Solution

MATHS

JEE main - Mathematics

MATHS (Section-A)

1. **(a)** $(-\infty, 4) \cup (5, \infty)$

Explanation:

Since, domain of $\frac{1}{\sqrt{(x-a)(x-b)}}$ is

$(-\infty, a) \cup (b, \infty)$, where a < b

$\Rightarrow$ The domain of $\frac{1}{\sqrt{(x-4)(x-5)}}$ is

$(-\infty, 4) \cup (5, \infty)$

2. **(d)** $\sqrt{2}, -\frac{\pi}{4}$

Explanation:

$\sqrt{2}, -\frac{\pi}{4}$

3. **(a)** 1680

Explanation:

There are letters of different kinds in the given word, namely, M, A, T, H, E, I, C, S.

Number of ways to choose 4 letters from 8 = $^8C_4$ = 70

These 4 letters can be arranged in ^4P = 24 ways

$\Rightarrow$ Required number of words = 70 $\times$ 24 = 1680

4. **(b)** 3

Explanation:

S = $\left(x^{1/2} + \frac{1}{2x^{1/4}}\right)^n$

= $\sum^n C_r \cdot \left(x^{1/2}\right)^{n-r} \cdot \left(\frac{1}{2x^{1/4}}\right)^r$

= $\sum^n \mathrm{C_r} \cdot \left(\frac{1}{2}\right)^{\mathrm{r}} \cdot \mathrm{x}^{(\mathrm{n-r})/2} \cdot \mathrm{x}^{-\mathrm{r}/4}$

S = $\sum_{\mathrm{r}=0}^{\mathrm{n}} {}^{\mathrm{n}}\mathrm{C_r} \left(\frac{1}{2}\right)^{\mathrm{P}} \mathrm{x}^{(2\mathrm{n}-3\mathrm{r})/4}$

We have to expand in decreasing powers of x.

$\therefore$ S = ${}^{\mathrm{n}}\mathrm{C}_0 \left(\frac{1}{2}\right)^0 \mathrm{x}^{\mathrm{n}/2} + {}^{\mathrm{n}}\mathrm{C}_1 \left(\frac{1}{2}\right)^1 \mathrm{x}^{(2\mathrm{n}-3)/4}$ + ${}^{\mathrm{n}}\mathrm{C}_2 \left(\frac{1}{2}\right)^2 \mathrm{x}^{(2\mathrm{n}-6)/4}$ + ...

First three terms form an A.P.

= ${}^{\mathrm{n}}\mathrm{C}_0, {}^{\mathrm{n}}\mathrm{C}_1 \left(\frac{1}{2}\right)$ and ${}^{\mathrm{n}}\mathrm{C}_2 \left(\frac{1}{2}\right)^2$ from an A.P.

$\Rightarrow {}^{\mathrm{n}}\mathrm{C}_0 + {}^{\mathrm{n}}\mathrm{C}_2 \left(\frac{1}{2}\right)^2$ = $2\left({}^nC_1\right) \cdot \frac{1}{2}$

$\Rightarrow$ 1 + $\frac{\mathrm{n}(\mathrm{n}-1)}{2} \times \frac{1}{4}$ = n

$\Rightarrow$ (n - 1)(n - 8) = 0

n $\neq$ 1 ($\because$ there are at least 3 term)

$\Rightarrow$ n = 8

$\therefore$ S = $\sum_{\mathrm{r}=0}^{8} {}^8\mathrm{C_r} \left(\frac{1}{2}\right)^2 \mathrm{x}^{4-3\mathrm{r}/4}$

$\Rightarrow 4 - \frac{3r}{4}$ = K; where K $\in$ I

Now of such values of r ranging from 0 to 8 are: 0, 4 and 8

$\therefore$ There will be three such terms.

Hence, the answer is 3.

5.

(c) a^2, b^2, c^2 are in G.P.

Explanation:

Given that, a, b, c are in G.P.

$\Rightarrow \frac{b}{a} = \frac{c}{b}$ = r ...[r : Common ratio]

$\Rightarrow \frac{b^2}{a^2} = \frac{c^2}{b^2}$ = r^2

$\Rightarrow a^2, b^2, c^2$ are in G.P.

6.

(d) 0

Explanation:

$x^y = e^{x - y}$

Taking log on both sides, we get

y log x = x - y

$\Leftrightarrow$ y(1 + logx) = x

$\Rightarrow y = \frac{x}{1+\log x}$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{(1+\log x)-x\left(\frac{1}{x}\right)}{(1+\log x)^2}$

$\Leftrightarrow y' = \frac{\log x}{(1+\log x)^2}$

$\Rightarrow$ y' (1) = 0

7.

(b) 0

Explanation:

Since, the function have extreme value at x = 1 and x = 2

$\therefore$ f'(x) = 0 at x = 1 and x = 2

$\Rightarrow$ f'(1) = 0 and f'(2) = 0

Also, it is given that,

$\lim_{x\to 0}\left[1 + \frac{f(x)}{x^2}\right] = 3$

$\Rightarrow 1 + \lim_{x\to 0} \frac{f(x)}{x^2} = 3$

$\Rightarrow \lim_{x\to 0} \frac{f(x)}{x^2} = 2$

$\Rightarrow$ f(x) will be of the form $ax^4 + bx^3 + 2x^2$ [$\because$ f(x) is four degree polynomial]

Let $f(x) = ax^4 + bx^3 + 2x^2$

$\Rightarrow f'(x) = 4ax^3 + 3bx^2 + 4x$

$\Rightarrow$ f'(1) = 4a + 3b + 4 = 0 ...(i)

and f'(2) = 32a + 12b + 8 = 0

$\Rightarrow$ 8a + 3b + 2 =0 ...(ii)

On solving Eqs. (i) and (ii),

We get $a = \frac{1}{2}, b = -2$

$\therefore f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$

$\Rightarrow$ f(2) = 8 - 16 + 8 = 0

8.

(d) $e^{x^2/4}$

Explanation:

$I_1 = \int_0^x e^{t(x-t)}dt$

$t(x-t) = -\left[t^2 - tx\right] = -\left[\left(t - \frac{x}{2}\right)^2 - \frac{x^2}{4}\right] = \frac{x^2}{4} - \left(t - \frac{x}{2}\right)^2$

$I_1 = \int_0^x e^{\frac{x^2}{4} - \left(t - \frac{x}{2}\right)^2} dt = e^{\frac{x^2}{4}} \int_0^x e^{-\frac{(2t-x)^2}{4}} dt;$
2t - x = y
$\Rightarrow \quad dt = \frac{dy}{2}$
$= \frac{e^{x^2/4}}{2} \int_{-x}^{x} e^{-\frac{y^2}{4}} dy$
$I_1 = e^{\frac{x^2}{4}} \int_0^x e^{-\frac{t^2}{4}} dt = e^{\frac{x^2}{4}} \cdot I_2$
$\frac{I_1}{I_2} = e^{\frac{x^2}{4}}$

9.

(b) $2(x^2 + y^2) = 4x + 3y$
Explanation:
$2(x^2 + y^2) = 4x + 3y$

10. **(a)** 1
Explanation:
1

11.

(c) -128
Explanation:
y = mx + 4 ...(i)
Tangent of $y^2 = 4x$ is
$\Rightarrow y = mx + \frac{1}{m}$ ...(ii) [$\because$ Equation of tangent of $y^2 = 4ax$ is $y = mx + \frac{a}{m}$]
From (i) and (ii)
$4 = \frac{1}{m} \Rightarrow m = \frac{1}{4}$
So, the line $y = \frac{1}{4}x + 4$ is also tangent to the parabola
$x^2 = 2by$, so solve both equations.
$x^2 = 2b\left(\frac{x+16}{4}\right)$
$\Rightarrow 2x^2 - bx - 16b = 0$
$\Rightarrow$ D = 0 [For tangent]
$\Rightarrow b^2 - 4 \times 2 \times (-16b) = 0$
$\Rightarrow b^2 + 32 \times 4b = 0$
b = -128, b = 0 (not possible)

12.

(d) $\sqrt{2} - 2$
Explanation:
$\frac{dy}{dx} + 2y \tan x = 2 \sin x$
I.F = $e^{\int 2 \tan x dx} = \sec^2 x$
The solution of the differential equation is
$y \times \text{I.F.} = \int I.F \cdot \times 2 \sin x \, dx + C$
$\Rightarrow y \cdot \sec^2 x = \int 2 \sin x \cdot \sec^2 x \, dx + C$
$\Rightarrow y \sec^2 x = 2 \sec x + C$ ...(i)
When $x = \frac{\pi}{3}$, y = 0; then C = -4
$\therefore$ From (i), $y \sec^2 x = 2 \sec x - 4$
$\Rightarrow y = \frac{2 \sec x - 4}{\sec^2 x} \Rightarrow y\left(\frac{\pi}{4}\right) = \sqrt{2} - 2$

13. **(a)** 6x + 4y + 3z = 36

Explanation:

Let A = (a, 0, 0), B = (0, b, 0), C = (0, 0, c), then the centroid of $\triangle$ABC is

$\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ = (2, 3, 4)

$\Rightarrow$ (a, b, c) = (6, 9, 12)

$\Rightarrow$ The equation of the required plane is

$\frac{x}{6} + \frac{y}{9} + \frac{z}{12}$ = 1

$\Leftrightarrow$ 6x + 4y + 3z = 36

14.

(c) -25

Explanation:

$(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})$

$= \{(\vec{a} + 2\vec{b}) \cdot \vec{a}\}\vec{b} - \{(\vec{a} + 2\vec{b}) \cdot \vec{b}\}\bar{a}$

$= \left\{|\vec{a}|^2 + 2\vec{b} \cdot \bar{a}\right\}\vec{b} - \left\{\vec{a} \cdot \vec{b} + 2|\vec{b}|^2\right\}\vec{a}$ ...(i)

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = \frac{\sqrt{2}}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}} = 0$

(i) $\Rightarrow (\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b}) = |\vec{a}|^2\vec{b} - 2|\vec{b}|^2\vec{a}$

$(2\vec{a} - \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$

$= (2\vec{a} - \vec{b}) \cdot \left(|\vec{a}|^2\vec{b} - 2|\vec{b}|^2\vec{a}\right)$

$= -4|\vec{b}|^2(\vec{a} \cdot \vec{a}) - |\vec{a}|^2(\vec{b} \cdot \vec{b})$ ...$[\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = 0]$

$= -5|\vec{a}|^2|\vec{b}|^2$

= -5 $\times$ 1 $\times$ 5 = -25

15.

(d) $\frac{5\sqrt{17}}{2}$

Explanation:

$\bar{x}_1$ = 6, $\bar{x}_2$ = 9, n_1 = n_2 = 10

$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{10(6+9)}{20} = \frac{15}{2}$

d_1 = $\bar{x}_1 - \bar{x}$ = 6 - $\frac{15}{2} = -\frac{3}{2}$

d_2 = $\bar{x}_2 - \bar{x}$ = 9 $- \frac{15}{2} = \frac{3}{2}$

$\sigma^2 = \frac{10}{20}$ [64 + $\frac{9}{4}$ + 144 + $\frac{9}{4}$]

= $\frac{1}{2}$ [208 + $\frac{9}{2}$] = $\frac{1}{4}$[425]

$= \frac{5\sqrt{17}}{2}$

16. **(a)** $\frac{1}{3}$

Explanation:

P(A ∩ B) = P(A) P (B | A)

$\Rightarrow P(A \cap B)$ = $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$

P(A | B) = $\frac{P(A \cap B)}{P(B)} \Rightarrow \frac{1}{2} = \frac{1}{6} \times \frac{1}{P(B)}$

$\Rightarrow$ P(B) = $\frac{1}{3}$

17.

(d) $\frac{1}{16}$

Explanation:

Since, $\sin\left(\frac{\pi}{22}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{22}\right) = \cos\frac{5\pi}{11} = -\cos\frac{16\pi}{11}$

$\sin\frac{3\pi}{22} = \cos\frac{4\pi}{11}$, $\sin\frac{5\pi}{22} = \cos\frac{3\pi}{11} = -\cos\frac{8\pi}{11}$

$\sin\frac{7\pi}{22} = \cos\frac{2\pi}{11}$, $\sin\frac{9\pi}{11} = \cos\frac{\pi}{11}$

Now, $2\sin\frac{\pi}{22} \cdot \sin\frac{3\pi}{22} \cdot \sin\frac{5\pi}{22} \cdot \sin\frac{7\pi}{22} \cdot \sin\frac{9\pi}{22}$

$= 2\cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{8\pi}{11}\cos\frac{16\pi}{11}$

$= \frac{2\times\sin 2^5\frac{\pi}{11}}{2^5\sin\frac{\pi}{11}} = \frac{1}{16}\ \left(\because \sin 2^5\frac{\pi}{11} = \sin\frac{\pi}{11}\right)$

18.

(d) $\frac{y^2}{4} - \frac{x^2}{12} = 1$

Explanation:

$\frac{y^2}{4} - \frac{x^2}{12} = 1$

19.

(d) $1 \in$ Q

Explanation:

$1 \in$ Q

20.

(c) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

Explanation:

Given

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\ldots\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$\because \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2+1 \\ 0 & 1 \end{bmatrix}$,

$\begin{bmatrix} 1 & 2+1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3+2+1 \\ 0 & 1 \end{bmatrix}$

.... and so on.

$\therefore \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\ldots\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & (n-1)+(n-2)+\ldots+3+2+1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & \frac{n(n-1)}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

Since, both matrices are equal, so equating corresponding element, we get

$\frac{n(n-1)}{2}$ = 78 ⇒ n (n - 1) = 156

= 13 × 12 = 13 (13 - 1)

⇒ n = 13

So, $A = \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = A^{-1} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

[∵ if |A| = 1 and A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A^{-1} = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$]

MATHS (Section-B)

21. 0

Explanation:

The maximum value of $f(x) = \cos x + \cos(\sqrt{2}x)$ is 2 which occurs at x = 0. Also, there is no other value of x for which this value will be attained again.

22. 4.0

Explanation:

Given, $|\vec{a}| = |\vec{b}| = |\vec{c}|$

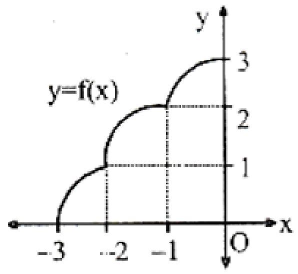
Let $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$

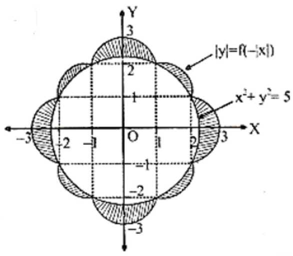
now $(\bar{a} + \bar{b} + \bar{c})^2$ = $a^2 + b^2 + c^2$ + $2(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a})$

$|\bar{a} + \bar{b} + \bar{c}| = \sqrt{3}\lambda$

now $\bar{a} \cdot (\bar{a} + \bar{b} + \bar{c}) = |\bar{a}| \cdot |\bar{a} + \bar{b} + \bar{c}| \cos\theta$
$\cos\theta = \frac{|\bar{a}||\bar{a}|}{|\bar{a}||\bar{a}+\bar{b}+\bar{c}|}$
$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \cos^2 2\theta = \frac{1}{9}$
$\Rightarrow 36\cos^2 2\theta = 36 \times \frac{1}{9} = 4$

23. 15
Explanation:
Area bounded by the curve y = f(x) and x-axis from r = - 3 to x = 0, is
$\int_{-3}^{-2} \sqrt{x+3}dx + \int_{-2}^{-1}(1+\sqrt{x+2})dx + \int_{-1}^{0}(2+\sqrt{x+1})dx$
$= \frac{2}{3}\left((x+3)^{\frac{3}{2}}\right)_{-3}^{-2} + \left(x + \frac{2}{3}(x+2)^{\frac{3}{2}}\right)_{-2}^{-1} + \left(2x + \frac{2}{3}(x+1)^{\frac{3}{2}}\right)_{-1}^{0}$
$= \frac{2}{3}(1 - 0) + (-1 + 2 + \frac{2}{3}(1 - 0)) + (0 + 2 + \frac{2}{3}(1 - 0))$
$= \frac{2}{3} + 1 + \frac{2}{3} + 2 + \frac{2}{3} = 5$
Required area = $4\left(5 - \frac{\pi}{4} \times 5\right)$
= 20 - 5π = p + π - q
$\Rightarrow$ p = 20, q = - 5
Hence, p + q = 15





24. 321.0
Explanation:
For sequence 3, 7, 11, 15..., 399
Common difference $d_1 = 4$
For sequence
2, 5, 8, 11, ..., 359
Common difference is $d_2 = 3$
For sequence
2, 7, 12, 17,..., 197
Common difference is $d_3 = 5$
LCM (d_1, d_2, d_3) = 60
Common terms are 47, 107, 167.
then sum = 47 + 107 + 167 = 321

25. 1
Explanation:
z = $\frac{-1+i\sqrt{3}}{2} \Rightarrow$ $z^3 = 1$ and $1 + z + z^2 = 0$
$P^2 = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix} \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$

$$= \begin{bmatrix} z^{2r} + z^{4s} & z^{2s}((-z)^r + z^r) \\ z^{2s}((-z)^r + z^r) & z^{4s} + z^{2r} \end{bmatrix}$$

For $P^2 = -1$, we should have

$z^{2r} + z^{4s} = -1$ and $z^{2s}((-z)^r + z^r) = 0$

$\Rightarrow z^{2r} + z^{4s} + 1 = 0$ and $(-z)^r + z^r = 0$

$\Rightarrow$ r is odd and s = r but not a multiple of 3,

which is possible when s = r = 1

$\therefore$ only one pair is there.