

Total No. of Questions—8]

[Total No. of Printed Pages—4+2

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F.E. EXAMINATION, 2016
ENGINEERING MATHEMATICS—II
(2012 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Attempt *four* questions : Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of electronic non-programable calculator is allowed.

(v) Assume suitable data, if necessary.

1. (a) Solve the following differential equations : [8]

(i)
$$\frac{dy}{dx} = \frac{2x - 3y + 1}{3x + 4y - 5}$$

(ii)
$$x \cos x \frac{dy}{dx} + (\cos x - x \sin x) y = 1.$$

(b) Assuming that the resistance to movement of a ship through water in the form of $(a^2 + b^2 v^2)$, where v is the velocity, a and b are constants, write down the differential equation

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for retardation of ship moving with engine stopped. Prove that the time in which the speed falls to one half its original value u is given by :

$$\frac{w}{abg} \tan^{-1} \left(\frac{abu}{2a^2 + b^2 u^2} \right)$$

where w is the weight of the ship. [4]

Or

2. (a) Solve : [4]

$$y \log y \, dx + (x - \log y) \, dy = 0$$

(b) Solve the following : [8]

(i) A body of temperature 80°F is placed in a room of constant temperature 50°F at time $t = 0$. At the end of 5 minutes the body was cooled to a temperature of 70°F . Find the time at which temperature of the body will be 60°F .

(ii) A capacitor $C = 0.01 \text{ F}$ in series with a resistor $R = 20 \, \Omega$ is charged from a battery 10 Volts. Assuming that initially the capacitor is completely uncharged, determine the charge $Q(t)$ and current $I(t)$ in the circuit.

3. (a) Find the Fourier series of : [5]

$$f(x) = x^3, \quad -\pi < x < \pi.$$

(b) Evaluate : [3]

$$\int_0^{\infty} \frac{x^9 (1 - x^5)}{(1 + x)^{25}} dx .$$

(c) Trace the following curve (any one) : [4]

(i) $x = a(t + \sin t), y = a(1 + \cos t)$

(ii) $r = a \cos 3\theta.$

Or

4. (a) If [4]

$$I_n = \int_0^{\pi/4} \sec^n \theta d\theta,$$

prove that :

$$I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n+2}{n-1} I_{n-2}.$$

(b) If [4]

$$f(x) = \int_2^x (x-t) G(t) dt$$

then show that :

$$\frac{d^2 f}{dx^2} - G(x) = 0.$$

(c) Find the length of arc of an Astroid : [4]

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}.$$

5. (a) Show that the plane

$$2x - 2y + z + 12 = 0$$

touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0.$$

Also find the point of contact. [5]

- (b) Find the equation of right circular cone passing through (2, -2, 1) with vertex at origin and axis parallel to the line : [4]

$$\frac{x - 2}{5} = \frac{y - 1}{1} = \frac{z + 2}{1}.$$

- (c) Find the equation of right circular cylinder whose axis is :

$$x = 2y = -z$$

and radius is 4. [4]

Or

6. (a) Find the equation of the sphere which has its centre at (2, 3, -1) and touches the line : [5]

$$\frac{x + 1}{-5} = \frac{y - 8}{3} = \frac{z - 4}{4}.$$

- (b) Find the equation of the cone with vertex at (1, 2, -3), semi-vertical angle $\cos^{-1}\frac{1}{\sqrt{3}}$ and the line :

$$\frac{x - 1}{1} = \frac{y - 2}{2} = \frac{z + 1}{-1}$$

as the axis of the cone. [4]

- (c) Find the equation of right circular cylinder of radius 2 with the axis : [4]

$$\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}.$$

7. Attempt any *two* of the following :

- (a) Evaluate : [6]

$$\iint \frac{1}{x^4 + y^2} dx dy$$

over the region

$$y \geq x^2, x \geq 1.$$

- (b) Evaluate : [7]

$$\iiint \frac{dx dy dz}{\sqrt{1 - x^2 - y^2 - z^2}}$$

taken throughout the volume of the sphere :

$$x^2 + y^2 + z^2 = 1$$

in the positive octant.

- (c) Find the area bounded by the parabola

$$y^2 = 4x$$

and the straight line : [6]

$$2x - 3y + 4 = 0.$$

Or

8. Attempt any *two* of the following :

(a) Evaluate : [6]

$$\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log_e(x^2 + y^2) dx dy .$$

(b) A rod of length l is divided into two parts at random. Find average of sum of squares of these parts. Also find mean value of rectangle contained by these two segments. [7]

(c) Find the volume common to the cylinders : [6]

$$x^2 + y^2 = a^2 \text{ and}$$

$$x^2 + z^2 = a^2 .$$