SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHS

JEE main - Mathematics

Time Allowed: 1 hour **Maximum Marks: 100**

General Instructions:

- All questions are compulsory.
- There are 25 questions where the first 20 questions are MCQs and the next 5 are numerical.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer.

MATHS (Section-A)

1. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is **[4]**

   a) $5! \times 6!$ b) $5^6 \times 15$

   c) 6^5 (15)! d) (15)! × 6!

2. Let z = x + iy be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the: **[4]**

   a) real axis b) line, y = -x

   c) imaginary axis d) line, y = x

3. The number of ways of selecting two numbers a and b, a ∈ {2, 4, 6,...,100} and b ∈{1, 3, 5,..., 99} such that 2 is the remainder when a + b is divided by 23 is **[4]**

   a) 186 b) 268

   c) 108 d) 54

4. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$ is 10k, then k is equal to: **[4]**

   a) 352 b) 336

   c) 176 d) 84

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and a + n = 33, then the value of n is **[4]**

   a) 21 b) 23

   c) 22 d) 24

6. Let f: R → R be a function such that $|f(x)| \leq x^2$, for all x ∈ R. Then, at x = 0, f is: **[4]**

   a) differentiable but not continuous. b) continuous as well as differentiable.

   c) neither continuous nor differentiable. d) continuous but not differentiable.

7. A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other **[4]**

into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is:

a) $\frac{66}{4+9\sqrt{3}}$ b) $\frac{22}{9+4\sqrt{3}}$

c) $\frac{22}{4+9\sqrt{3}}$ d) $\frac{66}{9+4\sqrt{3}}$

8. For $0 \leq x \leq \frac{\pi}{2}$, the value of $\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})dt$ equals: **[4]**

a) $-\frac{\pi}{4}$ b) 1

c) 0 d) $\frac{\pi}{4}$

9. If the two lines x + (a - 1)y = 1 and $2x + a^2y = 1$, ($a \in R - \{0, 1\}$) are perpendicular, then the distance of their point of intersection from the origin is **[4]**

a) $\frac{\sqrt{2}}{5}$ b) $\sqrt{\frac{2}{5}}$

c) $\frac{2}{5}$ d) $\frac{2}{\sqrt{5}}$

10. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the X-axis, lie on **[4]**

a) an ellipse which is not a circle b) a parabola

c) a hyperbola d) a circle

11. The locus of a point which divides the line segment joining the point (0, -1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is: **[4]**

a) $4x^2 - 3y = 2$ b) $9x^2 - 12y = 8$

c) $x^2 - 3y = 2$ d) $9x^2 - 3y = 2$

12. Let y = y(x) be the solution curve of the differential, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying y(0) = 1. This curve. This curve intersects the x-axis at a point whose abscissa is: **[4]**

a) -e b) 2 - e

c) 2 d) 2 + e

13. Let the line L pass through the point (0,1, 2), intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and be parallel to the plane 2x + y - 3z = 4 . Then the distance of the point P(1, -9, 2) from the line L is **[4]**

a) $\sqrt{54}$ b) $\sqrt{74}$

c) 9 d) $\sqrt{69}$

14. If $\vec{a} = \hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 7\hat{i} - 3\hat{k} + 4\hat{k}, \vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$ then $\vec{r} \cdot \vec{c}$ is equal to: **[4]**

a) 34 b) 36

c) 12 d) 30

15. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are co-prime, then m + n is equal to **[4]**

a) 317 b) 316

c) 315 d) 314

16. A person throws two fair dice. He wins Rs.15 for throwing a doublet (same numbers on the two dice), wins Rs.12 when the throw results in the sum of 9, and loses Rs.6 for any other outcome on the throw. Then, the expected gain/loss (in Rs.) of the person is **[4]**

a) 2 gain b) $\frac{1}{4}$loss

c) $\frac{1}{2}$gain d) $\frac{1}{2}$loss

17. If the lengths of the sides of a triangle are in AP and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is **[4]**

a) 5 : 9 : 13 b) 4 : 5 : 6

c) 3 : 4 : 5 d) 5 : 6 : 7

18. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3), then the area (in sq units) of $\triangle$PTQ is **[4]**

a) $60\sqrt{3}$ b) $36\sqrt{5}$

c) $54\sqrt{3}$ d) $45\sqrt{5}$

19. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true? **[4]**

a) $B \cap C \neq \phi$ b) If (A - C) $\subseteq$ B, then A $\subseteq$ B

c) $(C \cup A) \cap (C \cup B)$ = C d) If (A - B) $\subseteq$ C, then A $\subseteq$ C

20. Let A be any 3×3 invertible matrices. Then which one of the following is not always true? **[4]**

a) adj (adj (A)) = $|A| \cdot (\text{adj } (A))^{-1}$ b) adj (adj (A)) = $|A| \cdot A$

c) adj (adj (A)) = $|A|^2 \cdot (\text{adj } (A))^{-1}$ d) adj (A) = $|A| \cdot A^{-1}$

MATHS (Section-B)

21. Let a $\in$ Z and [t] be the greatest integer $\leq$ t. Then the number of points, where the function f(x) = [a + 13 sin x], x $\in (0, \pi)$ is not differentiable, is _____. **[4]**

22. If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ (a $\in$ R) are coplanar and $3(\vec{p} \cdot \vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$, then the value of λ is _____. **[4]**

23. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Define f : S $\rightarrow$ S as f(n) = $\begin{cases} 2n, & \text{if } n = 1,2,3,4,5 \\ 2n-11 & \text{if } n = 6,7,8,9,10 \end{cases}$ **[4]**

Let g: S $\rightarrow$ S be a function such that

fog(n) = $\begin{cases} n+1 & \text{, if } n \text{ is odd} \\ n-1 & \text{, if } n \text{ is even} \end{cases}$

g(10) ((g(1) +g(2) + g(3) + g(4) + g(5)) is equal to _____.

24. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is _____. **[4]**

25. Let z be those complex numbers which satisfy $|z + 5| \leq 4$ and $z(1 + i) + \bar{z}(1 - i) \geq -10$, $i = \sqrt{-1}$. If the maximum value of $|z+1|^2$ as $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is _____. **[4]**