SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHS

JEE main - Mathematics

Time Allowed: 1 hour **Maximum Marks: 100**

General Instructions:

- All questions are compulsory.
- There are 25 questions where the first 20 questions are MCQs and the next 5 are numerical.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer.

MATHS (Section-A)

1. If $f(x) = \frac{5x+3}{7x-5}$, then **[4]**

   a) $f^{-1}(x) = f(x)$ b) $f^{-1}(x) + f(x) = 0$

   c) (fof) (x) = -x d) $f^{-1}(x) = \frac{1}{3} f(x)$

2. If |z - 1| = 2 and $z \neq$ -1, 3 then all the values $\frac{3+2z-z^2}{z-1}$ lie on **[4]**

   a) |z - 1| = 4 b) the imaginary axis

   c) a line not passing through the origin d) the real axis

3. The number of 4 digit numbers (with no repetition) to be formed from 1, 2, 3, 4 and divisible by 4 are **[4]**

   a) 6 b) 44

   c) 64 d) 24

4. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to **[4]**

   a) (- 21, 714) b) (28, 861)

   c) (-54, 315) d) (28, 315)

5. If a, b, c are in A.P., then $\frac{1}{\sqrt{a}+\sqrt{b}}, \frac{1}{\sqrt{a}+\sqrt{c}}, \frac{1}{\sqrt{b}+\sqrt{c}}$ are in: **[4]**

   a) G.P. b) H.P.

   c) H.M. d) A.P.

6. The number of points of non-differentiability of f(x) = max. $(||x| - 1|, \frac{1}{2})$ is **[4]**

   a) 2 b) 3

   c) 5 d) 4

7. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is **[4]**

a) $\frac{1}{18\pi}$ b) $\frac{1}{9\pi}$

c) $\frac{1}{36\pi}$ d) $\frac{5}{6\pi}$

8. If $2\int_0^1 \tan^{-1} x dx = \int_0^1 \cot^{-1}(1 - x + x^2)dx$, then $\int_0^1 \tan^{-1}(1 - x + x^2)dx$ is equal to **[4]**

a) log 4 b) log 2

c) $\frac{\pi}{2}$ + log 2 d) $\frac{\pi}{2}$ - log 4

9. Two sides a and b of triangle ABC are given by the roots of the equation x^2 - 4x + 1 = 0 and the included angle between them is $\frac{\pi}{3}$ then the value of $\left(\frac{c^2}{a+b}\right)$ is : **[4]**

a) $\frac{4}{13}$ b) $\frac{1}{4}$

c) $\frac{13}{4}$ d) 4

10. P is a lattice point (a point having integer coordinates) in the 1st quadrant. The segment joining $(\sqrt{33}, \sqrt{17})$ and $(-\sqrt{33}, -\sqrt{17})$ subtends a right angle at P. The number of points which satisfy P are **[4]**

a) 4 b) 3

c) 2 d) 12

11. The locus of the middle points of the focal chords of the parabola, $y^2 = 4x$ is: **[4]**

a) $y^2 = 3(x - 1)$ b) $y^2 = 2(x - 1)$

c) $y^2 = 2(1 - x)$ d) $y^2 = x - 1$

12. Let $y'(x) + \frac{g'(x)}{g(x)}, y(x) = \frac{g'(x)}{1+g^2(x)}$ where f'(x) denotes $\frac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on R. If g(1) = y(1) = 1 and g(e) = $\sqrt{(2e-1)}$ then y(e) equals : **[4]**

a) $\frac{2}{3g(e)}$ b) $\frac{3}{2g(e)}$

c) $\frac{1}{3g(e)}$ d) $\frac{1}{2g(e)}$

13. A point moves such that the sum of the squares of its distances from the six faces of a cube given by x = ± 2, y = ± 2, z = ± 2 is 28 units. The locus of the point is: **[4]**

a) x + y + z = 1 b) x + y + z = 2

c) $x^2 + y^2 + z^2 = 1$ d) $x^2 + y^2 + z^2 = 2$

14. Let $\vec{a} = -\hat{i} - j + \hat{k}, \vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to **[4]**

a) $3(\hat{i} + \hat{j} + \hat{k})$ b) $3(\hat{i} - \hat{j} - \hat{k})$

c) $3(\hat{i} - \hat{j} + \hat{k})$ d) $3(\hat{i} + \hat{j} - \hat{k})$

15. If 25% of the items are less than 20 and 25% are more than 40, the quartile deviation is **[4]**

a) 40 b) 10

c) 20 d) 30

16. Numbers 1 to 100 are written on slips of papers and are kept in a box. A draws one slip randomly and replaces it. B then draws a slip randomly. What is the probability that B draws a bigger number? **[4]**

a) $\frac{99}{20000}$ b) $\frac{99}{500}$

c) $\frac{49}{500}$ d) $\frac{99}{200}$

17. If $\frac{\pi}{2} < \alpha < \pi$, then $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}} + \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$ is equal to: **[4]**

a) -2 sec α b) 2 cos α

c) -2 cos α d) 2 sec α

18. If the circle $x^2 + y^2 = k^2$ and the rectangular hyperbola xy = k have no points in common, then the number of integral values of k, is: **[4]**

a) 3 b) 0

c) 2 d) 1

19. The set of intelligent students in a class is **[4]**

a) A finite set b) A singleton set

c) Not a well defined collection d) A null set

20. If $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \frac{abc,}{2} \neq 0$, then the area of the triangle whose vertices are $\left(\frac{x_1}{a}, \frac{y_1}{a}\right), \left(\frac{x_2}{b}, \frac{y_2}{b}\right), \left(\frac{x_3}{c}, \frac{y_3}{c}\right)$ is **[4]**

a) $\frac{1}{4}$abc b) $\frac{1}{4}$

c) $\frac{1}{8}$ d) $\frac{1}{8}$abc

MATHS (Section-B)

21. Let f(x) be a cubic polynomial with f(1) = -10, f(-1) = 6, and has a local minima at x = 1, and f'(x) has a local minima at x = -1. Then f(3) is equal to _____. **[4]**

22. A line **L** passing through origin is perpendicular to the lines **[4]**

$L_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$L_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$

If the co-ordinates of the point in the first octant on $\mathbf{L_2}$ at a distance of $\sqrt{17}$ from the point of intersection of **L** and $\mathbf{L_1}$ are (a, b, c) then 18(a + b + c) is equal to _____.

23. Let the area of the region $\{(x,y) : |2x-1| \leq y \leq |x^2 - x|,\ 0 \leq x \leq 1\}$ be A. **[4]**

Then $(6A + 11)^2$ is equal to _____.

24. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1}$, $b_n = b_{n-1} + a_{n-1}$, $\forall \mathbf{n} \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to _____. **[4]**

25. Let a, b and c be three real numbers satisfying **[4]**

$$[a \quad b \quad c] \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0 \quad 0 \quad 0] \text{ ..... (i)}$$

If the point P (a, b, c), with reference to Eq. (i), lies on the plane 2x + y + z = 1, then the value of 7a + b + c is _____.