Total No. of Questions—8] [Total No. of Printed Pages—7

Seat No.	

[4757]-1001

S.E. (Civil) (First Semester) EXAMINATION, 2015

ENGINEERING MATHEMATICS III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator and steam tables is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4 + 2^x + 3e^{-x} + \cos x$$

(*ii*) $$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = x + x^{-1}$$

(*iii*) Use the method of variation of parameters to solve the linear differential equation :

$$\frac{d^2y}{dx^2} + y = \text{cosec } x.$$

(*b*) Solve the following system of linear equations by Gauss Elimination method : [4]

$$5x - 2y + 2z = 5, \quad 2x + y - z = 2, \quad x - y + z = 1.$$

Or

2. (*a*) Solve the system of simultaneous symmetric equations : [4]

$$\frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{4xy^2 - 2z}.$$

(*b*) Apply Runge-Kutta method of 4th order to solve the differential equation :

$$\frac{dy}{dx} = x + y^2, \qquad y(0) = 1$$

to find y for $0 \leq x \leq 0.2$ with $h = 0.1$. [4]

(*c*) Solve the following system of equations by Cholesky method : [4]

$$2x + 3y + z = 0$$

$$x + 2y - z = -2$$

$$-x + y + 2z = 0.$$

3. (*a*) The first four moments about the value 4 are –1.5, 17, –30 and 108. Calculate the moments about the mean. Also find coefficient of skewness and kurtosis. [4]

(*b*) Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 10,000 would you expect to be over 6 feet tall, where the data is normally distributed. (Given : $\phi(1.15) = 0.3749$) [4]

(*c*) Find the directional derivative of

$$\phi = xy^2 + yz^3$$

at the point (2, –1, 1) in the direction of vector $i + 2j + 2k$. [4]

Or

4. (*a*) Attempt any *one* : [4]

(*i*) Prove that $\frac{\bar{r}}{r^3}$ is solenoidal.

(*ii*) Show that :

$$\nabla \cdot \left[r \, \nabla \frac{1}{r^n} \right] = \frac{n(n-2)}{r^{n+2}}.$$

(*b*) Verify whether :

$$\bar{\mathrm{F}} = (2xyz^2)i + (x^2z^2 + z\cos yz)j + (2x^2yz + y\cos yz)k$$

is irrotational. [4]

(*c*) Two lines of regression are :

$$5y - 8x + 17 = 0 \quad \text{and} \quad 2y - 5x + 14 = 0.$$

If $\sigma_y^2 = 16$, find : [4]

(*i*) σ_x^2

(*ii*) Coefficient of correlation.

5. (*a*) Evaluate $\int_C \overline{F} . d\overline{r}$ for

$$\overline{F} = (2xy + 3x^2)\overline{i} + (x^2 + 4yz)\overline{j} + (2y^2 + 6yz)\overline{k}$$

where C is the curve $x = t$, $y = t^2$, $z = t^3$ joining (0, 0, 0) and (1, 1, 1). [4]

(*b*) Use divergence theorem to evaluate $\iint_S \overline{F} . d\overline{s}$

for $\overline{F} = 4xz\overline{i} - y^2\overline{j} + yz\overline{k}$ over the surface of cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$. [4]

(*c*) Using Stokes' theorem evaluate $\iint_S (\nabla \times \overline{F}) . d\overline{s}$

for $\overline{F} = (x^3 - y^3)\overline{i} - xyz\overline{j} + y^3\overline{k}$ where S is the surface $x^2 + ay^2 + z^2 - 2x = 4$ above the plane $x = 0$. [5]

Or

6. (*a*) Using Green's theorem, evaluate $\oint \overline{\mathrm{F}} \,.\, d\overline{r}$ for the field :

$$\overline{\mathrm{F}} = x^2 \,\overline{i} + xy\,\overline{j}$$

over the region R enclosed by $y = x^2$ and then line $y = x$. [4]

(*b*) Use divergence theorem to evaluate :

$$\iint_{\mathrm{S}} (x^3 \,\overline{i} + y^3 \,\overline{j} + z^3 \,\overline{k}) \,.\, d\overline{s}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$. [4]

(*c*) Evaluate $\int_{\mathrm{C}} \overline{\mathrm{F}} \,.\, d\overline{r}$ using Stokes' theorem for :

$$\overline{\mathrm{F}} = 4y\,\overline{i} + 2z\,\overline{j} + 6y\,\overline{k}$$

where C is the intersection of :

$$x^2 + y^2 + z^2 = 2z, \quad x = z - 1.$$ [5]

7. (*a*) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin\dfrac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by : [7]

$$y(x, t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right).$$

(*b*) Solve :

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

subject to : [6]

(*i*) $u(0, t) = 0$

(*ii*) $u_x(l, t) = 0$

(*iii*) $u(x, t)$ is bounded and

(*iv*) $u(x, 0) = \frac{u_0 x}{l}, \ 0 \le x \le l$.

Or

8. (*a*) Solve the equation :

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

subject to : [7]

(*i*) $v = 0$ when $y \to \infty$ for all x

(*ii*) $v = 0$ when $x = 0$ for all y

(*iii*) $v = 0$ when $x = l$ for all y

(*iv*) $v = x(l - x)$ when $y = 0$ for $0 < x < l\mathrm{T}$.

(*b*) Solve the wave equation : [6]

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

under the conditions :

(*i*) $u(0, t) = 0$

(*ii*) $u(\pi, t) = 0$

(*iii*) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

(*iv*) $u(x, 0) = x, \quad 0 < x < \pi$.