Solution

PHYSICS

JEE main - Physics

PHYSICS (Section-A)

1.

(b) $A \propto \frac{1}{\lambda^2}$

Explanation:

Suppose, $A = \frac{KA_0 V \lambda^x}{r}$

$\therefore L = \frac{KL^3 \cdot L^x}{L} = KL^{3+x}$, 3 + x = 1 or x = -2

$\therefore A \propto \lambda^{-2}$

2. **(a)** 30 km h^{-1}

Explanation:

Time taken by the car to cover first half of the distance is

$t_1 = \frac{100}{60}$

Time taken by the car to cover second half of the distance is

$t_2 = \frac{100}{v}$

Average speed, $v_{\text{av}} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$v_{\text{av}} = \frac{100+100}{t_1+t_2}$ or $40 = \frac{200}{\frac{100}{60} + \frac{100}{v}}$

$\frac{1}{60} + \frac{1}{v} = \frac{1}{20}$ or $\frac{1}{v} = \frac{1}{20} - \frac{1}{60}$

$\frac{1}{v} = \frac{2}{60} = \frac{1}{30}$ or, v = 30 km h^{-1}

3.

(d) 2l sin $\left(\frac{\theta}{2}\right)$

Explanation:

As the angle between 2 position vectors of same magnitude l is θ

so magnitude of change in position vectors is given as

$= \sqrt{l^2 + l^2 - 2l^2 \cos\theta} = l\sqrt{2(1 - \cos\theta)} = 2l\ \sin\frac{\theta}{2}$

4. **(a)** $\mu = \tan\theta\left(1 - \frac{1}{n^2}\right)$

Explanation:

Use the equation $v^2 - u^2 = 2ax$

On smooth inclined plane: $v^2 = 2g \sin\theta \times s$ ...(i)

On rough inclined plane:

$\left(\frac{v}{n}\right)^2 = 2g(\sin\theta - \mu\cos\theta) \times s$

Dividing eqn. (i) by eqn. (ii), $n^2 = \frac{\sin\theta}{\sin\theta - \mu\cos\theta}$

On solving, we get; $\mu = \left(1 - \frac{1}{n^2}\right)\tan\theta$

5. **(a)** 375 m/s

Explanation:

force = velocity $\times$ rate of flow of mass of water i.e., F = v $\times \frac{dm}{dt}$

F = v $\times$ 96 ...(i)

F = ma ...(ii)

equating equations (i) and (ii), we get,

v $\times$ 96 = ma

v $\times$ 96 = 1000 $\times$ 36

v = $\frac{1000 \times 36}{96}$

v = 375 m/s

6.

(d) 10

Explanation:

Angular acceleration is time derivative of angular speed and angular speed is time derivative of angular displacement.

By definition, $\alpha = \frac{d\omega}{dt}$

i.e., $d\omega = \alpha dt$

So, if in time t the angular speed of a body changes from ω_0 to ω.

$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha dt$

If α is constant,

$\omega - \omega_0 = \alpha t$

or $\omega = \omega_0 + \alpha t$ ...(i)

Now, as by definition

$\omega = \frac{d\theta}{dt}$

Now, eqn. (i) becomes

$\frac{d\theta}{dt} = \omega_0 + \alpha t$

i.e., $d\theta = (\omega_0 + \alpha t)\, dt$

So, if in the time t angular displacement is

$\int_0^{\theta} d\theta = \int_0^t (\omega_0 + \alpha)\, dt$

or $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ ...(ii)

Given, $\alpha = 3.0 \mathrm{rad/s}^2, \omega_0 = 2.0 \mathrm{rad/s}, t = 2$ s

Hence, $\theta = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2$

or θ = 4 + 6 = 10 rad

[**Note:** Eqn. (i) and (ii) are similar to first and second equations of linear motion.]

7.

(c) x = $\sqrt{2gh}\sqrt{\left(\frac{A^2 - a^2}{A^2}\right)}$

Explanation:

Applying Bernoulli's theorem, we have

$\frac{P}{\rho} + \frac{1}{2}(v')^2 + gh = \frac{P}{\rho} + \frac{1}{2}v^2 + 0$

where v' is the velocity of all surfaces of liquid and v the velocity of efflux.

Further, from the continuity equation,

Av' = av or v' = $\frac{av}{A}$

$\therefore \quad \frac{1}{2}\left(\frac{av}{A}\right)^2 + gh = \frac{1}{2}v^2$ = $\sqrt{2gh}\sqrt{\left(\frac{A^2 - a^2}{A^2}\right)}$

8.

(c) α_1 + 2α_2

Explanation:

V = $V_0(1 + \gamma\Delta\theta)$

or $L^3 = L_0(1 + \alpha_1\ \Delta\theta)L_0^2(1 + \alpha_2\ \Delta\theta)^2$

= $L_0^3(1 + \alpha_1\ \Delta\theta)(1 + \alpha_2\ \Delta\theta)^2$

Since, $L_0^3 = V_0$, hence

$1 + \gamma\Delta\theta = (1 + \alpha_1\ \Delta\theta)L_0^2(1 + \alpha_2\ \Delta\theta)^2$

$\cong (1 + \alpha_1\ \Delta\theta)L_0^2(1 + 2\alpha_2\ \Delta\theta) \cong 1 + \alpha_1\ \Delta\theta + 2\alpha_2\ \Delta\theta$

$\therefore \gamma = \alpha_1 + 2\alpha_2$

9. **(a)** $C_v (T_I - T_F)$

Explanation:

Work done in an adiabatic expansion is given by

W = $\frac{R}{\gamma - 1}[T_1 - T_2]$

The universal gas constant is given by:

R = $C_p - C_v$

W = $\frac{C_p - C_v}{\gamma - 1}[T_I - T_F]$

As $\gamma = \frac{C_p}{C_v}$

W = $\frac{C_p - C_v}{\frac{C_p}{C_v} - 1}[T_I - T_F]$

W = $\frac{C_p - C_v}{\frac{C_p - C_v}{C_v}}[T_I - T_F]$

W = $C_v (T_I - T_F)$

10.

(b) shift a little to the east as it moves to higher latitudes

Explanation:

Due to rotation of the earth about its axis, different points on its surface have the same angular velocity but different linear velocities, which is maximum at the equator and decreases at higher latitudes. The earth rotating from west to east imparts an eastward velocity to the wind mass. The wind mass moving northward from the equator retains its eastward velocity, which is greater than the eastward velocity of the surface at higher latitudes. Hence, relative to the earth's surface the wind mass shifts to the east.

11. **(a)** $\frac{U}{4}$

Explanation:

Energy stored in a capacitor is $\mathrm{U} = \frac{\mathrm{Q}^2}{2\mathrm{C}}$

As the battery is disconnected, total charge Q is shared equally by two capacitors.

So energy of each capacitor $= \frac{(\mathrm{Q}/2)^2}{2\mathrm{C}} = \frac{1}{4}\frac{\mathrm{Q}^2}{2\mathrm{C}} = \frac{\mathrm{U}}{4}$

12.

(d) a^2eB

Explanation:

a^2eB

13.

(b) 0.25

Explanation:

$e \propto NAB\omega$

So, $\frac{e_1}{e_2} = \frac{N_1\omega_1}{N_2\omega_2} = \frac{50 \times 50}{100 \times 100} = 0.25$

14.

(b) 1.0

Explanation:

The power factor for the LCR circuit will be :

When L is removed,

$\tan\phi = \frac{|X_c|}{R}$

$\Rightarrow \tan\frac{\pi}{3} = \frac{X_\mathrm{C}}{\mathrm{R}}$

When C is removed,

$\tan\phi = \frac{|\mathrm{X_L}|}{\mathrm{R}}$

$\Rightarrow \tan\frac{\pi}{3} = \frac{X_L}{R}$

Since, $X_L = X_c$, the circuit is in resonance,

Z = R

Power factor = $\cos\phi = \frac{R}{Z}$ = 1

15.

(d) 0.8

Explanation:

Power factor

$= \frac{R}{\sqrt{R^2+X_L^2}}$

$= \frac{8}{\sqrt{8^2+6^2}} = \frac{8}{10}$

= 0.8

16.

(b) $\sqrt{\frac{1}{\mu_0\varepsilon_0}}$

Explanation:

$\sqrt{\frac{1}{\mu_0\varepsilon_0}}$

17. **(a)** angular momentum

Explanation:

Energy of a photon, E = hν where h is the Planck's constant and ν is the frequency.

$\therefore$ [h] = $\frac{[\mathrm{E}]}{[\mathrm{v}]} = \frac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{T^{-1}}]}$ = $[ML^2T^{-1}]$

Angular momentum = Moment of inertia $\times$ Angular velocity (Angular momentum)

= $[ML^2]$ $[T^{-1}]$ = $[ML^2T^{-1}]$

18.

(c) 7 : 108

Explanation:

For Lyman series, n_1 = 1 and n_2 = 2 for first line

$\therefore \frac{1}{\lambda_1} = R\left[\frac{1}{l^2} - \frac{1}{2^2}\right] = R\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R}{4}$

For Paschen series, n_1 = 3 and n_2 = 4 for first line

$\therefore \frac{1}{\lambda_2} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = R\left[\frac{1}{9} - \frac{1}{16}\right] = \frac{7R}{144}$

$\frac{\lambda_1}{\lambda_2} = \frac{4/3R}{144/7R} = \frac{7}{108}$

19.

(d) $\frac{25}{9}$

Explanation:

Nucleus radius, R = $R_0A^{1/3}$

where R_0 is a constant and A is the mass number

$\therefore \quad \frac{R_{\mathrm{Te}}}{R_{\mathrm{A}}} = \left(\frac{A_{\mathrm{Te}}}{A_{\mathrm{A}}}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3}$

Assuming nucleus to be spherical

$\therefore$ Surface area, S = $4\pi R^2$

$\therefore \quad \frac{S_{\mathrm{Te}}}{S_{\mathrm{Al}}} = \left(\frac{R_{\mathrm{Te}}}{R_{\mathrm{Al}}}\right)^2 = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

20.

(b) there are no free electrons at 0 K

Explanation:

At absolute zero Kelvin temperature, covalent bonds are very strong and there are no free electrons and hence semiconductor behaves as a perfect insulator.

PHYSICS (Section-B)

21. 1

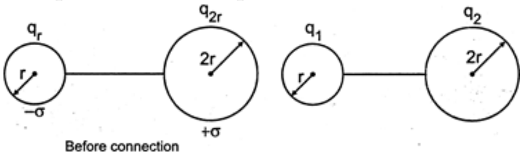
Explanation:

$q_r = -4\pi r^2\sigma, q_{2r} = 16\pi r^2\sigma, q_{\text{net}} = 12\pi r^2\sigma$

$\frac{q_1}{4\pi\varepsilon_0 r} = \frac{q_2}{4\pi\varepsilon_0 2r}$ or $q_1 = \frac{q_2}{2}$

and $q_1 + q_2 = q_{\text{net}} = 12\pi r^2\sigma$

So, $3q_1 = 12\pi r^2\sigma$ or $q_1 = 4\pi r^2\sigma$



So, surface charge density of smaller sphere after connection would be

$\sigma_1 = \frac{q_1}{4\pi r^2} = 1\sigma$

22. 04.00

Explanation:

At the surface of earth, g = $\frac{GM}{R^2}$

At point C $g_c = \frac{GM}{\left(R+\frac{R}{2}\right)^2} = \frac{4}{9}g$

At point A $g_A = g\left(1 - \frac{d}{R}\right)$ or, $g_A = g\left(1 - \frac{AB}{R}\right)$

From question,

$g_A = g_C \Rightarrow \frac{4}{9}g = g\left(1 - \frac{AB}{R}\right) \Rightarrow AB = \frac{5R}{9}$

$\therefore OA = OB - AB = R - \frac{5}{9}R = \frac{4K}{9}$

$\therefore \frac{OA}{AB} = \frac{x}{y} = \frac{\frac{4R}{9}}{\frac{5R}{9}} = \frac{4}{5}$ $\therefore$ x = 04.00

23. 8

Explanation:

The displacement of a simple harmonic oscillator is given by

x = A sin (ωt)

The potential energy of simple harmonic oscillator

U = $\frac{1}{2}\mathrm{kx}^2$

$\frac{\mathrm{dU}}{\mathrm{dt}} = \frac{1}{2}\mathrm{k2x}\frac{\mathrm{dx}}{\mathrm{dt}}$ $\left(\because \frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{A}\omega\cos\omega\mathrm{t}\right)$

= kA^2 $\omega\sin\omega t\cos\omega t$

$\left(\frac{\mathrm{dU}}{\mathrm{dt}}\right)_{\max}$ = $\frac{\mathrm{kA}^2\omega}{2}(\sin 2\omega\mathrm{t})_{\max}$

2ωt = $\frac{\pi}{2}$ $\Rightarrow$ t = $\frac{\pi}{4\omega}$ = $\frac{T}{8}$ $\Rightarrow \beta$ = 8

24. 7

Explanation:

$M = \frac{N\phi}{I} = \frac{2\left[\frac{\mu_0 IR^2}{2\left(8R^3\right)}\right]a^2\cos 45^\circ}{I}$

$= \frac{\mu_0 a^2}{8R2^{1/2}} = \frac{\mu_0 a^2}{R2^{7/2}}$

Hence, p = 7

25. 42.85

Explanation:

According to Newton's law of cooling, rate of loss of heat $\propto$ (T - T_0), where T is the average temperature in the given time interval. Hence, $mc\frac{(60-50)}{10} \propto \left(\frac{60+50}{2} - 25\right)$

and $mc\frac{(50-T)}{10} \propto \left(\frac{50+T}{2} - 25\right)$

Solving we get; T = 42.85^{o}C.