**Solution**

**MATHS**

**JEE main - Mathematics**

**MATHS (Section-A)**

1.

**(d)** (15)! $\times$ 6!

**Explanation:**

According to given information, we have if

k $\in${4, 8, 12, 16, 20}

Then, f (k) $\in$ {3, 6, 9, 12, 15, 18}

[$\because$ Codomain (f) = {1, 2, 3, ....20}]

Now, we need to assign the value of f(k) for

k $\in${4, 8, 12, 16, 20} this can be done in $^6C_5$ . 5! ways

= 6 . 5! = 6! and remaining 15 element can be associated by 15! ways.

$\therefore$ Total number of onto functions = 15! 6!

2.

**(d)** line, y = x

**Explanation:**

Let z = x + iy

$\because$ $z^2 = i|z|^2$

$\therefore$ $x^2 - y^2 + 2ixy = i(x^2 + y^2)$

$\Rightarrow$ $x^2 - y^2 = 0$ and $2xy = x^2 + y^2$

$\Rightarrow$ $(x - y)(x + y) = 0$ and $(x - y)^2 = 0$

$\Rightarrow$ x = y

3.

**(c)** 108

**Explanation:**

Given a $\in$ {2, 4, 6, 8, 10, ..., 100}

b $\in$ {1, 3, 5, 7, 9, ..., 99}

Now, a + b $\in$ {25, 71, 117, 163}

i. a + b = 25, no. of ordered pairs (a, b) is 12
ii. a + b = 71, no. of ordered pairs (a, b) is 35
iii. a + b = 117, no. of ordered pairs (a, b) is 42
iv. a + b = 163, no. of ordered pairs (a, b) is 19

$\therefore$ total = 12 + 35 + 42 + 19 = 108 pairs

4.

**(b)** 336

**Explanation:**

General term of

$\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10} = {}^{10}C_r\left(\alpha x^{\frac{1}{9}}\right)^{10-r}\left(\beta x^{\frac{-1}{6}}\right)^{r}$

$= {}^{10}C_r\, \alpha^{10-r}\beta^{r}(x)^{\frac{10-r}{9}-\frac{r}{6}}$

Term independent of x if $\frac{10-r}{9}-\frac{r}{6}=0 \Rightarrow$ r = 4

$\therefore$ Term independent of x = ${}^{10}C_4\alpha^6\beta^4$

Since $\alpha^3+\beta^2$ = 4

Then, by AM-GM inequality

$\frac{\alpha^3+\beta^2}{2} \geq \left(\alpha^3 b^2\right)^{\frac{1}{2}}$

$\Rightarrow (2)^2 \geq \alpha^3\beta^2 \Rightarrow \alpha^6\beta^4 \leq 16$

$\because$ The maximum value of the term independent of x = 10k

$\therefore$ 10k = $^{10}C_4 \cdot 16 \Rightarrow$ k = 336

5.

**(b)** 23

**Explanation:**

Here d = $\frac{100-a}{n+1}$

Now, $A_1$ = a + d and, $A_n$ = 100 - d

so, $\Rightarrow \frac{A_1}{A_n}=\frac{1}{7} \Rightarrow \frac{a+d}{100-d} = \frac{1}{7} \Rightarrow$ 7a + 8d = 100

$\Rightarrow$ 7a + 8$\left(\frac{100-a}{n+1}\right)$ = 100 ...(i)

$\because$ a + n = 33 ...(ii)

Now, by Eq, (i) and (ii)

$7n^2 - 132n - 667 = 0$

n = 23 and n = $\frac{-29}{7}$ reject.

6.

**(b)** continuous as well as differentiable.

**Explanation:**

Let $|f(x)| \leq x^2, \forall x \in R$

Now, at x = 0, | f(0) $\leq$ 0

$\Rightarrow$ f(0) = 0

$\therefore f'(0) = \lim_{h\to 0}\frac{f(h)-f(0)}{h-0} = \lim_{h\to 0}\frac{f(h)}{h}$ ...(i)

Now, $\left|\frac{f(h)}{h}\right| \leq |h| \left(\because |f(x)| \leq x^2\right)$

$\Rightarrow -|h| \leq \frac{f(h)}{h} \leq |h|$

$\Rightarrow \lim_{h\to 0}\frac{f(h)}{h} \to 0$ ...(ii)

(using sandwich Theorem)

$\therefore$ from (i) and (ii) we get f'(0) = 0,

i.e. - f (x) is differentiable, at x = 0

Since, differentiability $\Rightarrow$ Continuity

$\therefore$ | f(x) | $\leq x^2$, for all x $\in$ R is continuous as well as differentiable at x = 0.

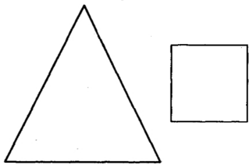
7.

**(d)** $\frac{66}{9+4\sqrt{3}}$

**Explanation:**

Since, length of the given wire = 22m

Let xm length is used in equilateral $\triangle$ and rest for the square



Let side of an equilateral $\triangle$ = a

and side of a square = b

$\Rightarrow$ 3a = x 4b = 22 - x

now, total area = $\frac{\sqrt{3}}{4}a^2 + b^2$

$\frac{\sqrt{3}}{4}\frac{x^2}{9} + \frac{(22-x)^2}{16}$
$\frac{dA}{dx} = 0 \Rightarrow x\left(\frac{\sqrt{3}}{2\times 9} + \frac{1}{8}\right) = \frac{22}{8} \Rightarrow x\left(\frac{4\sqrt{3}+9}{36}\right) = \frac{11}{2}$
a = $\frac{x}{3}$
$a = \left(\frac{11/2}{\frac{4\sqrt{3}+9}{36}}\right)\left(\frac{1}{3}\right) = \frac{66}{4\sqrt{3}+9}$

8.

**(d)** $\frac{\pi}{4}$
**Explanation:**
Consider
$\int_0^{\sin^2 x}$ sin$^{-1}$ ($\sqrt{t}$)dt + $\int_0^{\cos^2 x}$ cos$^{-1}$ ($\sqrt{t}$)dt
Let I = f(x) after integrating and putting the limits.
$f'(x) = \sin^{-1}\sqrt{\sin^2 x}$ (2 sin x cos x) - 0 + cos$^{-1}$ $\sqrt{\cos^2 x}$ (-2 cos x sin x) - 0
$\therefore f'(x) = 0 \Rightarrow$ f(x) = C (constant)
Now, we find f(x) at x = $\frac{\pi}{4}$
$\therefore I = \int_0^{\frac{1}{2}} \sin^{-1}\sqrt{t}dt + \int_0^{\frac{1}{2}} \cos^{-1}\sqrt{t}dt$
= $\int_0^{\frac{1}{2}} \left(\sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t}\right) dt = \int_0^{\frac{1}{2}} \frac{\pi}{2}dt = \frac{\pi}{4}$ = C
$\therefore f(x) = \frac{\pi}{4}$
∴ Required integration = $\frac{\pi}{4}$

9.

**(b)** $\sqrt{\frac{2}{5}}$
**Explanation:**
Given, lines x + (a - 1)y = 1 and 2x + $a^2$y = 1, where a ∈ R - {0, 1} are perpendicular to each other
∴ $\left(-\frac{1}{a-1}\right) \times \left(-\frac{2}{a^2}\right)$ = -1 [∵ If lines are perpendicular, then product of their slopes is -1]
⇒ $a^2$ (a - 1) = -2 ⇒$a^3$ - $a^2$ + 2 = 0
⇒ (a + 1) ($a^2$ - 2a + 2) = 0 ⇒a = -1
∴ Equation of lines are
x - 2y = 1 ...(i)
and 2x + y = 1 ...(iii)
On solving Eq. (i) and Eq. (ii), we get
x = $\frac{3}{5}$ and y = $-\frac{1}{5}$
∴ Point of intersection of the lines (i) and (ii) is $P\left(\frac{3}{5}, -\frac{1}{5}\right)$
Now, required distance of the point $P\left(\frac{3}{5}, -\frac{1}{5}\right)$ from origin $= \sqrt{\frac{9}{25} + \frac{1}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$

10.

**(b)** a parabola
**Explanation:**
Given equation of circle is $x^2$ + $y^2$ - 8x - 8y - 4 = 0, whose centre is C(4, 4) and radius $= \sqrt{4^2 + 4^2 + 4} = \sqrt{36}$ = 6
Let the centre of required circle be $C_1$ (x, y). Now, as it touch the X-axis, therefore its radius = |y|.
Also, it touch the circle $x^2$ + $y^2$ - 8x - 8y - 4 = 0, therefore $CC_1$ = 6 + |y|
⇒ $\sqrt{(x-4)^2 + (y-4)^2}$ = 6 + |y|
⇒ $x^2$ + 16 - 8x + $y^2$ + 16 - 8y = 36 + $y^2$ + 12 |y|

$\Rightarrow x^2 - 8x - 8y + 32 = 36 + 12|y|$

$\Rightarrow x^2 - 8x - 8y - 4 = 12|y|$

**Case I:** If y > 0, then we have

$x^2 - 8x - 8y - 4 = 12y$

$\Rightarrow x^2 - 8x - 20y - 4 = 0$

$\Rightarrow (x - 4)^2 - 20 = 20y$

$\Rightarrow (x - 4)^2 = 20(y + 1)$ which is a parabola.

**Case II:** If y < 0, then we have

$x^2 - 8x - 8y - 4 = -12y$

$\Rightarrow x^2 - 8x - 8y - 4 + 12y = 0$

$\Rightarrow x^2 - 8x + 4y - 4 = 0$

$\Rightarrow x^2 - 8x - 4 = -4y$

$\Rightarrow (x - 4)^2 = 20 - 4y$

$\Rightarrow (x - 4)^2 = -4(y - 5)$, which is again a parabola.

11.

**(b)** $9x^2 - 12y = 8$

**Explanation:**

Let point P be $(2t, t^2)$ and Q be (h, k)

Using section formula,

$h = \frac{2t}{3}, k = \frac{-2+t^2}{3}$

Hence, locus is 3k + 2 $= \left(\frac{3h}{2}\right)^2$

$\Rightarrow 9x^2 - 12y + 8$

12.

**(b)** 2 - e

**Explanation:**

The given differential equation is $\frac{dx}{dy}$ + x = $y^2$ Comparing with $\frac{dx}{dy}$ + Px = Q, where P = 1, Q = $y^2$

Now, I.F. = $e^{\int 1.dy} = e^y$

$x \cdot e^y = \int \left(y^2\right) e^y \cdot dy = y^2 \cdot e^y - \int 2y \cdot e^y \cdot dy$

= $y^2e^y - 2(y \cdot e^y - e^y) + C$

$\Rightarrow x \cdot e^y = y^2e^y - 2ye^y + 2e^y + C$

$\Rightarrow x = y^2 - 2y + 2 + C.e^{-y}$ ...(i)

As y(0) = 1, satisfying the given differential eqn,

$\therefore$ put x = 0,y= 1 in eqn. (i)

0 = 1 - 2 + 2 + $\frac{C}{e}$

C = -e

y = 0, x = 0 - 0 + 2 + (-e) $(e^{-0})$

x = 2 - e

13.

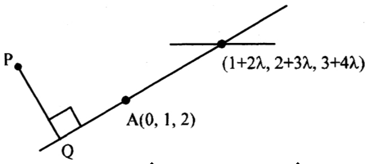
**(b)** $\sqrt{74}$

**Explanation:**

Let A(0, 1, 2)

B(1 + 2λ, 2 + 3λ, 3 + 4λ)

So, $\overrightarrow{AB} \cdot \vec{n} = 0$




$\Rightarrow [(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}] \cdot (2\hat{i} + \hat{j} - 3\hat{k})$

$2 + 4\lambda + 1 + 3\lambda - 3 - 12\lambda = 0$

$\Rightarrow 5\lambda = 0 \Rightarrow \lambda = 0$

Line $\overrightarrow{AB}$, $\overrightarrow{r} = \hat{j} + 2\hat{k} + \mu(\hat{i} + \hat{j} + \hat{k})$

General form: $Q(\mu, 1+\mu, 2+\mu)$

$\therefore \overrightarrow{PQ} \cdot \overrightarrow{AB} = 0$

$\Rightarrow (\mu - 1) + (10 + \mu) + \mu = 0 \Rightarrow 3\mu = -9 \Rightarrow \mu = -3$

$\therefore$ distance $= \sqrt{16 + 49 + 9} = \sqrt{74}$

14. **(a)** 34

**Explanation:**

Given, $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = 0 \Rightarrow \vec{r} \times \vec{b} - \vec{c} \times \vec{b} = 0$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = 0 \Rightarrow \vec{r} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$

And given that $\vec{r} \cdot \vec{a} = 0$

$\Rightarrow (\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 0 \Rightarrow \vec{c} \cdot \vec{a} + \lambda\vec{b} \cdot \vec{a} = 0$

$\Rightarrow \lambda = \frac{-\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}$

Now $\vec{r} \cdot \vec{c} = (\vec{c} + \lambda\vec{b}) \cdot \vec{c} = |\vec{c}|^2 + \lambda\vec{b} \cdot \vec{c}$

$= |\vec{c}|^2 - \left(\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\right)(\vec{b} \cdot \vec{c}) = 74 - \left[\frac{15}{3}\right] 8$ = 74 - 40 = 34

15. **(a)** 317

**Explanation:**

u = $\frac{\sum x}{12} = \frac{9}{2} \Rightarrow \sum x$ = 54

v = $\frac{\sum x^2}{12} = \left(\frac{9}{2}\right)^2 = 4 \Rightarrow \sum x^2$ = 291

$\sum x_{\text{new}}$ = 54 - (9 + 10) + 7 + 14 = 56

$\sum x^2_{\text{new}}$ = 291 - (81 + 100) + 49 + 196 = 355

$\sigma^2_{\text{new}} = \frac{355}{12} - \left(\frac{56}{12}\right)^2 = \frac{281}{36} = \frac{m}{n} \Rightarrow$ m + n = 317

16.

**(d)** $\frac{1}{2}$loss

**Explanation:**

It is given that a person wins ₹ 15 for throwing a doublet (1, 1) (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) and win ₹ 12 when the throw results in sum of 9, i.e., when (3, 6), (4, 5), (5, 4), (6, 3) occurs.

Also, losses ₹ 6 for throwing any other outcome, i.e., when any of the rest 36 - 6 - 4 = 26 outcomes occurs.

Now, the expected gain/loss

= 15 $\times$ P (getting a doublet) + 12 $\times$ P (getting sum 9) - 6 $\times$ P (getting any of rest 26 outcome)

$= \left(15 \times \frac{6}{36}\right) + \left(12 \times \frac{4}{36}\right) - \left(6 \times \frac{26}{36}\right)$

$= \frac{5}{2} + \frac{4}{3} - \frac{26}{6} = \frac{15+8-26}{6}$

$= \frac{23-26}{6} = -\frac{3}{6} = -\frac{1}{2}$ , means loss of ₹ $\frac{1}{2}$

17.

**(b)** 4 : 5 : 6

**Explanation:**

Let a, b and c be the lengths of sides of a $\triangle$ABC such that a < b < c.

Since, sides are in AP.

$\therefore$ 2b = a + c ....(i)

Let $\angle A=\theta$

Then, $\angle C=2\theta$ [according to the equation]

So, $\angle B=\pi-3\theta$ .....(ii)

On applying sine rule in Eqs.(i), we get

2sin B = sin A + sin C

$\Rightarrow 2\sin(\pi-3\theta)=\sin\theta+\sin 2\theta$ [from Eq.(ii)]

$\Rightarrow 2\sin 3\theta=\sin\theta+\sin 2\theta$

$\Rightarrow 2\left[3\sin\theta-4\sin^3\theta\right]=\sin\theta+2\sin\theta\cos\theta$

$\Rightarrow 6-8\sin^2\theta=1+2\cos\theta$ [$\because$ sin $\theta$ can not be zero]

$\Rightarrow 6-8\left(1-\cos^2\theta\right)=1+2\cos\theta$

$\Rightarrow 8\cos^2\theta-2\cos\theta-3=0$

$\Rightarrow (2\cos\theta+1)(4\cos\theta-3)=0$

$\Rightarrow \cos\theta=\frac{3}{4}$ or $\cos\theta=-\frac{1}{2}$ (rejected).

Clearly, the ratio of sides is a : b : c

= $\sin\theta : \sin 3\theta : \sin 2\theta$

= $\sin\theta : \left(3\sin\theta-4\sin^3\theta\right) : 2\sin\theta\cos\theta$

= $1 : \left(3-4\sin^2\theta\right) : 2\cos\theta$

= $1 : \left(4\cos^2\theta-1\right) : 2\cos\theta$

= $1 : \frac{5}{4} : \frac{6}{4}=4:5:6$

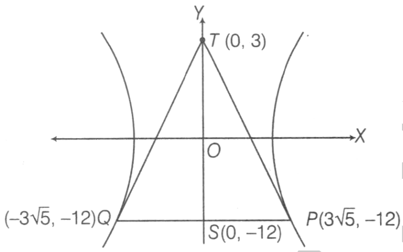
18.

**(d)** $45\sqrt{5}$

**Explanation:**

Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the point P and Q.

Tangent intersects at point T(0,3)




Clearly, PQ is chord of contact.

$\therefore$ Equation of PQ is -3y = 36

$\Rightarrow$ y = -12

Solving the curve $4x^2 - y^2 = 36$ and y = -12,

we get x = $\pm 3\sqrt{5}$

Area of $\triangle PQT$ = $\frac{1}{2}\times PQ\times ST$ = $\frac{1}{2}(6\sqrt{5}\times 15)=45\sqrt{5}$

19.

**(b)** If (A - C) $\subseteq$ B, then A $\subseteq$ B

**Explanation:**

If (A - C) $\subseteq$ B, then A $\subseteq$ B

If A = C then A - C = $\phi$

Clearly, $\phi \subseteq B$ but A $\subseteq$ B is not always true.

20.

**(b)** adj (adj (A)) = |A| · A

**Explanation:**

adj (adj (A)) = |A| · A

**MATHS (Section-B)**

21. 25.0

Explanation:

f(x) = [a + 13sin x] = a + [13sin x], x $\in$(0, $\pi$) We know that for [n sin x]; Total number of non-differentiable points are = 2n -1 for x $\in$ (0, $\pi$) So, number of non-differentiable points for [13 sin x] = 25 points

22. 1.0

Explanation:

$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix}$ = 0

$\Rightarrow$ 3a + 1 = 0 $\Rightarrow$ a = $-\frac{1}{3}$

The given vectors,

$\vec{p}=\frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}-\frac{1}{3}\hat{k}=\frac{1}{3}(2\hat{i}-\hat{j}-\hat{k})$

$\vec{q}=\frac{1}{3}(-\hat{i}+2\hat{j}-\hat{k})$

$\vec{r}=\frac{1}{3}(-\hat{i}-\hat{j}+2\hat{k})$

Now, $\vec{p}\cdot\vec{q}=\frac{1}{9}$ (-2 - 2 + 1) = $-\frac{1}{3}$

$\vec{r}\times\vec{q}=\frac{1}{9}\begin{vmatrix} i & j & k \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$

$=\frac{1}{9}$ (i(4 - 1) - j(-2 - 1) + k(1 + 2))

$=\frac{1}{9}$ (3i + 3j + 3k) = $\frac{i+j+k}{3}$

$|\vec{r}\times\vec{q}|=\frac{1}{3}\sqrt{3}\Rightarrow|\vec{r}\times\vec{q}|^2=\frac{1}{3}$

$3(\vec{p}\cdot\vec{q})^2-\lambda|\vec{r}\times\vec{q}|^2$ = 0

$\Rightarrow 3\cdot\frac{1}{9}-\lambda\cdot\frac{1}{3}$ = 0

$\Rightarrow \lambda$ = 1

23. 190.0

Explanation:

Given a function f(n) = $\begin{cases} 2n, & \text{if } n=1,2,3,5 \\ 2n-11, & \text{if } n=6,7,\ldots 10 \end{cases}$

Put n = 1, 2, 3, 4... 10

f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8,...

... f(6) = 1, f(7) = 3, f(8) = 5... f(10) = 9

Take fog (n) = $\begin{cases} (n+1), & \text{if } n \text{ is odd} \\ (n-1), & \text{if } n \text{ is even} \end{cases}$

Put n = 1, 2, 3... 10

f(g(1)) = 2, f(g(2)) = 1, f(g(3)) = 4, f(g(4)) = 3, f(g(5)) = 6, f(g(10)) = 9

As, f(g(10)) = 9, and f(10) = 9, then g(10) = 10

Similarly, g(1) = 1, g(2) = 6, g(3) = 2, g(4) = 7, g(5) = 3

Put the values in the required expression,

g(10) (g(1) + g(2) + g(3) + g(4) + g(5)

$\Rightarrow$ 10(1 + 6 + 2 + 7 + 3)

$\Rightarrow$ 10 $\times$ (19) = 190

24. 3000.0

Explanation:

Since 54 = $3^3\times 2$

Given that number whose G.C.D. with 54 is 2.

$\therefore$ Numbers should be divisible by 2 but not by 3

N = (Numbers divisible by 2) - (Numbers divisible by 6)

N = $\frac{9000}{2}-\frac{9000}{6}$ = 4500 - 1500 = 3000

25. 48

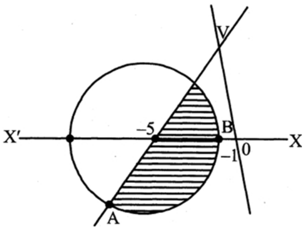
Explanation:

$z(1 + i) + \bar{z}(1 - i) \geq -10$

$\Rightarrow (z + \bar{z}) + i(z - \bar{z}) \geq -10 \Rightarrow x - y + 5 \geq 0$

And $|z + 5| \leq 4$ is interior of a circle with centre (-5, 0) and radius 4.

$\therefore |z + 1|$ represents the distance of z from -1.




$|z + 1|$ is maximum at A.

On solving equation of circle and line we get

$A(-2\sqrt{2} - 5, -2\sqrt{2})$

$|z + 1|^2 = AB^2 = (2\sqrt{2} + 4)^2 + (2\sqrt{2})^2$

$\alpha + \beta\sqrt{2} = 32 + 16\sqrt{2}$

So, $\alpha + \beta = 32 + 16 = 48$