x]$
$\Rightarrow -4 + x = \frac{437 \times 2}{19}$
$\Rightarrow -4 + x = 46$
$\Rightarrow x = 50$

73. The sequence formed by the given numbers is 103,107,111,115, ...,999.
This is an AP in which a = 103 and d = (107 - 103) = 4.
Let the total number of these terms be n. Then,
$T_n = 999 \Rightarrow a + (n-1)d = 999$
$\Rightarrow 103 + (n -1) \times 4 = 999$
$\Rightarrow (n-1)\times 4 = 896 \Rightarrow (n-1) = 224 \Rightarrow n = 225$.
$\therefore$ middle term = $(\frac{n+1}{2})$th term = $(\frac{225+1}{2})$th term = 113th term.
$T_{113} = (a + 112d) = (103 + 112 \times 4) = 551$.
$\therefore T_{112} = (551 - 4) = 547$.
So, we have to find $S_{112}$ and $(S_{225} - S_{113})$.
Using the formula $S_m = \frac{m}{2}(a + l)$ for each sum, we get
$s_{112} = \frac{112}{2}(103 + 547) = (112 \times 325) = 36400$
$(S_{225} - S_{113}) = \frac{225}{2}(103 + 999) - \frac{113}{2}(103 + 551)$
$= (225 \times 551) - (113 \times 327)$
= 123975 - 36951 =87024.
Sum of all numbers on LHS of the middle term is 36400.
Sum of all numbers on RHS of the middle term is 87024.

74. The given AP is 3, 8, 13, ..., 253
Here, a = 3
d = 8 - 3 = 5
l = 253
Let the number of terms of the AP be n.
Term, nth term = l
$\Rightarrow 3 + (n - 1)5 = 253 \because a_n = a + (n - 1)d$
$\Rightarrow (n - 1)5 = 253 - 3$
$\Rightarrow (n - 1)5 = 250$
$\Rightarrow n - 1 = \frac{250}{5}$
$\Rightarrow n - 1 = 50$
$\Rightarrow n = 50 + 1$

$\Rightarrow$ n = 51

So, there are 51 terms in the given AP.

Now, 20th term from the last term

= (51 - 20 + 1)th term from the beginning

= 32th term from the beginning

= 3 + (32 - 1)5 $\because a_n = a + (n - 1)d$

= 3 + 155

= 158

Hence, the 20th term from the last term of the given AP is 158.

**Aliter.** Let us write the given AP in the reverse order.

Then the AP becomes 253, 248, 243, ...., 3

Here, a = 253

d = 248 - 253 = -5

Therefore, required term

= 20th term of the AP

= 253 + (20 - 1) (-5) $\because a_n = a + (n - 1)d$

= 253 - 95

= 158

Hence, the 20th term from the last term of the given AP is 158.

75. Here, (-4) + (-1) + 2 + 5 + ---- + x = 437.

Now,

-1 - (-4) = -1 + 4 = 3

2 - (-1) = 2 + 1 = 3

5 - 2 = 3

Thus, this forms an A.P. with a = -4, d = 3, l = x

Let their be n terms in this A.P.

Then,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 437 = \frac{n}{2}[2 \times (-4) + (n - 1) \times 3]$

$\Rightarrow$ 874 = n[-8 + 3n - 3]

$\Rightarrow$874 = n[3n - 11]

$\Rightarrow$874 = $3n^2$ - 11n

$\Rightarrow$$3n^2$ - 11n - 874 = 0

$\Rightarrow$$3n^2$ - 57n + 46n - 874 = 0

$\Rightarrow$3n(n - 19) + 46(n - 19) = 0

$\Rightarrow$3n + 46 = 0 or n = 19

$\Rightarrow n = -\frac{46}{3}$ or n = 19

Numbers of terms cannot be negative or fraction.

$\Rightarrow$ n = 19

Now, $S_n = \frac{n}{2}[a + l]$

$\Rightarrow 437 = \frac{19}{2}[-4 + x]$

$\Rightarrow -4 + x = \frac{437 \times 2}{19}$

$\Rightarrow -4 + x = 46$

$\Rightarrow x = 50$