Total No. of Questions : 9]

SEAT No. :

PC1675

[Total No. of Pages : 4

[6351]-101

F.E.

ENGINEERING MATHEMATICS - I

(2019 Pattern) (Semester- I) (107001) (Credit System)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt Q.1 (Compulsory); Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8 or Q.9.*
2) *Use of electronic pocket calculator is allowed.*
3) *Assume suitable data wherever necessary.*
4) *Figures to the right indicate full marks.*

Q1) Write the correct option for the following multiple choice questions. **[10]**

a) If $u = x^4 + y^4 + z^4$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$ **[2]**

i) u ii) $4u$

iii) $2u$ iv) 0

b) If $x = u^2 - v^2$ and $y = 2uv$ and $\frac{\partial(x,y)}{\partial(u,v)} = 4(u^2 + v^2)$ then $\frac{\partial(u,v)}{\partial(x,y)} =$ **[2]**

i) $4(u^2 + v^2)$ ii) $4(x^2 + y^2)$

iii) $\frac{1}{4(x^2 + y^2)}$ iv) $\frac{1}{4(u^2 + v^2)}$

c) For square matrix P to be an orthogonal matrix, **[2]**

i) $PP^T = A^{-1}$ ii) $PP^T = I$

iii) $P^2 = I$ iv) $P = P^T$

d) The quadratic form corresponding to the matrix $A = \begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 2 & 1 \\ 3/2 & 1 & -3 \end{bmatrix}$ is **[2]**

i) $Q(x) = x_1^2 + 2x_2^2 - x_3^2$ ii) $x_1^2 + x_2^2 - 3x_3^2 + 3x_1x_2 - 2x_2x_3$

iii) $x_1^2 + 2x_2^2 - 3x_3^2 + 3x_1x_3 + 2x_2x_3$ iv) $x_1^2 + 2x_2^2 - 3x_3^2 + \frac{3}{2}x_1x_3 + x_2x_3$

e) If $u = \ln\left[\dfrac{\sqrt{x^2+y^2}}{x+y}\right]$ then u is a homogeneous function of degree. **[1]**

i) 1 ii) 1/2

iii) 2 iv) 0

f) For a square matrix A, sum of the eigen values is 3 and product of the eigen values is 2 then characteristic equation of A is **[1]**

i) $\lambda^2 - 3\lambda - 2 = 0$ ii) $\lambda^2 - 3\lambda + 2 = 0$

iii) $\lambda^2 + 2\lambda + 3 = 0$ iv) $\lambda^2 + 2\lambda - 3 = 0$

Q2) a) If $u = \log(x^3 + y^3 - y^2x - x^2y)$ then show that $\left(\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y}\right)^2 u = \dfrac{-4}{(x+y)^2}$ **[5]**

b) If $u = \sin^{-1}\left(x^2+y^2\right)^{1/5}$ then prove that **[5]**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{2}{5}\tan u\left[\frac{2}{5}\tan^2 u - \frac{3}{5}\right]$$

c) If $z = f(x,y)$ where $x = u + v$; $y = uv$ then prove that **[5]**

$$u\frac{\partial z}{\partial u} + v\frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y}.$$

OR

Q3) a) If $u = ax + by$; $v = bx - ay$ find value of $\left(\dfrac{\partial u}{\partial x}\right)_y \left(\dfrac{\partial x}{\partial u}\right)_v \left(\dfrac{\partial y}{\partial v}\right)_x \left(\dfrac{\partial v}{\partial y}\right)_u$. **[5]**

b) If $T = \sin\left(\dfrac{xy}{x^2+y^2}\right) + \sqrt{x^2+y^2} + \dfrac{x^2y}{x+y}$. Find the value of $x \cdot \dfrac{\partial T}{\partial x} + y \cdot \dfrac{\partial T}{\partial y}$. **[5]**

c) If $Z = F(x, y)$ where $x = e^u \cos v$; $y = e^u \sin v$ then prove that $y \cdot \dfrac{\partial z}{\partial u} + x \cdot \dfrac{\partial z}{\partial v} = e^{2u}\dfrac{\partial z}{\partial y}$. **[5]**

Q4) a) If $x = uv,\ y = \frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. **[5]**

b) A power dissipated in a resistor is given by $P = \frac{E^2}{R}$. Find the approximate percentage error in P if E is increased by 3% and R is increased by 2%. **[5]**

c) Find stationary point of $f(x, y) = x^3 + y^3 - 3axy$ where $a < 0$. **[5]**

OR

Q5) a) If $u^3 + v^3 = x + y,\ u^2 + v^2 = x^3 + y^3$, find $\frac{\partial(u,v)}{\partial(x,y)}$. **[5]**

b) Examine for functional dependence $u = y + z,\ v = x + 2z^2,\ w = x - 4yz - 2y^2$. **[5]**

c) Find stationary value of $u = x^2 + y^2 + z^2$ under the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ using Lagrange's method. **[5]**

Q6) a) Examine for consistency the following system of equations and solve if consistent. **[5]**

$x + 2y + z = 2$

$2x - y - z = 2$

$4x - 7y - 5z = 2$

b) Examine whether the vectors $x_1 = (2, -1, 3, 2)$, $x_2 = (1, 3, 4, 2)$ and $x_3 = (3, -5, 2, 2)$ are linearly independent or dependent. If dependent, find the relation between them. **[5]**

c) For which values of a, b, c the matrix A is orthogonal where **[5]**

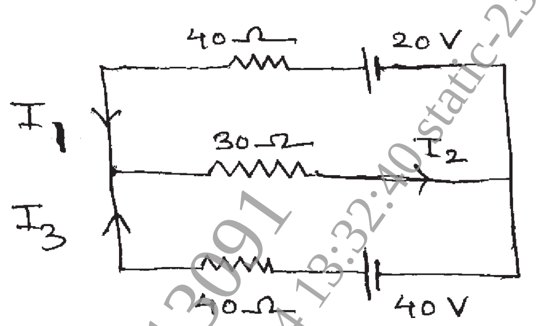
$$A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & a \\ \frac{2}{3} & \frac{1}{3} & b \\ \frac{2}{3} & -\frac{2}{3} & c \end{bmatrix}$$

OR

Q7) a) Determine values of λ for which the system of equations $3x - y + \lambda z = 0$, $2x + y + z = 2$, $x - 2y - \lambda z = -1$ is inconsistent. **[5]**

b) Examine whether the vectors $x_1 = (3, 1, 1)$, $x_2 = (2, 0, -1)$ and $x_3 = (4, 2, 1)$ are linearly independent or dependent. If dependent find the relation between them. **[5]**

c) Determine the currents in the following network. **[5]**



Q8) a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **[5]**

b) By using cayley Hamilton theorem find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. **[5]**

c) Reduce the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ to its diagonal form by finding modal matrix P. **[5]**

OR

Q9) a) Find the eigen values of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. Also find eigen vector corresponding to smallest eigen value of A. **[5]**

b) Verify cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$. Hence find A^{-1}, if it exists. **[5]**

c) Find the transformation which reduces the quadratic form $3x^2 + 5y^2 + 2z^2 - 2yz + 2zx - 2xy$ to the canonical form by using congruent transformations. Also write the canonical form. **[5]**