Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5151]-101

F.E. EXAMINATION, 2017

ENGINEERING MATHEMATICS–I

(2015 PATTERN)

Time : Three Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6,** Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Show that system of Linear equations is consistent. Find solution : [4]

$$x + 2y + 3z = 6$$
$$2x + 3y = 11$$
$$4x + y - 5z = -3 .$$

(*b*) Find eigen values and eigen vectors of the matrix : [4]

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

(*c*) Prove that : [4]

$$(\cosh x - \sinh x)^n = \cosh nx - \sinh nx \cdot$$

P.T.O.

Or

2. (*a*) Are the following vectors are linearly dependent ? If so find relation : [4]

$$X_1 = (3, 2, 7),$$
$$X_2 = (2, 4, 1),$$
$$X_3 = (1, -2, 6).$$

(*b*) If α, β are roots of equation $x^2 - 2x + 4 = 0$, prove that : [4]

$$\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}.$$

(*c*) If [4]

$$(a + ib)^p = m^{x+iy},$$

prove that :

$$\frac{y}{x} = \frac{2 \tan^{-1} \frac{b}{a}}{\log (a^2 + b^2)}.$$

3. (*a*) Test the convergence of the series (any *one*) : [4]

(*i*) $\sum \frac{x^n}{a + \sqrt{n}}$

(*ii*) $\frac{1}{\sqrt{5}} - \frac{1}{2\sqrt{6}} + \frac{1}{3\sqrt{7}} - ......$

(*b*) Show that : [4]

$$\log \left[\frac{1 + e^{2x}}{e^x}\right] = \log 2 + \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{45} - ....$$

(*c*) Find the *n*th derivative of : [4]

$$y = e^x \cos x . \cos 2x .$$

Or

4. (*a*) Solve any *one* : [4]

(*i*) $\lim_{x\to 0}\left[\frac{\pi}{4x}-\frac{\pi}{2x(e^{\pi x}+1)}\right]$

(*ii*) $\lim_{x\to 0}\left(\frac{2^x+5^x+7^x}{3}\right)^{1/x}$.

(*b*) Using Taylor's theorem expand x^3-2x^2+3x+1 in powers of $(x - 1)$. [4]

(*c*) If $y=e^{a\sin^{-1}x}$, prove that : [4]

$$(1-x^2)y_{n+2}-(2n+1)x\ y_{n+1}-(n^2+a^2)\,y_n=0\,.$$

5. Solve any *two* :

(*a*) Find the value of *n* for which $z=t^n e^{-r^2/4t}$ satisfies the partial differential equation : [6]

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial z}{\partial r}\right)\right]=\frac{\partial z}{\partial t}\,.$$

(*b*) If

$$\mathrm{T}=\sin\left(\frac{xy}{x^2+y^2}\right)+\sqrt{x^2+y^2}+\frac{x^2y}{x+y},$$

find the value of $x\frac{\partial \mathrm{T}}{\partial x}+y\frac{\partial \mathrm{T}}{\partial y}$. [7]

(*c*) If $z = f(x, y)$, where

$$x = u\cos\alpha - v\sin\alpha,$$

$$y = u\sin\alpha - v\cos\alpha,$$

where α is constant, show that : [6]

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2.$$

Or

6. Solve any *two* **:**

(*a*) If [6]

$$x^2 = a\sqrt{u} + b\sqrt{v} \text{ and}$$

$$y^2 = a\sqrt{u} - b\sqrt{v}$$

where a and b are constants, prove that :

$$\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u.$$

(*b*) If

$$u = \tan^{-1}\left(\frac{\sqrt{x^3 + y^3}}{\sqrt{x} + \sqrt{y}}\right),$$

then show that : [7]

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2\sin^3 u \cos u.$$

(*c*) If

$$u = x^2 - y^2, \quad v = 2xy \quad \text{and} \quad z = f(u, v),$$

then show that : [6]

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2\sqrt{u^2 + v^2}\ \frac{\partial z}{\partial u}.$$

7. (*a*) If [4]

$$u + v = x^2 + y^2, \quad u - v = x + 2y$$

Find $\frac{\partial u}{\partial x}$ treating y constant.

(*b*) Examine for functional dependence : [4]

$$u = \frac{x - y}{x + z}, \quad v = \frac{x + z}{y + z}.$$

(*c*) Finsd stationary points of : [5]

$$f(x, y) = x^3 y^2 (1 - x - y)$$

and find $f_{\max}$ where it exists.

Or

8. (*a*) If [4]

$$x = v^2 + w^2, \quad y = w^2 + u^2, \quad z = u^2 + v^2,$$

prove that JJ' = 1.

(*b*) Find the percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3 from the formula : [4]

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

if r_1, r_2, r_3 are in error by 2% each.

(*c*) Find the stationary points of :

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600$$

if the condition $4x^2 + y^2 + 4z^2 = 16$ is satisfied. [5]