SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHS

JEE main - Mathematics

Time Allowed: 1 hour **Maximum Marks: 100**

General Instructions:

- All questions are compulsory.
- There are 25 questions where the first 20 questions are MCQs and the next 5 are numerical.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer.

MATHS (Section-A)

1. The domain of function f defined as $\frac{1}{\sqrt{(x-4)(x-5)}}$ is **[4]**

a) $(-\infty, 4) \cup (5, \infty)$
b) $(-\infty, 4] \cup [5, \infty)$
c) $(-\infty, 4) \cup [5, \infty)$
d) $(-\infty, 4] \cup (5, \infty)$

2. Let $\omega = \frac{z^2 - 3z + 6}{z + 1}$ and z = 1 + i, then $|\omega|$ and amp ω respectively are: **[4]**

a) 2, $\frac{3\pi}{4}$
b) 2, $-\frac{\pi}{4}$
c) $\sqrt{2}$, $\frac{3\pi}{4}$
d) $\sqrt{2}, -\frac{\pi}{4}$

3. How many 4-letter words can be made from the word **MATHEMATICS** such that no letter is repeated? **[4]**

a) 1680
b) 330
c) 990
d) 7920

4. Arrange the expansion of $\left(x^{\frac{1}{2}} + \frac{1}{2x^{\frac{1}{4}}}\right)^n$ in decreasing powers of x. Suppose the coefficient of the first three terms (taken in that order) form an arithmetic progression. Then the number of terms in the expansion having integral powers of x, is: **[4]**

a) more than 3
b) 3
c) 1
d) 2

5. If a, b, c are in G.P., then: **[4]**

a) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in G.P.
b) $\frac{a^2}{b+c}, \frac{b^2}{c+a}, \frac{c^2}{a+b}$ are in G.P.
c) a^2, b^2, c^2 are in G.P.
d) $a^2(b + c), c^2(a + b), b^2(a + c)$ are in G.P.

6. If $x^y = e^{x - y}$, then y' (1) equals: **[4]**

a) $\frac{1}{1+\log 2}$
b) 1
c) -1
d) 0

7. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2. If $\lim_{x \to 0}\left[1 + \frac{f(x)}{x^2}\right] = 3$, then **[4]**

f(2) is equal to

a) -8
b) 0
c) -4
d) 4

8. Let $I_1 = \int_0^x e^{tx} \cdot e^{-t^2}$ dt and $I_2 = \int_0^x e^{-t^2/4}$ dt where x > 0 then the value of $\frac{I_1}{I_2}$ is : **[4]**

a) $e^{-x^2/4}$
b) $e^{-x^2/2}$
c) $e^{x^2/2}$
d) $e^{x^2/4}$

9. A variable line L passes through the point of intersection of the lines 3x + 4y = 12 and x + 2y - 5 = 0 meeting the coordinate axes at A and B, then the locus of foot of perpendicular from origin to it, is : **[4]**

a) $\frac{3}{y} + \frac{4}{x} = 4$
b) $2(x^2 + y^2) = 4x + 3y$
c) $x^2 + y^2 = 4x + 3y$
d) $\frac{3}{y} + \frac{4}{x} = 6$

10. If the circles **[4]**

$x^2 + y^2 + 2ax + 2by + c = 0$

and $x^2 + y^2 + 2bx + 2ay + c = 0$

where c > 0, have exactly one point in common then the value of $\frac{(a+b)^2}{2c}$ is:

a) 1
b) $\sqrt{2}$
c) $\frac{1}{2}$
d) 2

11. If y = mx + 4 is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to: **[4]**

a) -64
b) -32
c) -128
d) 128

12. Let y = y(x) be the solution of the differential equation cos x$\frac{dy}{dx}$ + 2y sin x = sin 2x, x $\in \left(0, \frac{\pi}{2}\right)$. If $y(\frac{\pi}{3})$ = 0, then $y(\frac{\pi}{4})$ is equal to: **[4]**

a) $\frac{1}{\sqrt{2}}$ - 1
b) 2 + $\sqrt{2}$
c) 2 - $\sqrt{2}$
d) $\sqrt{2}$ - 2

13. A plane meets the axes in A, B and C such that the centroid of the triangle ABC is (2, 3, 4). The equation of the plane is: **[4]**

a) 6x + 4y + 3z = 36
b) 9x + 16y + 25z = 36
c) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9}$ = 1
d) $\frac{x}{2} + \frac{y}{3} + \frac{z}{4}$ = 1

14. If $\vec{a} = \frac{1}{\sqrt{3}}(\hat{i} - \sqrt{2}\hat{j})$ and $\vec{b} = \sqrt{2}\hat{i} + \hat{j} - \sqrt{2}\hat{k}$, then the value of $(2\vec{a} - \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is **[4]**

a) -5
b) 25
c) -25
d) $\sqrt{5}$

15. For two data sets, each of size 10, the variances are given to be 8 and 12, their corresponding means are given to be 6 and 9 respectively. The standard deviation of the combined data set is **[4]**

a) $\frac{11\sqrt{17}}{2}$
b) $\frac{11}{4}$
c) $\frac{7}{4}$
d) $\frac{5\sqrt{17}}{2}$

16. It is given that the events A and B are such that P(A) = $\frac{1}{4}$, P(A|B) = $\frac{1}{2}$ and P(B|A) = $\frac{2}{3}$. Then, P(B) is: **[4]**

a) $\frac{1}{3}$
b) $\frac{1}{6}$
c) $\frac{1}{2}$
d) $\frac{2}{3}$

17. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to **[4]**

a) $\frac{3}{16}$
b) $\frac{1}{32}$
c) $\frac{9}{32}$
d) $\frac{1}{16}$

18. If (0, $\pm$4) and (0, $\pm$2) be the foci and vertices of a hyperbola, then its equation is **[4]**

a) $\frac{x^2}{12}-\frac{y^2}{4}=1$
b) $\frac{x^2}{4}-\frac{y^2}{12}=1$
c) $\frac{y^2}{12}-\frac{x^2}{4}=1$
d) $\frac{y^2}{4}-\frac{x^2}{12}=1$

19. If Q = {x : x = $\frac{1}{y}$, where y $\in$ N}, then **[4]**

a) $\frac{2}{3} \in$ Q
b) 2 $\in$ Q
c) 0 $\in$ Q
d) 1 $\in$ Q

20. If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdots\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is **[4]**

a) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$
b) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$
c) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$
d) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$

MATHS (Section-B)

21. The number of values of x, where the function f(x) = cos x + cos ($\sqrt{2}$x) attains its maximum, is ________. **[4]**

22. Let $\vec{a},\vec{b},\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to ________. **[4]**

23. Let y = f(x) = $\begin{cases}\sqrt{x+3}, & -3\le x<-2\\ -1+\sqrt{x+2}, & -2\le x<-1\\ -2+\sqrt{x+1}, & -1\le x\le 0\end{cases}$. If |y| = f(- |x|) be a curve and area enclosed between the curve and the circle $x^2+y^2=5$ equals p + πq, where p and q are integers then find the value of (p + q). **[4]**

24. The sum of the common terms of the following three arithmetic progressions. 3, 7, 11, 15,.., 399, 2, 5, 8, 11,.., 359 and 2, 7, 12, 17,..., 197, is equal to ________. **[4]**

25. Let z = $\frac{-1+\sqrt{3}i}{2}$, where i = $\sqrt{-1}$, and r, s $\in$ {1, 2, 3}. Let P = $\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which P^2 = -1 is: **[4]**