

Total No. of Questions—8]

[Total No. of Printed Pages—4+2

Seat No.	
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**[4756]-201**

**F.E. (Second Semester) EXAMINATION, 2015**

**ENGINEERING MATHEMATICS-II**

**(2012 PATTERN)**

**Time : Two Hours**

**Maximum Marks : 50**

**N.B.** :— (i) Attempt *four* questions : Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Neat diagrams must be drawn whenever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of electronic non-programmable calculator is allowed.

(v) Assume suitable data whenever necessary.

1. (a) Solve the following differential equations : [8]

(i)  $\frac{dy}{dx} = \cos x \cos y + \sin x \sin y$

(ii)  $(x^2 + y^2 + 1)dx - 2xy dy = 0$ .

(b) In a circuit containing inductance L, resistance R and voltage E, the current I is given by :

$$E = RI + L \frac{dI}{dt}.$$

P.T.O.

Given :

$L = 640 \text{ H}$ ,  $R = 250 \text{ } \Omega$ ,  $E = 500 \text{ Volts}$ .

$I$  being zero when  $t = 0$ . Find the time that elapses before it reaches 80% of its maximum value. [4]

*Or*

2. (a) Solve : [4]

$$x \frac{dy}{dx} + y = y^2 \log x$$

(b) Solve the following : [8]

(i) A body at temperature  $100^\circ\text{C}$  is placed in a room whose temperature is  $20^\circ\text{C}$  and cools to  $60^\circ\text{C}$  in 5 minutes. Find its temperature after a further interval of 3 minutes.

(ii) A steam pipe 20 cm in diameter is protected with a covering 6 cm thick for which the coefficient of thermal conductivity is  $k = 0.003 \text{ cal/cm deg. sec}$  in steady state. Find the heat lost per hour through a meter length of the pipe, if the surface of pipe is at  $200^\circ\text{C}$  and outer surface of the covering is at  $30^\circ\text{C}$ .

3. (a) Find a half range cosine series of  $f(x) = \pi x - x^2$  in the interval  $0 < x < \pi$ . [5]

(b) Evaluate : [3]

$$\int_0^{\infty} \frac{x^3}{3^x} dx .$$

(c) Trace the following curve (any one) : [4]

(i)  $y^2 = x^5 (2a - x)$

(ii)  $r = a \sin 2\theta$ .

Or

4. (a) If [4]

$$I_n = \int_{\pi/4}^{\pi/2} \cot^n \theta d\theta$$

prove that  $I_n = \frac{1}{n-1} - I_{n-2}$ . Hence evaluate  $I_3$ .

(b) Using differentiation under Integral sign prove that : [4]

$$\int_0^{\infty} \frac{e^{-x} - e^{-ax}}{x \sec x} dx = \frac{1}{2} \log \left( \frac{a^2 + 1}{2} \right)$$

for  $a > 0$

(c) Find the length of the curve [4]

$$x = a(\theta - \sin \theta), y = a(1 - \cos \theta)$$

between  $\theta = 0$  to  $\theta = 2\pi$ .

5. (a) Show that the plane  $4x - 3y + 6z - 35 = 0$  is tangential to the sphere  $x^2 + y^2 + z^2 - y - 2z - 14 = 0$  and find the point of contact. [5]

(b) Find the equation of the right circular cone whose vertex is given by  $(1, -1, 2)$  and axis is the line  $\frac{x - 1}{2} = \frac{y + 1}{1} = \frac{z - 2}{-2}$  and semi-vertical angle is  $45^\circ$ . [4]

(c) Find the equation of right circular cylinder of radius 2 and axis is given by : [4]

$$\frac{x - 1}{2} = \frac{y - 2}{-3} = \frac{z - 3}{6}$$

*Or*

6. (a) Find the equation of the sphere through the circle  $x^2 + y^2 + z^2 = 1$ ,  $2x + 3y + 4z = 5$  and which intersects the sphere  $x^2 + y^2 + z^2 + 3(x - y + z) - 56 = 0$  orthogonally. [5]

(b) Find the equation of right circular cone with vertex at origin making equal angles with the co-ordinate axes and having generator with direction cosines proportional to 1, -2, 2. [4]

- (c) Obtain the equation of the right circular cylinder of radius 5 where axis is : [4]

$$\frac{x - 2}{3} = \frac{y - 3}{1} = \frac{z + 1}{1}.$$

7. Attempt any *two* of the following :

- (a) Change the order of integration in the double integral : [6]

$$\int_0^5 \int_{2-x}^{2+x} f(x, y) dy dx$$

- (b) Evaluate : [7]

$$\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z} dx dy dz$$

- (c) Find the centroid of the loop of the curve : [6]

$$r^2 = a^2 \cos 2\theta.$$

Or

8. Attempt any *two* of the following :

- (a) Evaluate : [6]

$$\int_0^a \int_0^{\sqrt{a^2 - x^2}} e^{-x^2 - y^2} dx dy.$$

(b) Evaluate : [6]

$$\iiint \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}} \, dx \, dy \, dz$$

throughout the volume of ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .

(c) Prove that the moment of inertia of the area included between the curves  $y^2 = 4ax$  and  $x^2 = 4ay$  about  $x$ -axis is  $\frac{144}{35}Ma^2$ , where  $M$  is the mass of the area included between the curves. [7]