Total No. of Questions :8]

SEAT No. :

P4021

[Total No. of Pages : 4

[5351]-101

F.E.

ENGINEERING MATHEMATICS-I

(2015 Pattern) (Semester - I & II) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Solve Q. 1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6 and Q. 7 or Q. 8.*
2) *Neat diagram must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Use of, electronic pocket calculator allowed.*
5) *Assume suitable data, if necessary.*

Q1) a) Reduce the following matrix to its normal form and hence find its rank. **[4]**

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

b) Find the eigen values and eigen vectors of : **[4]**

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

c) If $\tan(\alpha + i\beta) = x + iy$, then prove that $x^2 + y^2 + 2x\cot 2\alpha = 1$ and $x^2 + y^2 - 2y\coth(2\beta) + 1 = 0$. **[4]**

OR

Q2) a) If α and β are roots of equation $Z^2 \sin^2\theta - Z\sin 2\theta + 1 = 0$, prove that $\alpha^n + \beta^n = 2\cos n\theta . \text{cosec}^n\theta$, where n is integer. **[4]**

b) Prove that $\tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\} = \frac{2ab}{a^2 - b^2}$ using principal value of logarithm. **[4]**

c) Examine Linear dependence of vectors $x_1 = (2, -1, 3, 2)$, $x_2 = (1, 3, 4, 2)$, $x_3 = (-1, -4, 1, 0)$. If dependent find the relation among them. **[4]**

P.T.O.

Q3) a) Solve any one : **[4]**

i) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)x^n}{n^2}$.

ii) Test the convergence of the series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots\dots$

b) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x+2)$ using Taylor's theorem. **[4]**

c) Find the n[th] derivative of $y = \cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right)$. **[4]**

Q4) a) Solve any one : **[4]**

i) $\lim_{x\to 0}\left(\frac{2^x + 3^x}{2}\right)^{1/x}$

ii) $\lim_{x\to 0}\frac{xe^x - \log(1+x)}{x^2}$

b) Show that : **[4]**

$$(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \frac{3}{4}x^5 + \dots\dots$$

c) If $y = e^{\tan^{-1} x}$, then prove that

$$(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$$ **[4]**

Q5) Solve any two :

a) If $u = 2x + 3y,\ v = 3x - 2y$ find the value of $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v \left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial v}{\partial y}\right)_u$. **[6]**

b) If $u = \operatorname{cosec}^{-1}\left(\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}\right)$, show that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{12}\right).$$ **[7]**

c) If $z = f(x, y)$ where $x = u + v$, $y = uv$, then prove that

$$u\frac{\partial z}{\partial u}+v\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}+2y\frac{\partial z}{\partial y}$$ **[6]**

OR

Q6) Solve any two :

a) If $u = \log\left(x^3+y^3-x^2y-xy^2\right)$, prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2 u = \frac{-4}{(x+y)^2}$. **[6]**

b) If $u = \frac{x^3+y^3}{x+y}+\frac{1}{x^5}\sin^{-1}\left[\frac{x^2+y^2}{x^2+2xy}\right]$, find the value of

$x^2 = \frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ at the point (1, 2). **[7]**

c) If $u = x^2 - y^2$, $v = 2xy$ and $z = f(u, v)$ then show that

$$x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=2\sqrt{u^2+v^2}\,\frac{\partial z}{\partial u}.$$ **[6]**

Q7) a) If $ux + vy = a, \frac{u}{x}+\frac{v}{y}=1$, prove that $\left(\frac{\partial u}{\partial x}\right)_y - \left(\frac{\partial v}{\partial y}\right)_x = \frac{x^2+y^2}{y^2-x^2}$. **[4]**

b) In calculating the volume of a right circular core, errors of 2% and 1% are made in measuring the height and radius base respectively. Find the error in the calculated volume. **[4]**

c) Find the stationary points of the function

$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. Examine for maxima and minima at there points. **[5]**

OR

Q8) a) If $u = x + y^2, v = y + z^2, w = z + x^2$ find $\frac{\partial x}{\partial u}$. **[4]**

b) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = xy + yz + zx$. Examine whether u, v, w are functionally dependent. If so find the relation between them. **[4]**

c) Find the stationary value of $u = x^m y^n z^p$ under the condition $x + y + z = a$. **[5]**