SEAT No. :

P4396

[5251]-1008

F.E.

ENGINEERING MATHEMATICS - II
(2015 Pattern) (Credit System)

Time : 3 Hours]   *[Max. Marks : 50*

Instructions to the candidates:

1) *Answer Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8,*

2) *Neat diagrams must be drawn wherever necessary.*

3) *Figures to the right indicate full marks.*

4) *Use of electronic non-programmable calculator is allowed.*

5) *Assume suitable data, if necessary.*

Q1) a) Solve the following differential equations :   **[8]**

     i) $(1 + xy)\, y\, dx + (1 - xy)x\, dy = 0$

     ii) $\dfrac{dy}{dx} = \dfrac{x - y + 3}{2x - 2y + 5}$

   b) A body of mass *m* falling from rest is subjected to the force of gravity and an air resistance proportional to the square of the velocity (Kv^2). If it falls through distance '*x*' and possesses a velocity '*v*' at that instant,

prove that $\dfrac{2Kx}{m} = \log\left(\dfrac{a^2}{a^2 - v^2}\right)$, where mg = Ka^2 .   **[4]**

OR

Q2) a) Solve : $(1 + y^2) + (x - e^{-\tan^{-1} y})\dfrac{dy}{dx} = 0$ **[4]**

   b) Solve the following :   **[8]**

     i)   If the temperature of the body drops from 100°C to 60°C in one minute when the temperature of the surrounding is 20°C, what will be the temperature of body at the end of third minute.

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ii) A constant electromotive force E volts is applied to a circuit containing a constant resistance R ohms in a series and a constant inductance L henries. If the initial current is zero, show that the current builts up to half its theoretical maximum in $\dfrac{L}{R}\log 2$.

Q3) a) Find half range cosine series for $f(x) = \sin^2 x,\ 0 < x < \pi$ **[5]**

b) Evaluate $\displaystyle\int_0^\infty \dfrac{dx}{1 + x^4}$. **[3]**

c) Trace the curve (Any one) : **[4]**
i) $y^2(a - x) = x^3$
ii) $x = a(t + \sin t),\ y = a(1 - \cos t)$

<div align="center">OR</div>

Q4) a) Evaluate

$$\int_0^{2a} x\sqrt{2ax - x^2}\ dx.$$ **[4]**

b) Prove that

$$\int_0^\infty \dfrac{e^{-ax} - e^{-bx}}{x}\,dx = \log\dfrac{b}{a},\ a > 0,\ b > 0.$$ **[4]**

c) Find the arc length of the curve $x = e^\theta\cos\theta,\ y = e^\theta\sin\theta$ from $\theta = 0$ to

$\theta = \dfrac{\pi}{2}$. **[4]**

Q5) a) Find the centre and radius of the circle which is the intersection of the sphere $x^2 + y^2 + z^2 - 2x + 4y + 2z - 6 = 0$ & the plane $x + 2y + 2z - 4 = 0$. **[5]**

b) Obtain the equation of a right circular cone which passes through the point $(2, 1, 3)$ which the vertex $(1, 1, 2)$ and axis parallel to $\dfrac{x-2}{2} = \dfrac{y-1}{-4} = \dfrac{z+2}{3}$. **[4]**

c) Obtain the equation of a right circular cylinder of radius 5 and axis the line $\dfrac{(x-2)}{2} = \dfrac{(y-3)}{1} = \dfrac{(z+1)}{1}$. **[4]**

Q6) a) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$; $z = 0$ and the point (α, β, γ). **[5]**

b) Find the equation of right circular cone whose vertex is $(1, -1, 2)$, axis the line $\dfrac{x-1}{2} = \dfrac{y+1}{1} = \dfrac{z-2}{-2}$ and semi-vertical angle $45°$. **[4]**

c) Find the equation of the right circular cylinder whose axis is $\dfrac{x-2}{2} = \dfrac{y-1}{1} = \dfrac{z}{3}$ and which passes through the point $(0, 0, 3)$. **[4]**

Q7) Attempt any two of the following :

a) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{\cos^{-1} x \, dx \, dy}{\sqrt{(1-x^2-y^2)(1-x^2)}} .$$ **[6]**

b) Evaluate

$$\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) \, dx \, dy \, dz .$$ **[7]**

c) Find the moment of inertia of the portion of the parabola $y^2 = 4ax$ bounded by x - axis and latus rectum, about x axis, if density at each point varies as the cube of the abscissa. **[6]**

OR

Q8) Attempt any two of the following :

a) Find the area outside the circle $r = a \sin\theta$ and outside the cardioide $r = a (1 - \cos\theta)$. **[6]**

b) Find the volume of the region enclosed by the cone $z = \sqrt{x^2 + y^2}$ and paraboloid $z = x^2 + y^2$. **[7]**

c) Find the centroid of the one loop of the curve $r = a \sin2\theta$. **[6]**

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