Total No. of Questions—**8**] [Total No. of Printed Pages—**7**

Seat No.	

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S.E. (Civil Engg.) (I Sem.) EXAMINATION, 2019

ENGINEERING MATHEMATICS–III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagram must be drawn wherever necessary.

(*iii*) Assume suitable data, if necessary.

(*iv*) Use of non-programmable calculator is allowed.

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1. (*a*) Solve any *two* of the following : [8]

(*i*) $(D^2 + 2D + 1)y = x \cos x$

(*ii*) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} = e^x \sin x$

(by variation of parameters)

(*iii*) $x^2 \dfrac{d^2y}{dx^2} - 4x \dfrac{dy}{dx} + 6y = x^5 + 2x^3$.

(*b*) Apply Gauss-Jordan method to solve the following system of equations : [4]

$$x_1 + 2x_2 + x_3 = 8$$
$$2x_1 + 3x_2 + 4x_3 = 20$$
$$4x_1 + 3x_2 + 2x_3 = 16.$$

P.T.O.

Or

2. (*a*) The deflection of a strut with one end built in ($x = 0$) and other supported and subjected to end thrust P satisfies the equation :

$$\frac{d^2y}{dx^2} + a^2 y = \frac{a^2 R}{P}(l - x).$$

Given that $\frac{dy}{dx} = y = 0$ when $x = 0$ and $y = 0$ when $x = l$, prove that :

$$y = \frac{R}{P}\left[\frac{\sin ax}{a} - l\cos ax + l - x\right],$$

where $\tan al = al$. [4]

(*b*) Use Runge-Kutta method of fourth order to solve :

$$\frac{dy}{dx} = \frac{1}{x + y};\ x_0 = 0,\ y_0 = 1,$$

to find y at $x = 0.2$ taking $h = 0.2$. [4]

(*c*) Solve the following system of equations by Cholesky's method : [4]

$$9x_1 + 6x_2 + 12x_3 = 15$$

$$6x_1 + 13x_2 + 11x_3 = 25$$

$$12x_1 + 11x_2 + 26x_3 = 40.$$

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3. (*a*) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Find first four central moments, β_1 and β_2. [4]

(*b*) 5000 candidates appeared in a certain paper carrying a maximum of 100 marks. The marks were normally distributed with mean 39.5 and standard deviation of 12.5. Find approximately the number of candidates with minimum 60 marks. (Area corresponding to Z = 1.64 is 0.4495) [4]

(*c*) If the directional derivative of $\phi = axy + byz + czx$ at (1, 1, 1) has maximum magnitude 4 in a direction parallel to *x*-axis find the values of *a*, *b*, *c*. [4]

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\bar{b} \times \nabla\, [\bar{a}.\nabla \log r] = \dfrac{\bar{b} \times \bar{a}}{r^2} - \dfrac{2(\bar{a}.\bar{r})\,(\bar{b} \times \bar{r})}{r^4}$.

(*ii*) $\nabla^2\, [\nabla \left(\dfrac{\bar{r}}{r}\right)] = 0$.

(*b*) Show that : [4]

$$\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k},$$

is irrotational. Also find scalar potential ϕ such that $\bar{F} = \nabla\phi$.

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(*c*) Find the correlation coefficient for the following data : [4]

x	*y*
1	1
3	2
4	4
6	4
8	5
9	7
11	8
14	9

5. Solve any *two* :

(*a*) Evaluate

$$\int_C \bar{F}.d\bar{r}$$

where $\bar{F} = (2x + y^2)i + (3y - 4x)j$ and C is the curve $y = x^2$ joining points (0, 0) and (1, 1). [6]

(*b*) Use Gauss-Divergence theorem to evaluate $\iint_S \bar{F}.d\bar{s}$ for

$$\bar{F} = 4xzi - y^2 j + yzk$$

over the surface of cube bounded by the planes :

$x = 0,\ x = 2,\ y = 0,\ y = 2,\ z = 0,\ z = 2.$ [7]

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(*c*) Using Stokes theorem to evaluate $\iint_S (\nabla \times \overline{F}) \cdot d\overline{s}$ for

$$\overline{F} = yi + zj + xk$$

over the surface $x^2 + y^2 = 1 - z,\ z \geq 0$. [6]

Or

6. Solve any *two* :

(*a*) Using Green's theorem to evaluate :

$$\int_C [\cos y i + x(1 - \sin y) j] . d\overline{r}$$

where C is the boundary of a circle $x^2 + y^2 = 1,\ z = 0$. [6]

(*b*) Using Stokes theorem, evaluate $\iint_S (\nabla \times \overline{F}) \cdot d\overline{S}$ where

$$\overline{F} = -y^3 i + x^3 j$$

and the boundary of surface S is given by $x^2 + \frac{y^2}{4} = 1$. [7]

(*c*) Using Gauss-Divergence theorem to evaluate $\iint_S \overline{F}.\overline{n}\, dS$ where

$$\overline{F} = \sin x\ i + (2 - \cos x) j$$

and S is the total surface area of paraboloid bounded by :

$x = 0,\ x = \frac{\pi}{2},\ y = 0,\ y = 2,\ z = 0$ and $z = 3$. [6]

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7. Solve any *two* of the following :

(*a*) A string is stretched and fastened to two points distance l apart is displaced into the form $y(x, 0) = 3(lx - x^2)$ from which it is released at $t = 0$. Find the displacement of a string at a distance x from one end. [7]

(*b*) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if :

(*i*) u is finite for all t

(*ii*) $u(0, t) = 0$ for all t

(*iii*) $u(\pi, t) = 0$ for all t

(*iv*) $u(x, 0) = \pi x - x^2, \ 0 \le x \le \pi$. [6]

(*c*) A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along short edge $y = 0$ is given :

$$u(x,0) = 100\sin\left(\frac{\pi x}{10}\right), \ \ 0 \le x \le 10,$$

while the two long edges $x = 0$ and $x = 10$ as well as the other short edge are kept at 10°C. Find steady-state temperature $u(x, 4)$. [6]

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Or

8. Solve any *two* of the following :

(*a*) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by :

$$y(x, 0) = y_0 \sin^3 \left(\frac{\pi x}{l}\right).$$

If it is released from rest from this position, find the displacement y at any distance x from one end at any time t. [7]

(*b*) The equation for the conduction of heat along a bar of length l is $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ neglecting radiation. Find an expression for $u(x, t)$ if the ends of the bar are maintained at zero temperature and if initially the temperature is T at the centre of the bar and falls uniformly to zero at its ends. [6]

(*c*) An infinitely long plane uniform plate is bounded by two parallel edges in the y direction and an end at right angles to them. The breadth of the plate is π. This end is maintained at temperature u_0 at all points and other edges at zero temperature. Find the steady-state temperature function $u(x, y)$. [6]

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