F.E. 2012 pattern sem-I
Nov-Dec-2012

Seat No.	

Total No. of Questions : 8] [Total No. of Printed Pages : 4

[4261]-1

F. E. Examination - 2012

ENGINEERING MATHEMATICS - I

(2012 Pattern)

Time : 2 Hours] **[Max. Marks : 50**

Instructions :

*(1) Attempt **four** questions : Q. No. **1** or **2**, Q. No. **3** or **4**, Q. No. **5** or **6** and Q. No. **7** or **8**.*

(2) Figures to the right indicate full marks.

(3) Assume suitable data if necessary.

(4) Neat diagrams must be drawn wherever necessary.

(5) Use of electronic non-programmable calculator is allowed.

Q.1) (A) Examine the consistency of the system of the following equations. If consistent, solve system of equations : **[04]**

$$x + y + z = 3$$

$$x + 2y + 3z = 4$$

$$x + 4y + 9z = 6$$

(B) Find Eigen Values and Eigen Vector corresponding to highest Eigen Value for the matrix : **[04]**

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

(C) If $\tan \log (x - iy) = a - ib$ and $a^2 + b^2 \neq 1$, then prove that $\tan \log (x^2 + y^2) = \frac{2a}{1 - a^2 - b^2}$. **[04]**

OR

Q.2) (A) Examine for Linear Dependence or Independence of Vectors (2, 2, 7, –1), (3, –1, 2, 4) and (1, 1, 3, 1). **[04]**

(B) Solve : $x^7 + x^4 + i (x^3 + 1) = 0$. **[04]**

(C) A square lies above real axis in Argand diagram and two of its adjacent vertices are the origin and the point 2 + 3i. Find the complex number representing other vertices. **[04]**

Q.3) (A) Test convergence of the series : **(Any One)** **[04]**

(a) $\frac{2}{9} + \frac{2 \cdot 5}{9 \cdot 13} + \frac{2 \cdot 5 \cdot 8}{9 \cdot 13 \cdot 17} + \ldots$

(b) $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right) \sin\left(\frac{1}{n}\right)$

(B) Expand $3x^3 - 2x^2 + x - 6$ in powers of $(x - 2)$. **[04]**

(C) Find n^{th} derivative of $x^2 e^x \cos x$. **[04]**

OR

Q.4) (A) Solve : **(Any One)** **[04]**

(a) $\lim_{x \to 1} \left(1 - x^2\right)^{\frac{1}{\log(1-x)}}$

(b) Find a and b, if $\lim_{x \to 0} \frac{a \sinh x + b \sin x}{2x^3} = \frac{8}{6}$.

(B) Show that : **[04]**

$$x \operatorname{cosec} x = 1 + \frac{x^2}{6} + \frac{7}{360} x^4 + \ldots$$

(C) Prove that n^{th} derivative of $y = \tan^{-1}x$ is

$$(-1)^{n-1} (n - 1) ! \sin n \left(\frac{\pi}{2} - y\right) \sin^{n} \left(\frac{\pi}{2} - y\right).$$ **[04]**

Q.5) Solve **any two** :

(a) If $u = \log\left(\sqrt{x^2 + y^2 + z^2}\right)$,

then prove that

$$\left(x^2 + y^2 + z^2\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1.$$ **[06]**

(b) If $u = \dfrac{x^2 y^2 z^2}{x^2 + y^2 + z^2} + \cos\left(\dfrac{xy + yz}{x^2 + y^2 + z^2}\right)$,

then find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}.$$ **[07]**

(c) If $u = x^2 - y^2$, $v = 2xy$ and $z = f(u, v)$, then show that

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2}\,\frac{\partial z}{\partial u}.$$ **[06]**

OR

Q.6) Solve **any two** :

(a) If $ux + vy = 0$ and $\dfrac{u}{x} + \dfrac{v}{y} = 1$,

then show that

$$\left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial v}{\partial y}\right)_x = \frac{x^2 + y^2}{y^2 - x^2}.$$ **[06]**

(b) If $u = \operatorname{cosec}^{-1}\left(\sqrt{\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}}\right)$,

then show that

$$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right).$$ **[07]**

(c) If $x = r\cos\theta$, $y = r\sin\theta$, where r and θ are functions of t, then prove that :

$$x\frac{dy}{dt} - y\frac{dx}{dt} = r^2\frac{d\theta}{dt}.$$ **[06]**

Q.7) (A) If $u = x(1 - y)$ and $v = xy$,

find $\dfrac{\partial(x, y)}{\partial(u, v)}$. **[04]**

(B) Examine for functional dependence for $u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$. **[04]**

(C) Discuss the maxima and minima of

$$f(x, y) = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right).$$ **[05]**

OR

Q.8) (A) If $x = u^2 - v^2$, $y = uv$, find $\dfrac{\partial u}{\partial x}$. **[04]**

(B) Find the percentage error in computing the parallel resistance r of two resistances r_1 and r_2 from the formula $\dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2}$, where r_1 and r_2 are both in error by 2% each. **[04]**

(C) Find the points on the surface $z^2 = xy + 1$ nearest to the origin, by using Lagrange's Method. **[05]**