Solution

AREAS RELATED TO CIRCLES

Class 10 - Mathematics

Section A

1.

(c) 32 m^2

Explanation:

The area of the segment $= \left(\frac{x^\circ}{360^\circ} \times \pi r^2\right) - \frac{bh}{2}$

= Area of the sector - Area of the triangle

= 44 - 12

= 32 m^2

2. **(a)** $\frac{60\sqrt{10}}{7}$ units

Explanation:

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$\Rightarrow \frac{49}{360} \times \pi r^2 = 100\pi$

$\Rightarrow \frac{49}{360} \times r^2 = 100$

$\Rightarrow r^2 = \frac{100 \times 360}{49}$

$\Rightarrow r = \frac{10 \times 6\sqrt{10}}{7}$

$\Rightarrow r = \frac{60\sqrt{10}}{7}$ units

3.

(d) $\frac{\alpha}{360} \times \pi \mathrm{R}^2$

Explanation:

$\frac{\alpha}{360} \times \pi \mathrm{R}^2$

4.

(c) $\frac{60}{\pi}$ cm

Explanation:

Given: Length of arc = 20 cm

$\Rightarrow \frac{\theta}{360^\circ} \times 2\pi$r= 20

$\Rightarrow \frac{60^\circ}{360^\circ} \times 2\pi$r= 20

$\Rightarrow \frac{\pi r}{3}$ = 20

$\Rightarrow r\left(\frac{\pi}{3}\right)$ = 20

$\Rightarrow r\left(\frac{\pi}{3}\right)$ = 20

$\Rightarrow$ r = $\frac{60}{\pi}$ cm

5.

(c) 21.99 m

Explanation:

The area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$

$= \frac{70^\circ}{360^\circ} \times \frac{22}{7} \times 6^2$

21.99 m

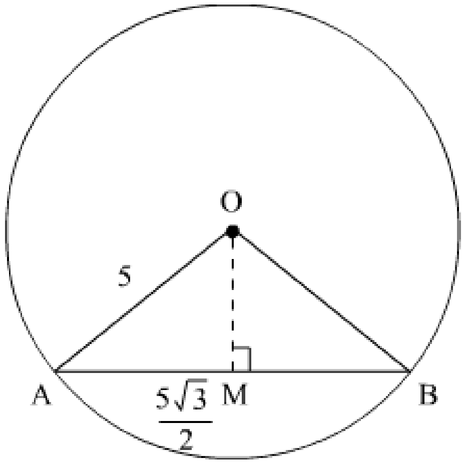
6.

(b) 72^{o}

Explanation:

It is given that area of the sector = 69.3 cm^2
and Radius = 10.5 cm
Now, Area of the sector $= \frac{\pi r^2 \theta}{360}$
$\Rightarrow \frac{\pi \times (10.5)^2 \times \theta}{360}$ = 69.3
$\Rightarrow \theta = \frac{69.3 \times 360 \times 7}{10.5 \times 10.5 \times 22}$ = 72^{o}
Therefore, Central angle of the sector = 72^{o}

7. **(a)** $\frac{25\pi}{3} \mathrm{cm}^2$
Explanation:
We have to find the area of the sector OAB.



We have,
$\mathrm{AM} = \frac{5\sqrt{3}}{2}$
So,
$\sin \angle \mathrm{AOM} = \frac{5\sqrt{3}}{2(5)}$
Hence,
$\angle \mathrm{AOM} = 60^{\circ}$
$\Rightarrow \angle AOB = 120^{\circ}$
Area of sector AOB $= \frac{120}{360} \times \pi \times 5^2 \mathrm{cm}^2 = \frac{25\pi}{3}$ cm^2

8.
(d) 8 cm
Explanation:
We have given length of the arc and area of the sector bounded by that arc and we are asked to find the radius of the circle.
We know that area of the sector $= \frac{\theta}{360} \times \pi r^2$.
Length of the arc $= \frac{\theta}{360} \times 2\pi r$
Now we will substitute the values.
Area of the sector $= \frac{\theta}{360} \times \pi r^2$
$20\pi = \frac{\theta}{360} \times \pi r^2$ ……..(1)
Length of the arc $= \frac{\theta}{360} \times 2\pi r$
$5\pi = \frac{\theta}{360} \times 2\pi r$ ……..(2)
$\frac{20\pi}{5\pi} = \frac{\frac{\theta}{360} \times \pi r^2}{\frac{\theta}{360} \times 2\pi r}$
$\frac{20}{5} = \frac{r^2}{2r}$
$\therefore 4 = \frac{r}{2}$
$\therefore$ r = 8
Therefore, radius of the circle is 8 cm.

9.

(d) $\frac{p}{720} \times 2\pi R^2$

Explanation:

Area of the sector of angle p of a circle with radius R

$= \frac{\theta}{360} \times \pi r^2 = \frac{p}{360} \times \pi R^2$

$= \frac{p}{2(360)} \times 2\pi R^2 = \frac{p}{720} \times 2\pi R^2$

10.

(b) 3696 cm^2

Explanation:

Clearly, each wiper sweeps a sector of a circle of radius 42 cm and sector angle 120°.

$\therefore$ Total area cleaned at each sweep = 2 $\times \frac{\theta}{360^\circ} \times \pi r^2$

= 2 $\times \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 42 \times$ 42 cm^2 = 3696 cm^2

11.

(d) 77 cm^2

Explanation:

For a minute hand, 60 minutes is equivalent to 360° and so 30 minutes will be 180°.

Area swept in 60 minutes is area of full circle.

So area swept in 30 minutes will be area of half circle.

Thus, area swept $= \frac{1}{2} \times \left(\frac{22}{7}\right) \times 7^2$ = 77 cm^2

12.

(c) 22 cm

Explanation:

Arc length $= \frac{2\pi r\theta}{360} = \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right)$ cm $= 22$cm

13. **(a)** $\sqrt{3\pi}$ cm

Explanation:

Let the length of side of square be x cm

Then area of square = x^2 cm^2

Area of sector of circle

= $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{90^\circ}{360^\circ} \times \pi r^2$ [$\because$ angle of square = θ = 90°]

$\therefore$ Shaded area = $\frac{\pi \times 4}{4}$ = π

According to question, Area of square = 3 $\times$ shaded area

$\Rightarrow 3\pi$ = x^2 $\therefore$ x = $\sqrt{3\pi}$ cm

14.

(b) 25.66 cm^2

Explanation:

In 1 minute, minute hand makes an angle of 6°

so, in 10 minutes, minute hand will make an angle of 60°

Area swept by minute hand in 10 minutes = Area of sector of radius 7 cm with angle 60°

Area of sector $= \frac{x^\circ}{360^\circ} \times \pi \times r^2$

So, Area swept by minute hand in 10 minutes = $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 7^2$

= 25.66 cm^2

15. **(a)** $\frac{x}{360} \times \pi r^2$

Explanation:

Area of a sector of a circle with radius r and making an angle of x^0
at the centre = $\frac{x}{360} \times \pi r^2$

16. **(a)** 308 cm^2

Explanation:

We know that the area A of a sector of a circle of radius r and central angle θ(in degrees) is given by
$A = \frac{\theta}{360} \times \pi r^2$
Here, r = 28 cm and θ = 45.
$\therefore A = \frac{45}{360} \times \pi \times (28)^2$ $= \frac{1}{8} \times \frac{22}{7} \times 28 \times 28\text{cm}^2$ = 308 cm^2

17.

(d) 148 cm^2

Explanation:

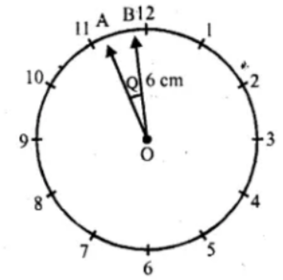
$Area\ of\ the\ sector = \frac{Arc\ length}{2\pi r} \times \pi r^2$
$= \frac{18.5}{2\pi(16)} \times [\pi 16^2]$
$\mathbf{= 148\ cm^2}$

18.

(c) 5.5 cm^2

Explanation:

Length of hour hand of a clock (r) = 6 cm
Time 11.20 am to 11.55 am = 35 minute = $\frac{35}{60}\text{h}$



$\therefore$ In 1 hour the hour hand rotates 30°.
Thus, central angle of the sector = $30 \times \frac{35}{60} = 17.5^\circ$
$\therefore$ Area of the sector swept by the hour hand = $\frac{17.5}{360} \times \frac{22}{7} \times 6 \times 6$ cm^2
$= \frac{2.5 \times 22}{10}\text{cm}^2 = 5.5\text{cm}^2$

19.

(c) 40882.8 m^2

Explanation:

The area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$
$= \frac{123^\circ}{360^\circ} \times \frac{22}{7} \times 138^2$
= 20441.4 m^2
Area covered by the man of the walking track in a day = 20441.4 + 20441.4
= 40882.8 m^2

20. **(a)** 10.90 cm

Explanation:

The area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$
$= \frac{50^\circ}{360^\circ} \times \frac{22}{7} \times 5^2$
= 10.90 cm

21.

(c) A is false but R is true.
Explanation:
Area of a sector = $\frac{\theta}{180^\circ} \times \pi r^2$ not $\frac{\theta}{360^\circ} \times 2\pi r$

22.

(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
Area of a segment = $\frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

23.

(d) A is false but R is true.
Explanation:
Area of a segment = Area of sector – Area of triangle
i.e., = $\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

24.

(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
Area of the sector = $\frac{\theta}{360} \times \pi r^2$
= $\frac{60}{360} \times \frac{22}{7} \times 6 \times 6$
= $\frac{132}{7} = 18\frac{6}{7}$ cm^2

25.

(d) A is false but R is true.
Explanation:
We have,
2πr = length of wire
$2 \times \frac{22}{7} \times 28$ = length of wire
length of wire = 176 cm
Now, the perimeter of square = 176
4a = 176
a = 44
Area of square = $(44)^2$ = 1936 cm^2

Section B

26. 359.33
Explanation:
Angle subtended in a minute = 6^o
Then, θ = angle subtended in 35 minutes = 35 $\times$ 6 = 210^o
Therefore, Area swept by the minute hand = Area of a sector = $\frac{\pi r^2\theta}{360^o} = \frac{22}{7} \times \frac{14\times14\times210}{360} = \frac{1078}{3}$ = 359.33 cm^2

27. 183.3
Explanation:
We know that:
Angle described by the minute hand in 60 minutes = 360^o
Therefore, Angle described by the minute hand in one minute = $\frac{360}{60}$ = 6^o

Angle described by the minute hand in 35 minutes = $(6 \times 35)^{o}$ = 210^{o}

Area swept by the minute hand in 35 minutes = Area of a sector of angle 210^{o} in a circle of radius 10 cm

$= \left\{\frac{210}{360} \times \frac{22}{7} \times (10)^2\right\}$ cm^2 = 183.3 cm^2

28. 189.97

Explanation:

We have, r = 16.5 km and θ = 80°.

Let A be the area of the sea over which the ships are warmed. Then,

$A = \frac{\theta}{360} \times \pi r^2 = \frac{80}{360} \times 3.14 \times 16.5 \times 16.5\ km^2$ = 189.97 km^2

29. 16.8

Explanation:

Here, θ = 30^{o}, l = arc = 8.8 cm

$\therefore \quad l = \frac{\theta}{360} \times 2\pi r$

$\Rightarrow 8.8 = \frac{30}{360} \times 2 \times \frac{22}{7} \times r$

$\Rightarrow r = \frac{8.8 \times 6 \times 7}{22}\text{cm}$ = 16.8 cm

30. 22

Explanation:

Diameter of a circle = 42 cm

$\Rightarrow$ Radius of a circle = r $= \frac{42}{2}$

= 21 cm

Central angle $= \theta = 60^\circ$

$\therefore$ Length of the arc $= \frac{2\pi r\theta}{360}$

$= \frac{2 \times \frac{22}{7} \times 21 \times 60^\circ}{360^\circ}\text{cm}$

= 22 cm

31. Radius of cirlce = 4cm

$\theta = 30^\circ$

$\therefore$ Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{30^\circ}{360^\circ} \times \pi \times 4 \times 4$

$= \frac{4\pi}{3} cm^2$

32. Let the radius of the large circle be R.

Then, we have

Area of large circle of radius R = Area of a circle of radius 5 cm+ Area of a circle of radius 12 cm

$\Rightarrow \pi R^2 = \left(\pi \times 5^2 + \pi \times 12^2\right)$

$\Rightarrow \pi R^2 = (25\pi + 144\pi)$

$\Rightarrow \pi R^2 = 169\pi$

$\Rightarrow R^2 = 169$

$\Rightarrow$ R = 13 cm

$\Rightarrow$ Diameter = 2R

= 26 cm

33. We have given the radius of the circle and angle subtended at the center of the circle.

r = 10 cm

$\theta = 108^\circ$

Now we will find the area of the sector,

Area of the sector $= \frac{\theta}{360} \times \pi r^2$

Substituting the values we get,

$= \frac{108}{360} \times \pi \times 10^2$ ...(1)

Now we will simplify the equation (1) as below,

$= \frac{3}{10} \times \pi \times 100$

$= 3 \times \pi \times 10$

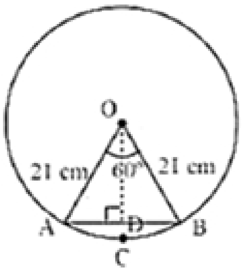
= 30π

Therefore, the area of the sector is 30π cm^2

34. Radius (r) of circle = 21 cm

Angle subtended by the given arc = 60^{o}

Length of an arc of a sector of angle $\theta = \frac{\theta}{360^\circ} \times 2\pi r$



Area of sector OACB = $\frac{60^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

= 231 cm^2

Length of arc ACB = $\frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$

= $\frac{1}{6} \times 2 \times 22 \times 3$

= 22 cm

35. We have

R = 6 cm

Length of the arc = $3\pi \mathrm{cm}$

as we know that arc length $= \frac{\theta}{360} \times 2\pi r$

Substituting the values we get,

$3\pi = \frac{\theta}{360} \times 2\pi \times 6$ ...(1)

Now we will simplify the equation (1) as below,

$3\pi = \frac{\theta}{360} \times 12\pi$

$3\pi = \frac{\theta}{30} \times \pi$

$3 = \frac{\theta}{30}$

$\theta = 90^\circ$

Therefore, the angle subtended at the centre of the circle is 90^{o}.

36. Radius of the circular piece of cardboard(r) = 3 cm

$\therefore$Two sectors of 90° each have been cut off

$\therefore$We get a semicircular cardboard piece

$\therefore$Perimeter of arc ACB

=$\frac{1}{2}(2\pi r) = \pi r$

= $\frac{22}{7} \times 3 = \frac{66}{7}$ = 9.428 cm

37. Long hand makes 24 rounds in 24 hours

Short hand makes 2 round in 24 hours

radius of the circle formed by long hand = 6 cm.

and radius of the circle formed by short hand = 4 cm.

Distance travelled by long hand in one round = circumference of the circle = $2 \times \pi \times r$

= $2 \times 6 \times \pi$

= 12π cm

Distance travelled by long hand in 24 rounds$= 24 \times 12\pi$

$= 288\pi$

Distance travelled by short hand in a round = $2 \times \pi \times r$

=$2 \times 4\pi$

= 8π cm

Distance travelled by short hand in 2 round

$= 2 \times 8\pi$

$= 16\pi$ cm

Sum of the distances = $288\pi + 16\pi = 304\pi$

$= 304 \times 3.14$

$= 954.56\ \text{cm}.$

Thus, the sum of distances travelled by their tips in 24 hours is 954.56 cm.

38. Angle described by the minute hand in 60 minutes = 360^0

$\therefore$Angle described by the minute hand in 56 minutes = $\left(\frac{360}{60} \times 56\right)^0 = 336^\circ$

$\therefore \theta = 336^\circ$ and r = 7.5 cm

$\therefore$Area swept by the minute hand in 56 minutes $= \left(\frac{\pi r^2\theta}{360}\right)$

$= \left(3.14 \times 7.5 \times 7.5 \times \frac{336}{360}\right) \text{cm}^2$

= 165 cm^2

39. We have given an angle subtended by an arc at the centre of the circle and radius of the circle.

r = 14 cm

$\theta = 90^\circ$

Now we will find the area of the minor sector,

Area of the minor sector = $\frac{\theta}{360} \times \pi r^2$

Substituting the values we get,

Area of the minor sector = $\frac{90}{360} \times \pi \times 14^2$ …… (1)

Now we will simplify the equation (1) as below,

Area of the minor sector = $\frac{1}{4} \times \pi \times 14^2$

Area of the minor sector = $\frac{1}{4} \times \pi \times 14 \times 14$

Area of the minor sector = $\pi \times 7 \times 7$

Area of the minor sector = 49π

Therefore, area of the minor sector is 49π cm^2.

40. Angle described by the minute hand in 60 minutes = 360°

Angle described by the minute hand in 20 minutes $= \left(\frac{360}{60} \times 20\right)$

= 120°

Required area swept by the minute hand in 20 minutes

= Area of the sector (with r = 15 cm and $\theta = 120^\circ$)

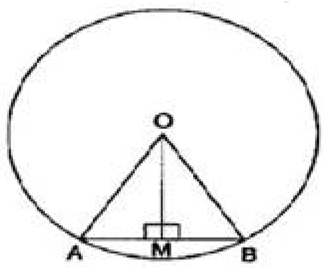
$= \left(\frac{\pi r^2\theta}{360^\circ}\right) \text{cm}^2$

$= \left(3.14 \times 15 \times 15 \times \frac{120^\circ}{360^\circ}\right)$

= 235.5 cm^2

Section C

41.



Given, r = 21 cm and θ = 60^o

i. Length of arc = {tex}=\;\frac\theta{360}2\mathrm{πr}\;=\frac{60}{360}\times2\times\frac{22}7\times21\;=\;22\;\mathrm{cm}{/tex}
ii. Area of the sector = {tex}=\;\frac\theta{360}\mathrm{πr}^2\;=\frac{60}{360}\times\frac{22}7\times21\times21\;=\;231\;\mathrm{cm}^2{/tex}
iii. Area of segment formed by corresponding chord = Area of the sector - Area of $\triangle$OAB

= {tex}\frac\theta{360}\mathrm{πr}^2{/tex} - Area of $\triangle$OAB

⇒ Area of segment = 231 - Area of $\triangle$OAB .... (1)

In right angled triangle OMA and OMB,

OM = OB [Radii of the same circle]

OM = OM [Common]

$\therefore$ $\triangle$OMA $\cong$ $\triangle$OMB [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\therefore$ M is the mid-point of AB and $\angle$AOM = $\angle$BOM

$\Rightarrow$ $\angle$AOM = $\angle$BOM = $\frac{1}{2}\angle$AOB =$\frac{1}{2}$ $\times$ 60^o = 30^o

Therefore, in right angled triangle OMA,

$\cos 30^o$ = $\frac{OM}{OA}$

$\Rightarrow$ $\frac{\sqrt{3}}{2}$= $\frac{OM}{21}$

$\Rightarrow$ OM = $\frac{21\sqrt{3}}{2}$ cm

Also, $\sin 30^o$ = $\frac{AM}{OA}$

$\Rightarrow$ $\frac{1}{2}$= $\frac{AM}{21}$

$\Rightarrow$ AM = $\frac{21}{2}$ cm

$\therefore$ AB = 2 AM = 2 $\times$ $\frac{21}{2}$ = 21 cm

$\therefore$ Area of $\triangle$OAB = $\frac{1}{2}$ $\times$ AB $\times$ OM = $\frac{1}{2}$ $\times$ 21 $\times$ $\frac{21\sqrt{3}}{2}$ =$\frac{441\sqrt{3}}{4}$ cm ^{2}

Using eq. (1),

Area of segment formed by corresponding chord = [231 - $\frac{441\sqrt{3}}{4}$] cm ^{2}

= 40.05 cm^2

42. Radius OA = OB = 2m

$\angle$AOB = 90°

In $\angle$ AOB, by pythagoras theorem

AB^2 = OA^2 + OB^2

$\Rightarrow$ AB^2 = 2^2 + 2^2

$\Rightarrow$ AB^2 = 8

$\Rightarrow$ AB = $\sqrt{8} = 2\sqrt{2}$m

Area of $\triangle$AOB = $\frac{1}{2}\times$ OA $\times$ OB = $\frac{1}{2}\times$ 2 $\times$ 2 = 2m^2

Again area of $\triangle$AOB = $\frac{1}{2}\times$ AB $\times$ OC = $\frac{1}{2}$ $\times$ $2\sqrt{2}\times$ OC = $\sqrt{2}\times$ OC m^2

$\therefore \sqrt{2}\times$ OC = 2

$\Rightarrow$ OC = $\frac{2}{\sqrt{2}} = \sqrt{2}$ m

i. $\therefore$ height of tunnel = DO + OC

= (2 + $\sqrt{2}$)m

ii. perimeter of cross- section = AB + area of major arc AB

$= 2\sqrt{2} + \frac{270}{360} \times 2\pi r$

= $2\sqrt{2} + \frac{3}{4} \times 2\pi\times$ 2

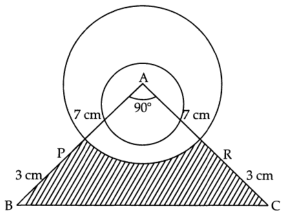
$= (2\sqrt{2} + 3\pi)m$

iii. the area of cross-section = Area of major sector + Area of $\triangle$AOB

= $\frac{270^\circ}{360} \times \pi(2)^2$ + 2

= $\frac{3}{4}\pi\times$ 4 + 2

= (3π + 2) m^2

43. 

Base = 7 + 3 = 10cm and height = 7 + 3 =10 cm

From the given figure

Area of right-angled$\triangle ABC = \frac{1}{2} \times base \times height$

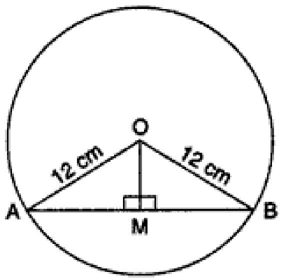
$= \frac{1}{2} \times 10 \times 10$

$= 50$ cm^2

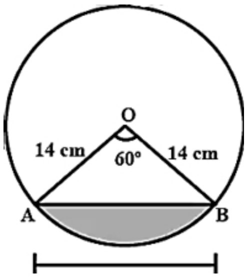
Area of quadrant APR of the circle of radius 7 cm

$= \frac{1}{4} \times \pi \times (7)^2$
Area of quadrant$= \frac{1}{4} \times \frac{22}{7} \times 49 = 38.5\text{cm}^2$
Area of base PBCR = Area of $\triangle$ABC - Area of quadrant APR
= $50 - 38.5$ = 11.5 cm^2.
So, Area of shaded portion is 11.5 cm^2.

44. Radius of circle = 14
$\angle AOB = 60^\circ$
$\therefore$ Area of minor segment =Area of sector OAB- Area of ΔOAB
$= \frac{\theta}{360^\circ} \pi r^2 - \frac{1}{2} r^2 \sin\theta$
$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14 \times \sin 60^\circ$
$= \frac{1}{6} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$
$= 14 \times 14 \left[\frac{11}{21} - \frac{1.73}{4}\right]$
$= 14 \times 14 \left[\frac{44-36.33}{84}\right]$
$= 14 \times 14 \times \frac{7.67}{84} = \frac{1503.32}{84} = 17.89\text{cm}^2$

45. 

r = 12 cm, θ = 120°
$\therefore$ Area of the corresponding sector of the circle = $\frac{\theta}{360^\circ} \pi r^2 = \frac{120}{360^\circ} \times 3.14 \times 12 \times 12$ = 150.72 cm^2
Area of $\triangle$AOB
Draw OM $\perp$ AB
In right triangle OMA and OMB,
OA = OB .... Radii of the same circle
OM = OM .... Common side
$\therefore$ $\triangle$OMA $\cong$ $\triangle$OMB ....RHS congruence criterion
$\therefore$ AM = BM .... CPCT
$\Rightarrow$ AM = BM = $\frac{1}{2}$AB
and $\angle$AOM = $\angle$BOM [CPCT]
$\Rightarrow$ $\angle$AOM = $\angle$BOM = $\frac{1}{2}\angle$AOB
= $\frac{1}{2} \times 120^\circ = 60^\circ$
$\therefore$ In right triangle OMA,
cos60° = $\frac{OM}{OA}$
$\Rightarrow$ $\frac{1}{2} = \frac{OM}{12}$
$\Rightarrow$ OM = 6 cm
sin60° = $\frac{AM}{OA}$
$\Rightarrow$ $\frac{\sqrt{3}}{2} = \frac{AM}{12}$
$\Rightarrow$ AM = $6\sqrt{3}$ cm
$\Rightarrow$ 2AM = $12\sqrt{3}$ cm
$\Rightarrow$ AB =$12\sqrt{3}$ cm
$\therefore$ Area of $\triangle$AOB =$\frac{1}{2} \times$ AB $\times$ OM
= $\frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3}$ cm^2
= 36 $\times$ 1.73 cm^2 = 62.28 cm^2
So, Area of the corresponding segment of the circle = Area of the correspoding sector of circle - Area of $\triangle$AOB
= 150.72 - 62.28 = 88.44 cm^2

46. 

In $\triangle$AOB, OA = OB = 14 cm

$\therefore$, $\angle$OBA = $\angle$OAB [angles opposite to equal side]

$\angle$AOB = 60^o

$\therefore$ $\angle$AOB + $\angle$OAB = $\frac{120^\circ}{2}$ = 60^o = $\angle$OBA

All angles are 60^o, therefore $\triangle$OAB is an equilateral $\triangle$

Now, area of minor segment = area of sector - ar$\triangle$OAB

$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{\sqrt{3}}{4}$ $(OA)^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$

$= \frac{308}{3} - 49\sqrt{3} = \frac{308 - 147\sqrt{3}}{3}$

= 17.80 cm^2

47. 

We know that, in 60 min, angle swept by minute hand = 360°

In 1 min, angle swept by minute hand = $\frac{360^\circ}{60^\circ}$

Nor minutes in (6: 05 am to 6:40) = 35 min,

So, Angle swept by minute hand = $\frac{360^\circ}{60^\circ} \times 35^\circ = (6 \times 35)^\circ$

Given that, length of minute hand(r) = 5 cm

$\therefore$ Area of sector AOBA with $\angle O = (6 \times 35)^\circ$ is

$= \frac{\pi r^2}{360^\circ} \times \angle O$

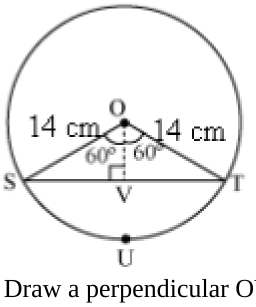
$= \frac{22}{7} \times \frac{(5)^2}{360} \times 6 \times 35$

$= \frac{22}{7} \times \frac{5 \times 5}{360} \times 6 \times 35$

$= \frac{22 \times 5 \times 5 \times 5}{60} = \frac{22 \times 5 \times 5}{12}$

$= \frac{11 \times 5 \times 5}{6} = \frac{275}{6} = 45\frac{5}{6}\mathrm{cm}^2$

Hence, the area swept by the minute hand is $45\frac{5}{6}$ cm^2

48. 

Draw a perpendicular OV on chord ST. It will bisect the chord ST so that SV = VT

In triangle OVS,

$\frac{OV}{OS}$ = cos 60°

$\frac{OV}{14} = \frac{1}{2}$

OV = 7

$\frac{SV}{SO} = \sin 60^0 = \frac{\sqrt{3}}{2}$

$\frac{SV}{14} = \frac{\sqrt{3}}{2}$

SV = $7\sqrt{3}$

ST = 2SV = $2 \times 7\sqrt{3} = 14\sqrt{3}$

Area of triangle OST = $\frac{1}{2} \times ST \times OV$

= $\frac{1}{2} \times 14\sqrt{3} \times 7$

= $49\sqrt{3}$ = 84.77 cm^2

Area of sector OSUT = $\frac{120^0}{360^0} \times \pi(14)^2$

= $\frac{1}{3} \times \frac{22}{7} \times 196$ = 205.33 cm^2

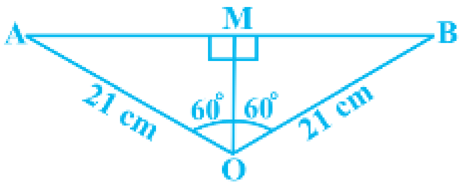
=Area of segment SUT = Area of sector OSUT - Area of $\triangle$OST

= 205.33 - 84.77 = 120.56 cm^2

49. Area of the segment AYB = Area of sector OAYB - Area of Δ OAB

Now, area of the sector OAYB = $\frac{120}{360} \times \frac{22}{7} \times 21 \times 21\text{cm}^2 = 462\text{cm}^2$

For finding the area of Δ OAB, draw OM ⊥ AB as shown in Fig.



Note that OA = OB. Therefore, by RHS congruence, Δ AMO ≅ Δ BMO.

So, M is the mid-point of AB and $\angle\text{AOM} = \angle\text{BOM} = \frac{1}{2} \times 120^\circ = 60^\circ$

Let OM = x cm

So, from Δ OMA, $\frac{\text{OM}}{\text{OA}} = \cos 60^\circ$

or, $\frac{x}{21} = \frac{1}{2} \quad \left(\cos 60^\circ = \frac{1}{2}\right)$

or, $x = \frac{21}{2}$

So, $\text{OM} = \frac{21}{2}\text{cm}$

Also, $\frac{\text{AM}}{\text{OA}} = \sin 60^\circ = \frac{\sqrt{3}}{2}$

So, $\text{AM} = \frac{21\sqrt{3}}{2}\text{cm}$

Therefore, $AB = 2AM = \frac{2\times21\sqrt{3}}{2}\text{cm} = 21\sqrt{3}\text{cm}$

So, area of Δ OAB $= \frac{1}{2}\text{AB} \times \text{OM} = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}\text{cm}^2$

$= \frac{441}{4}\sqrt{3}\text{cm}^2$

Therefore, area of the segment AYB = $\left(462 - \frac{441}{4}\sqrt{3}\right)$ cm^2 [From (1), (2) and (3)]

$= \frac{21}{4}(88 - 21\sqrt{3})\text{cm}^2$

50. Given,

AB is the diameter of the circle

$\angle COB = \theta$

According to question

Area of minor segment cut off by AC = 2 × Area of sector BOC

$\Rightarrow \quad \frac{\angle AOC}{360^\circ} \times \pi r^2 - \frac{1}{2}r^2 \sin \angle AOC = 2 \times \frac{\theta}{360^\circ} \times \pi r^2$

$\Rightarrow \quad \frac{180-\theta}{360^\circ} \times \pi r^2 - \frac{1}{2}r^2 \sin(180 - \theta) = 2 \times \frac{\theta}{360^\circ}\pi r^2$

$\Rightarrow \quad \frac{180-\theta}{360^\circ} \times \pi r^2 - 2 \times \frac{\theta}{360^\circ}\pi r^2 = \frac{1}{2}r^2 \sin\theta$

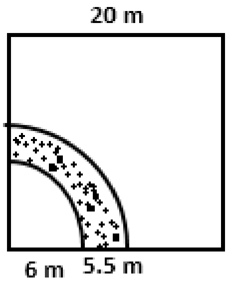
$\Rightarrow \quad \pi r^2 \left[\frac{180-\theta}{360^\circ} - \frac{2\theta}{360^\circ}\right] = \frac{1}{2}r^2 \sin\theta$

$\Rightarrow \quad \pi r^2 \left[\frac{180-\theta-2\theta}{360^\circ}\right] = \frac{1}{2}r^2 \sin\theta$

$\Rightarrow \quad \pi \left[\frac{180-3\theta}{360^\circ}\right] = \frac{1}{2}\sin\theta$ [Cancel r^2 from both side]

$\Rightarrow \quad \pi \left[\frac{180}{360^\circ} - \frac{3\theta}{360^\circ}\right] = \frac{1}{2} \times 2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$ [We know that $\sin 2\theta = 2\sin\theta\cos\theta$]

$\Rightarrow \quad \pi \left[\frac{1}{2} - \frac{\theta}{120^\circ}\right] = \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$ Hence Proved

51. 

Shaded portion indicates the increased area.

$\therefore$ Required area $= \frac{1}{4}\pi(6+5.5)^2 - \frac{1}{4}\pi(6)^2$

$= \frac{1}{4}\pi(11.5)^2 - \frac{1}{4}\pi(6)^2$

$= \frac{1}{4} \times \frac{22}{7}\left(11.5^2 - 6^2\right)$

$= \frac{11}{14}(11.5+6)(11.5-6)$

$= \frac{11}{14}(17.5)(5.5)$

= 75.625 cm^2

52. Given that

If Angle subtended at centre is 54°, then length of the Arc = 16.5 cm

so if Angle subtended at centre is 1°, then length of the Arc = (16.5/54)cm

so if Angle subtended at centre is 360° (full circle)

then length of Arc that is circumference = (16.5/54) $\times$ 360°

= 110 cm

So Circumference of circle = 110 cm

circumference = $2\pi r$

$\Rightarrow$ 110 = $2\pi r$

$\Rightarrow$ 110 = 2 $\times$ (22/7) $\times$ r

$\Rightarrow$ r = (110 $\times$ 7)/(2 $\times$ 22) = 17.5 cm

so Radius = 17.5 cm

Area of circle = πr^2 = 22/7 $\times$ 17.5 $\times$ 17.5 = 962.5 cm^2

Hence radius = 17.5 cm,

circumference = 110 cm and

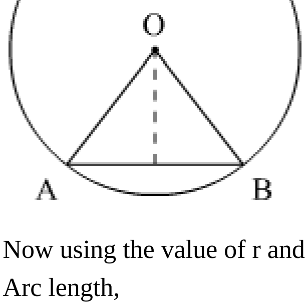
area = 962.5 cm^2

53. We know that the arc length l and area A of a sector of circle at an angle θ of radius r is given by

$l = \frac{\theta}{360^\circ} \times 2\pi r$ and area, $A = \frac{\theta}{360^\circ}\pi r^2$.

Let OAB is the given sector.

It is given that OA = 21 cm and angle $\angle AOB = 120^\circ$.



Now using the value of r and θ, we will find the value of l and A,

Arc length,

$l = \frac{120^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21\text{cm}$

= 44 cm

Area of sector,

$A = \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$

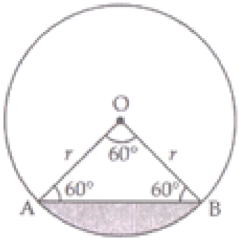
= 462 cm^2

54. Shaded region, is a minor segment.

In $\triangle$OAB,

θ = 60°, OA = OB = r = 14 cm

Therefore, Area of minor segment = Area of sector – Area of △ OAB



In △OAB,

OA = OB [Radii same circle]

Let ∠A = ∠B = x° [since Angles opposite to equal sides are equal]

∠O + ∠A + ∠B = 180° [using Angle sum property of a triangle]

⇒ 60° + x + x = 180°

⇒ 2x = 180° – 60°

⇒ x = $\frac{120^\circ}{2}$

⇒ x = 60°

Therefore, △OAB is an equilateral △ with side 14 cm.

Area of minor segment = $\frac{\pi r^2\theta}{360^\circ}$ - Area of ΔOAB

Therefore, Area of minor segment = $\frac{\pi r^2\theta}{360} - \frac{\sqrt{3}}{4}r^2$

$= \frac{22\times14\times14\times60^\circ}{7\times360^\circ} - \frac{\sqrt{3}}{4} \times 14 \times 14$

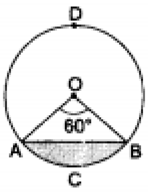
$= \frac{22\times14}{3} - 49\sqrt{3} = \left(\frac{308}{3} - 49\sqrt{3}\right)$ cm^2

= (102.666 – 84.870) cm^2 = 17.796 cm^2

55. Let ACB be the given arc subtending an angle of 60° at the centre.

Then, r = 21 cm

and θ = 60°.



i. Length of the arc ACB $= \frac{2\pi r\theta}{360}$ cm

$= \left(2 \times \frac{22}{7} \times 21 \times \frac{60}{360}\right)$ cm

= 22 cm.

ii. Area of the sector OACBO $= \frac{\pi r^2\theta}{360}$ cm^2

$= \left(\frac{22}{7} \times 21 \times 21 \times \frac{60}{360}\right)$ cm^2

= 231 cm^2.

iii. Area of the minor segment ACBA

= (area of the sector OACB) - (area of the $\triangle OAB$)

$= \left(231 - \frac{1}{2}r^2 \sin\theta\right)$ cm^2

$= \left[231 - \left(\frac{1}{2} \times 21 \times 21 \times \sin 60^\circ\right)\right]$ cm^2

$= \left(231 - \frac{1}{2} \times 21 \times 21 \times \frac{\sqrt{3}}{2}\right)$ cm^2

$= \left(231 - \frac{1}{4} \times 441 \times 1.72\right)$ cm^2

= (231 - 189.63) cm^2

= 41.37 cm^2

iv. Area of the major segment BDAB

= (area of the circle) - (area of the minor segment)

$= \left\{\left(\frac{22}{7} \times 21 \times 21\right) - 41.37\right\}$ cm^2

= (1386 - 41.37) cm^2

= 1344.63 cm^2

Section D

56. i. Area of square ABCD = $(Side)^2$,

= $(8)^2$

= 64 cm^2

ii. $\triangle$ABC, $\angle$B = 90^o

$\therefore$ $AC^2 = AB^2 + BC^2 = 2AB^2$

AC = $\sqrt{2}$ AB

Diagonal AC = $8\sqrt{2}$ cm

iii. Area of Sector OPRQO

= $\frac{\theta}{360}\pi r^2$

= $\frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 4 \times 4$ cm^2

[Radius of inscribed circle = $\frac{1}{2}$ side of square]

Area of sector OPRQO = $\frac{88}{7}$

= $12\frac{4}{7}$ cm^2

OR

Area of circle = πr^2

= $\frac{22}{7} \times (4)^2$

= $\frac{352}{7}$ cm^2

$\therefore$ Required Area = $64 - \frac{352}{7}$

= $\frac{448-352}{7}$

= $\frac{96}{7}$ cm^2

= $13\frac{5}{7}$ cm^2

57. i. Area of sector ODCO = $\frac{22}{7} \times 7 \times 7 \times \frac{90}{360} = \frac{77}{2}$ or 38.5

$\therefore$ Area of sector ODCO is $\frac{77}{2}$ or 38.5 cm^2

ii. Area of ($\triangle$AOB) = $\frac{1}{2} \times 10 \times 10 = 50$

$\therefore$ Area of ($\triangle$AOB) is 50 cm^2

iii. Required cost = (50 - 38.5) $\times$ 20

= 230

$\therefore$ required cost is ₹ 230.

OR

Length of are CD = $\frac{90}{360} \times 2 \times \frac{22}{7} \times 7$

$\therefore$ Length of arc CD is 11 cm.

58. i. Total area of two segments = $\frac{1}{4}\pi r^2$ - $\frac{1}{2}r^2$ + $\frac{1}{6}\pi r^2$ - $\frac{\sqrt{3}}{4}r^2 = 256\frac{2}{3}$

ii. $\left(\frac{1}{4}\pi - \frac{1}{2} + \frac{1}{6}\pi - \frac{\sqrt{3}}{4}\right)r^2 = \frac{770}{3}$

$\Rightarrow$ r = 26.1 cm (approx.)

iii. Area of segment with red roses = $\frac{1}{4}\pi r^2$ - $\frac{1}{2}r^2$ sq m

= 194.63 sq m (approx.)

OR

Area of segment with yellow roses = $\frac{1}{6}\pi r^2$ - $\frac{\sqrt{3}}{4}r^2$ sq m

= 62.03 sq m (approx.)

59. i. Let r and R be the radii of each smaller circle and larger circle respectively.

d and D are diameter of each smaller circle and larger circle respectively.

We have, d = $\frac{1}{4}$D

$\Rightarrow$ r = $\frac{1}{4}$R $\Rightarrow$ r = $\frac{1}{4} \times 16 \Rightarrow$ r = 4 cm

Area of smaller circle = πr^2

= $\frac{22}{7} \times 4 \times 4$ = 50.28 cm^2

ii. Let r and R be the radii of each smaller circle and larger circle respectively.

We have, d = $\frac{1}{4}$D

$\Rightarrow$ r = $\frac{1}{4}$R $\Rightarrow$ r = $\frac{1}{4} \times 16 \Rightarrow$ r = 4 cm

Area of larger circle = $\pi r 2$

= $\frac{22}{7} \times 16 \times 16$ = $\frac{5632}{7}$ = 804.57 cm^2

iii. Let r and R be the radii of each smaller circle and larger circle respectively.

We have, d = $\frac{1}{4}$D

$\Rightarrow$ r = $\frac{1}{4}$R $\Rightarrow$ r = $\frac{1}{4} \times 16 \Rightarrow$ r = 4 cm

Area of quadrant of a smaller circle

= $\frac{1}{4} \times 50.28$ = 12.57 cm^2

OR

Let r and R be the radii of each smaller circle and larger circle respectively.

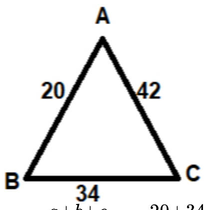
We have, d = $\frac{1}{4}$D

$\Rightarrow$ r = $\frac{1}{4}$R $\Rightarrow$ r = $\frac{1}{4} \times 16 \Rightarrow$ r = 4 cm

Area of black colour region = Area of larger circle - Area of 4 smaller circles

= 804.57 - 4 $\times$ 50.28 = 603.45 cm^2

60. i.



S = $\frac{a+b+c}{2} = \frac{20+34+42}{2}$

= 48 m

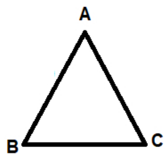
Area of $\triangle$ lawn = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{48(48-20)(48-34)(48-42)}$

$= \sqrt{48 \times 28 \times 14 \times 6}$

= 336 m^2

ii.



As $\angle A + \angle B + \angle C$ = 180^o

Area grazed by horses = $\frac{180}{360} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

= 77 m^2

iii. Area which is not grazed = 336 - 77

= 259 m^2

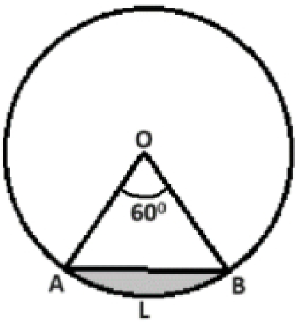
OR

Area grazed = $\frac{120}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{154}{3}$

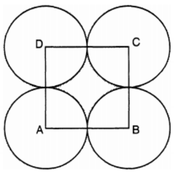
= 51.34 m^2

Section E

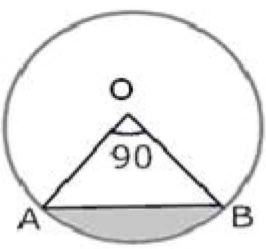
61. 

i. Length of the arc AB $= 2 \times \frac{22}{7} \times 21 \times \frac{60}{360}$
= 22 cm

ii. Area of sector OALB $= \frac{22}{7} \times 21 \times 21 \times \frac{60}{360} = 231 \text{ cm}^2$
Area of $\triangle\text{OAB} = \frac{\sqrt{3}}{4} \times 21 \times 21 = \frac{441\sqrt{3}}{4} \text{ cm}^2$
Area of minor segment $= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$
or (231 - 190.95) = 40.05 cm^2

62. 

Let r cm be the radius of each circle.
Area of square - Area of 4 sectors = $\frac{24}{7}\text{cm}^2$
$(\text{side})^2$ - {tex}4\left[\frac\theta{360}\mathrm{πr}^2\right]{/tex} = $\frac{24}{7}\text{cm}^2$
or, $(2r)^2 - 4\left(\frac{90^\circ}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$
or, $(2r)^2 - 4\left(\frac{1}{4^\circ} \times \pi r^2\right) = \frac{24}{7}$
or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$
or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$
or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$
or, $6r^2 = 24$
or, $r^2 = 4$
or, $r = \pm 2$
or, Radius of each circle is 2 cm (r cannot be negative)

63. 

i. Area of minor sector = {tex}\frac\theta{360}\mathrm{πr}^2{/tex}
$= \frac{90}{360}(3.14)(10)^2$
$= \frac{1}{4} \times 3.14 \times 100$
$= \frac{314}{4}$
= 78.50 = 78.5 cm^2

ii. Area of major sector = Area of circle - Area of minor sector
= $\pi(10)^2$ - $\frac{90}{360}$ $\pi(10)^2$ = 3.14 (100) - $\frac{1}{4}$(3.14) (100)
= 314 - 78.50 = 235.5 cm^2

iii. We know that area of minor segment
= Area of minor sector OAB - Area of ΔOAB
$\because$ area of $\triangle\text{OAB} = \frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\,(\because \angle AOB = 90^\circ)$

Area of sector = $\frac{\theta}{360}\pi r^2$

= $\frac{1}{4}$(3.14) (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5 cm^2

iv. Area of major segment = Area of the circle - Area of minor segment

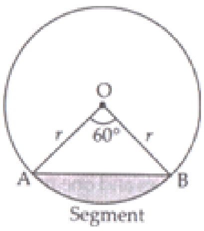
= $\pi(10)^2$ - 28.5

= 100(3.14) - 28.5

= 314 - 28. 5 = 285.5 cm^2

64. Area of minor segment = Area of sector – Area of $\triangle$OAB

In $\triangle$OAB,



θ = 60°

OA = OB = r = 12 cm

$\angle$B = $\angle$A = x [$\angle$s opp. to equal sides are equal]

$\Rightarrow$ $\angle$A + $\angle$B + $\angle$O = 180°

$\Rightarrow$ x + x + 60° = 180°

$\Rightarrow$ 2x = 180° – 60°

$\Rightarrow$ x = $\frac{120^\circ}{2}$ = 60°

$\therefore$ $\triangle$OAB is equilateral $\triangle$ with each side (a) = 12 cm

Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

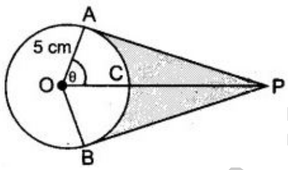
Area of minor segment = Area of the sector – Area of $\triangle$OAB

$= \frac{\pi r^2\theta}{360^\circ} - \frac{\sqrt{3}}{4}a^2$

$= \frac{3.14\times12\times12\times60^\circ}{360^\circ} - \frac{\sqrt{3}}{4}\times 12\times 12$

= 6.28 $\times$ 12 – $36\sqrt{3}$

$\therefore$ Area of minor segment = (75.36 – $36\sqrt{3}$) cm^2.

65. 

$\cos\theta = \frac{1}{2}$ or, $\theta = 60^\circ$

Reflex $\angle AOB = 120^\circ$

$\therefore$ ADB = $\frac{2\times3.14\times5\times240}{360}$ = 20.93 cm

Hence length of elastic in contact = 20.93 cm

Now, AP = $5\sqrt{3}\mathrm{cm}$

a ($\triangle$OAP) = $\frac{1}{2}\times$ base $\times$ height = $\frac{1}{2}\times 5\times 5\sqrt{3} = \frac{25\sqrt{3}}{2}$

Area ($\triangle$OAP + $\triangle$OBP) = 2$\times$ $\frac{25\sqrt{3}}{2} = 25\sqrt{3}$ = 43.25 cm^2

Area of sector OACB = {tex}\;\frac{\mathrm\theta}{360}\times\mathrm{πr}^2\;{/tex}

= $\frac{25\times3.14\times120}{360}$ = 26.16 cm^2

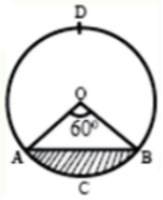
Shaded Area = 43.25 - 26.16 = 17.09 cm^2

66. Let ACB be the given arc subtending an angle of 60^o at the centre.

$\because$ θ = 60^o and OA = OB.

$\therefore$ $\triangle$OAB is an equilateral triangle.

Here, r = 14cm and θ = 60^o.



Area of the minor segment ACBA = (Area of the sector OACBO) - (Area of $\triangle$OAB)

$\frac{\pi r^2\theta}{360^\circ} - \frac{\sqrt{3}}{4}r^2$

= $\frac{22}{7} \times 14 \times 14 \times \frac{60^o}{360^o}$ - $\frac{\sqrt{3}}{4} \times 14 \times 14$

= $\frac{308}{3}$ - $49\sqrt{3}$

= 17.79 cm^2

Area of the major segment BDAB = Area of circle - Area of minor segment ACBA

= πr^2 - 17.79

= $\frac{22}{7} \times 14 \times 14$ - 17.79

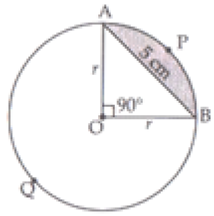
= 616 - 17.79

= 598.21 cm^2

67. Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, we are given that θ = 90°

Area of $\triangle$OAB = $\frac{1}{2}$Base $\times$ Altitude = $\frac{1}{2}$r $\times$ r = $\frac{1}{2}r^2$



Area of minor segment APB

$= \frac{\pi r^2\theta}{360^\circ}$ – Area of $\triangle$AOB

$= \frac{\pi r^2 90^\circ}{360^\circ} - \frac{1}{2}r^2$

$\Rightarrow$ Area of minor segment = $\left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$ …(i)

Area of major segment AQB = Area of circle – Area of minor segment

= πr$^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$

$\Rightarrow$ Area of major segment AQB $= \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right]$ …(ii)

Difference between areas of major and minor segment

$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$

$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$

$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$

In right $\triangle$OAB,

$r^2 + r^2 = AB^2$

$\Rightarrow 2r^2 = 5^2$

$\Rightarrow r^2 = \frac{25}{2}$

Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right]$ cm^2

68. Area of sector AOB = $\frac{22}{7} \times 7 \times 7 \times \frac{90}{360}$

= $\frac{77}{2}$ cm^2

Area of $\triangle$AOB = $\frac{1}{2} \times 7 \times 7 = \frac{49}{2}$ cm^2

$\therefore$ Shaded area = $\frac{77}{2}$ - $\frac{49}{2}$ = $\frac{28}{2}$ = 14 cm^2

Length of arc AB = $2 \times \frac{22}{7} \times 7 \times \frac{90}{360}$ = 11 cm

69. Assuming AOB to be a straight line and hence the diameter of the circle.

$\Rightarrow \angle ACB = 90^o$

Then in $\triangle$ ACB, $AC^2 + BC^2 = 28^2 + 21^2 = (35)^2 = AB^2$

$\therefore$ AB = 35 cm is the diameter and $\Rightarrow$ r = $\frac{35}{2}$ cm

Area of shaded region

= area of quadrant + ($\frac{1}{2} \times \pi r^2$ - area of $\triangle$ACB)

$= \left(\frac{3}{4} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}\right) - \frac{1}{2} \times 28 \times 21$

= 721.9 - 294 = 427.9 (approx)

70. Area of minor segment $= \left(\frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right) - \left(\frac{1}{2} \times 14 \times 14\right)$

= 154 - 98 = 56 sq. cm.

Area of major segment $= \left(\frac{22}{7} \times 14 \times 14\right) - 56 = 560$ sq. cm

71. i. $\triangle \mathrm{OAB}$ is an equilateral triangle.

$\therefore$ AB = OA = 10 cm

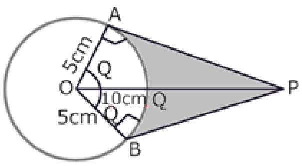
ii. Area of segment APB (A_1) = 3.14 $\times$ 100 $\times$ $\frac{60}{360} - \frac{1.73}{4} \times 100$

= 9.08 cm^2 approx.

Area of sector OBC (A_2) = 3.14 $\times$ 100 $\times$ $\frac{120}{360}$

= 104.67 cm^2 approx.

Area of shaded region = $A_1 + A_2$ = 113.75 cm^2 approx.

72. 

$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta = 60^o$

$\Rightarrow \angle AOB = 2 \times \theta = 120^o$

$\therefore$ ARC AB = {tex}\;\frac{120\times\;2\times\mathrm\pi\times5}{360}cm=\;\frac{10\mathrm\pi}3cm\;\left[\because l=\frac\theta{360}\times2\mathrm{πr}\right]{/tex}

Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim - length of arc AB

= $2\pi \times 5$ cm - $\frac{10\pi}{3}$cm

= $\frac{20\pi}{3}$cm

Now, the area of sector OAQB = {tex}\;\frac{120\times\mathrm\pi\times5\times5}{360}cm^2=\;\frac{25\mathrm\pi}3cm^2\;\left[\because Area=\frac\theta{360}\times\mathrm{πr}^2\right]{/tex}

Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = $25\sqrt{3}$ cm^2

$\left[\because AP = \sqrt{100\ - 25} = \sqrt{75}\ = 5\sqrt{3}\ cm\right]$

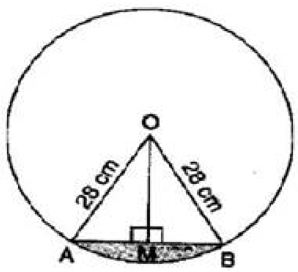
Hence, shaded area $= 25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3}\ - \pi]\ cm^2$

73. r = 28 cm and θ = $\frac{360}{6} = 60^\circ$

Area of minor sector = {tex}\frac\theta{360}\mathrm{πr}^2\;=\frac{\displaystyle60}{\displaystyle360}\times\frac{22}7\times\;28\times\;28\;=\frac{1232}3{/tex}

= 410.67 cm^2

For, Area of $\triangle$AOB,



Draw OM $\perp$ AB.

In right triangles OMA and OMB,

OA = OB [Radii of same circle]

OM = OM [Common]

$\therefore \triangle OMA \cong OMB$ [RHS congruency]

$\therefore$ AM = BM [By CPCT]

$\Rightarrow$ AM = BM = $\frac{1}{2}$AB and $\angle AOM = \angle BOM = \frac{1}{2}AOB = \frac{1}{2} \times 60^o = 30^o$

In right angled triangle OMA, $\cos 30^o = \frac{OM}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{28}$

$\Rightarrow$ OM = $14\sqrt{3}$ cm

Also, $\sin 30° = \frac{AM}{OA}$

$\Rightarrow \frac{1}{2} = \frac{AM}{28}$

$\Rightarrow$ AM = 14 cm

$\Rightarrow$ 2 AM = 2 $\times$ 14 = 28 cm

$\Rightarrow$ AB = 28 cm

$\therefore$ Area of $\triangle AOB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 28 \times 14\sqrt{3} = 196\sqrt{3} = 196 \times 1.7 = 333.2$ cm ^{2}

$\therefore$ Area of minor segment = Area of minor sector - Area of $\triangle AOB$

= 410.67 - 333.2 = 77.47 cm^2

$\therefore$ Area of one design = 77.47 cm^2

$\therefore$ Area of six designs = 77.47 $\times$ 6 = 464.82 cm^2

Cost of making designs = 464.82 $\times$ 0.35 = Rs. 162.68

74. Here, OA = 3.5 cm, OC = 10.5 cm

Shaded area = $\pi \times \frac{60}{360}\left(10 \cdot 5^2 - 3 \cdot 5^2\right)$

= $\frac{22}{7} \times \frac{1}{6} \times 98$

= $\frac{154}{3}$ cm^2 or 51.3 cm^2

Length of arc CD = $2 \times \frac{22}{7} \times 10.5 \times \frac{60}{360}$

= 11 cm

75. Area of that part of the field in which the horse can graze by means of a 5 m long rope = $\frac{1}{4} \times 3.14 \times (5)^2 = 19.625$ m^2

Area of that part of the field in which the horse can graze by means of a 10 m long rope = $\frac{1}{4} \times 3.14 \times (10)^2 = 78.5$ m^2

Increase in grazing area = 78.5 m^2 - 19.625 m^2 = 58.875 m^2