Total No. of Questions : 9]

SEAT No. :

P-9074

[Total No. of Pages : 4

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F.E.

ENGINEERING MATHEMATICS - II

(2019 Pattern) (Semester - II) (107008)

Time : 2½ Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Question No. 1 is compulsory.*
2) *Solve Q. No. 2 or Q. No. 3, Q. No. 4 or Q. No. 5, Q. No. 6 or Q. No. 7, Q. No. 8 or Q. No. 9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*

Q1) Write the correct option for the following multiple choice questions.

a) $\int_0^{2\pi} \sin^3\theta \cos^4\theta \, d\theta$ **[2]**

i) $\frac{2}{35}$ ii) $\frac{1}{15}$

iii) 0 iv) $\frac{2\pi}{35}$

b) The equation of tangents to the curve $3ay^2 = x(x-a)^2$, at the origin, if exist is **[2]**

i) $x = a$ ii) $x = 0, y = 0$

iii) $x = 0$ iv) $y = 0$

c) $\int_{\theta=0}^{\pi/2}\int_{r=0}^{2} r \, dr \, d\theta =$ **[2]**

i) π ii) 1

iii) 2 iv) $\frac{\pi}{2}$

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d) Radius r of a sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ is **[2]**

i) $r = 9$ ii) $r = 2$

iii) $r = 4$ iv) $r = 3$

e) The total number of loops for the curve $r = a \sin 3\theta$ are **[1]**

i) 2 ii) 3

iii) 6 iv) 4

f) $\iint \rho P^2 dxdy$ where ρ-density and p^2 is distance of particle from axis, represents **[1]**

i) Area ii) Mass

iii) Moment of Inertia iv) Volume

Q2) a) If $u_n = \int_0^{\pi/4} \sin^{2n} x\, dx$ then prove that $u_n = \left(1 - \frac{1}{2n}\right) u_{n-1} - \frac{1}{n 2^{n+1}}$. **[5]**

b) Prove that : $\beta(m, n) = \beta(m, n+1) + \beta(m+1, n)$ **[5]**

c) If $f(x) = \int_0^x (x-t)^2 G(t)\, dt$ then prove that $\frac{d^3 f}{dx^3} = 2G(x)$ **[5]**

OR

Q3) a) If $U_n = \int_0^{\pi/4} \tan^n \theta\, d\theta$, then prove that $n\left[U_{n+1} + U_{n-1}\right] = 1$ **[5]**

b) Evaluate : $\int_0^{\infty} 2^{-9x^2} dx$ **[5]**

c) Evaluate : **[5]**

i) $\frac{d}{dt}\left[erf\left(\sqrt{t}\right)\right]$

ii) $\frac{d}{dt}\left[erf_c\left(\sqrt{t}\right)\right]$

Q4) a) Trace the curve $y^2(2a - x) = x^3$, $a > 0$. **[5]**

b) Trace the curve $r = a(1 - \cos\theta)$ **[5]**

c) Find the arc length of cycloid $x = a(\mathrm{t} + \sin t)$, $y = a(1 - \cos t)$ from one cusp to another cusp. **[5]**

OR

Q5) a) Trace the curve $xy^2 = a^2(a - x)$, $a > 0$ **[5]**

b) Trace the curve $r = a\cos 3\theta$. **[5]**

c) Trace the curve **[5]**

$$x^{2/3} + y^{2/3} = a^{2/3}$$

Q6) a) Show that the plane $2x + y + 2z = 6$ touches the sphere $x^2 + y^2 + z^2 - 6x - 6y - 6z + 18 = 0$. Also find the point of contact. **[5]**

b) Find the equation of right circular cone whose vertex is at origin, axis is the line $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ and has a semi-vertical angle of 30°. **[5]**

c) Find the equation of right circular cylinder of radius 4 and axis is the line

$$\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$$ **[5]**

OR

Q7) a) If the sphere $x^2 + y^2 + z^2 + 2\lambda x + 3\lambda y + 4\lambda z - 1 - 5\lambda = 0$ cuts the sphere $x^2 + y^2 + z^2 + 3x - 3y + 3z - 56 = 0$, orthogonally, then find the value of λ. **[5]**

b) Find the equation of right circular cone whose vertex is at origin, generator is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and axis is the line $\frac{x}{-1} = \frac{y}{1} = \frac{z}{2}$. **[5]**

c) Find the equation of right circular cylinder of radius 2, whose axis passes through the origin and has direction ratios 1, 1, 1. **[5]**

Q8) a) Change order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dx\, dy$ **[5]**

b) Find the area of cardioide $r = a(1 + \cos\theta)$ using double integration. **[5]**

c) Prove that moment of inertia of the area included between curves $y^2 = 4ax$ and $x^2 = 4ay$ about x-axis is $\frac{144}{35}\mathrm{Ma}^2$, given that density $\rho = \frac{3\mathrm{M}}{16a^2}$ and M is the mass. **[5]**

OR

Q9) a) Change following double integration to its polar form and evaluate $\iint_R \frac{x^2y^2}{x^2+y^2}\,dx\,dy$, where R is annulus between $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. **[5]**

b) Prove that the volume bounded by cylinders $y^2 = x$ and $x^2 = y$ and planes $z = 0$, $x + y + z = 2$ is $\frac{11}{30}$. **[5]**

c) Find the x - co-ordinate of centre of gravity of a loop of $r = a\sin 2\theta$ in first quadrant, given that area of loop is $\mathrm{A} = \frac{\pi a^2}{8}$. **[5]**