SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHS

JEE main - Mathematics

Time Allowed: 1 hour **Maximum Marks: 100**

General Instructions:

- All questions are compulsory.
- There are 25 questions where the first 20 questions are MCQs and the next 5 are numerical.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer.

MATHS (Section-A)

1. The range of the function f : R - {1, 3} $\rightarrow$ R defined by f(x) = $\frac{2x^2-5x-3}{x^2-4x+3}$ is **[4]**

a) R - {1, 2} b) R - {3}

c) R - {2} d) R - {1}

2. A complex number z satisfies |z| = 1. If Im(z) < 0 then arg $\left(\frac{z+1}{z+i}\right)$ is **[4]**

a) $-\frac{\pi}{4}$ b) $\frac{3\pi}{4}$

c) $-\frac{3\pi}{4}$ d) $\frac{\pi}{4}$

3. In how many ways can the letters of the word **INTERMEDIATE** be arranged so that the two vowels do not occur together? **[4]**

a) 151200 b) 5040

c) 51200 d) 15120

4. If |x| is small so that x^2 and higher powers of x may be neglected, then an approximate value of $\frac{\left(1+\frac{2}{3}x\right)^{-3}(1-15x)^{-\frac{1}{5}}}{(2-3x)^4}$ is **[4]**

a) $\frac{1}{16}$(1 + 7x) b) $\frac{1}{8}$(1 + 7x)

c) $\frac{1}{16}$(1 - 7x) d) 1 - 7x

5. If A_1, A_2; G_1, G_2 and H_1, H_2 are two A.M.'s, G.M.'s and H.M.'s between two numbers respectively, then $\frac{G_1G_2}{H_1H_2} \times \frac{H_1+H_2}{A_1+A_2}$ = **[4]**

a) 1 b) 2

c) 0 d) 3

6. If $f(x) = \begin{cases} x+a, & x \leq 0 \\ |x-4|, & x > 0 \end{cases}$ and $g(x) = \begin{cases} x+1 & ,x < 0 \\ (x-4)^2+b & ,x \geq 0 \end{cases}$ are continuous on R, then (gof) (2) + (fog) (- 2) is equal to: **[4]**

a) -8 b) 8

c) 10 d) -10

7. If a_1, a_2, a_n are positive real numbers whose product is a fixed number e, the minimum value of $a_1 + a_2 + a_3 + \ldots a_{n-1} + 2a_n$ is: **[4]**

a) $2ne^{1/n}$ b) $n(2e)^{1/n}$

c) $(n + 1)\, e^{1/n}$ d) $(n + 1)\,(2e)^{1/n}$

8. If f(a + b + 1 - x) = f(x), for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x(f(x) + f(x + 1))dx$ is equal to: **[4]**

a) $\int_{a-1}^{b-1} f(x + 1)dx$ b) $\int_{a+1}^{b+1} f(x + 1)dx$

c) $\int_{a+1}^{b+1} f(x)dx$ d) $\int_{a-1}^{b-1} f(x)dx$

9. If the line 2x + 3y = 3 intersects the circle $x^2 + y^2 - 4 = 0$ at , A and B and M (α, β) is point of intersection of the tangents at A and B, then $\frac{\alpha}{\beta}$ is equal to : **[4]**

a) $\frac{3}{2}$ b) $\frac{2}{3}$

c) $\frac{3}{4}$ d) $\frac{4}{3}$

10. Locus of the point of intersection of the pair of perpendicular tangents to the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 7$ is the director circle of the circle with radius. **[4]**

a) $\sqrt{2}$ b) 4

c) 2 d) $2\sqrt{2}$

11. In a square matrix A = $[a_{ij}]$ of order 3, $a_{ii} = m_i + i$, where i = 1, 2, 3 m_i's are slopes ($|M_1| < |M_2| < |M_3|$) of the 3 normals concurrent at the point (9, - 6) to the parabola $y^2 = 4x$. Rest of all other entries are one. The value of tr. (A) is equal to: **[4]**

a) -6 b) 6

c) 3 d) -3

12. Let the solution curve of the differential equation $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, y(1) = 3 be y = y(x). Then y(2) is equal to: **[4]**

a) 17 b) 13

c) 11 d) 15

13. A vector $\vec{r}$ has length 6 units and direction ratios proportional to 2, -1, 2. Given that $\vec{r}$ makes an obtuse angle with x-axis. The component of $\vec{r}$ along x-axis is: **[4]**

a) -6 b) -4

c) 4 d) 6

14. Let A($\vec{a}$), B($\vec{b}$), C($\vec{c}$) and D($\vec{d}$) be the vertices of a convex quadrilateral. Consider the following statements: **[4]**

I. $\frac{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}|}{\overrightarrow{BA}\cdot\overrightarrow{BC}} + \frac{|\vec{c}\times\vec{d}+\vec{d}\times\vec{a}+\vec{a}\times\vec{c}|}{\overrightarrow{DA}\cdot\overrightarrow{DC}} = 0$

II. ☐ABCD is cyclic

III. tan B + tan D = 0

a) I and III only

b) I and II only

c) II and III only

d) I, II and III

15. The variance of first 50 even natural numbers is: **[4]**

a) 437

b) $\frac{437}{4}$

c) 833

d) $\frac{833}{4}$

16. For three events A, B and C, if P (exactly one of A or B occurs) = P(exactly one of B or C occurs) = P (exactly one of C or A occurs) = $\frac{1}{4}$ and P (all the three events occur simultaneously) = $\frac{1}{16}$, then the probability that atleast one of the events occurs, is **[4]**

a) $\frac{7}{64}$

b) $\frac{7}{16}$

c) $\frac{3}{16}$

d) $\frac{7}{32}$

17. If cos α + cos β = $\frac{3}{2}$ and sin α + sinβ $\frac{1}{2}$ and θ is the arithmetic mean of α and β, then sin 2θ + cos 2θ is equal to: **[4]**

a) $\frac{8}{5}$

b) $\frac{3}{5}$

c) $\frac{7}{5}$

d) $\frac{4}{5}$

18. If the latus rectum of a hyperbola subtends right angle at its centre, then its eccentricity is: **[4]**

a) $\sqrt{2}$

b) $\frac{\sqrt{3}+\sqrt{5}}{2}$

c) $\frac{\sqrt{3}+1}{2}$

d) $\frac{\sqrt{5}+1}{2}$

19. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be: **[4]**

a) 37

b) 55

c) 29

d) 65

20. Let A and B be two invertible matrices of order 3 $\times$ 3. If det(ABA^T) = 8 and det(AB^{-1}) = 8, then det($BA^{-1}B^T$) is equal to **[4]**

a) $\frac{1}{4}$

b) 1

c) $\frac{1}{16}$

d) 16

MATHS (Section-B)

21. Let a_n (e $\geq$ 1) be the value of x for which $\int\limits_{x}^{2x} e^{-t^n}$ dt (x > 0) is maximum. If L = $\lim\limits_{n\to\infty}$ ln (a_n) then find the value of e^{-L}. **[4]**

22. Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\vec{r} = (\alpha\hat{i} + \hat{\beta}\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to ________. **[4]**

23. Let f(x) be a polynomial of degree 3. If the curve y = f(x) has relative extrema at x = $\frac{\pm 2}{\sqrt{3}}$ and passes through (0, 0) and (1,-2) dividing the circle $x^2 + y^2 = 4$ in two parts, then the area bounded by $x^2 + y^2 = 4$ and y $\geq f(x)$ is $\frac{k\pi}{2}$. Find the value of k. **[4]**

24. Suppose f is a function satisfying f(x + y) = f(x) + f(y) for all x, y $\in \mathbb{N}$ and f(1) = $\frac{1}{5}$. If $\sum\limits_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to ________. **[4]**

[4]

25. If $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, Y = aI + BX + gX^2 and $Z = \alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2$, $\alpha, \beta, \gamma \in \mathbb{R}$.

If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$, then $(\alpha - \beta + \gamma)^2$ is equal to ________.