MATHS

JEE main - Mathematics

Time Allowed: 1 hour   **Maximum Marks: 100**

General Instructions:

- All questions are compulsory.

- There are 25 questions where the first 20 questions are MCQs and the next 5 are numerical.

- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer.

MATHS (Section-A)

1. If a polynomial function f satisfies the relation $\log_2 [f(x)] = \log_2$ **[4]**

   $\left(2 + \frac{2}{3} + \frac{2}{9} + \ldots + \infty\right) \cdot \log_3 \left(1 + \frac{f(x)}{f\left(\frac{1}{x}\right)}\right)$ and f (10) = 1001, then the value of f (20) is:

   a) 7999   b) 16001

   c) 8001   d) 2002

2. Let the complex number z = x + iy be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal **[4]**
   to:

   a) $\frac{4}{3}$ b) $\frac{3}{2}$

   c) $\frac{3}{4}$ d) $\frac{2}{3}$

3. The value of $\sum_{r=1}^{15} r^2 \left(\frac{^{15}C_r}{^{15}C_{r-1}}\right)$ is equal to: **[4]**

   a) 1085   b) 560

   c) 680   d) 1240

4. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^{11}$ is **[4]**

   a) 990   b) 605

   c) 660   d) 330

5. Let a_r be the rth term of an A.P. If a_{11} = 45 then the common difference that would make the value of $a_2 a_6 a_{11}$ **[4]**
   least's equal to:

   a) 3   b) 4

   c) 14   d) 7

6. If $p^2 = a \cos^2\theta + b^2 \sin^2\theta$, then $p^2 + p\frac{d^2p}{d\theta^2}$ equals: **[4]**

   a) $\frac{a^4 b^4}{p^3}$ b) $\frac{a^2 b^2}{p^2}$

   c) $\frac{a^2 b^2}{p^4}$ d) $\frac{ab}{p^2}$

7. Maximum value of $\left(\frac{1}{x}\right)^x$ is: **[4]**

   a) $(e)^{\frac{1}{e}}$

   b) $(e)^{-e}$

   c) 1

   d) $(e)^e$

8. $\lim\limits_{n \to \infty} \frac{3}{n}\left\{4 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + .. + \left(3 - \frac{1}{n}\right)^2\right\}$ is equal **[4]**

   a) 12

   b) 19

   c) $\frac{19}{3}$

   d) 0

9. Let ABC be a right triangle with $\angle$BAC = 90º then $\left(\frac{r^2}{2R^2} + \frac{r}{R}\right)$ is equal to : (where r and R have usual **[4]**
   meaning in triangle.)

   a) sin B sin C

   b) tan B tan C

   c) cot B cot C

   d) sec B sec C

10. If the tangent at (1, 7) to the curve x^2 = y - 6 touches the circle $x^2 + y^2 + 16x + 12y + c = 0$, then the value of c is **[4]**

   a) 185

   b) 95

   c) 195

   d) 85

11. The point at which the line y = mx + c touches the parabola y^2 = 4ax is **[4]**

   a) $\left(-\frac{a}{m^2}, \frac{2a}{m}\right)$

   b) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

   c) $\left(\frac{a}{m^2}, \frac{-2a}{m^2}\right)$

   d) $\left(-\frac{a}{m^2}, -\frac{2a}{m^2}\right)$

12. The general solution of the differential equation $\frac{dy}{dx}$ - $3x^2y$ = 2y is: **[4]**

   a) $y = ce^{2x+x^3}$

   b) $y = ce^{x^3-2x}$

   c) $y = ce^{2x}$

   d) $y = ce^{x^3}$

13. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1, 1, 2). **[4]**
    Then the equation of the line through this centroid and perpendicular to the plane P is:

    a) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

    b) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

    c) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

    d) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

14. In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i} + \beta\hat{j}$ on $3\hat{i} - 4\hat{j}$ is 1 and if $\alpha - 2\beta$ = 1, then **[4]**
    the value of β is equal to

    a) 1

    b) 0

    c) 2

    d) -1

15. If the mean of 11 observations of a data is 70 and that of first six observations is 72 and of last six observations **[4]**
    is 60, then the middlemost observation is

    a) 22

    b) 20

    c) 18

    d) 24

16. If P(C) = $\frac{5}{6}$, $P(A \cap \bar{B} \cap C) = \frac{2}{7}$ and P($\overline{A} \cap \overline{B} \cap$ C) = $\frac{1}{7}$, then $P(B \cap C)$ = **[4]**

    a) $\frac{3}{14}$

    b) $\frac{17}{42}$

c) $\frac{4}{7}$ d) $\frac{2}{21}$

17. The value of sum $\sin 1^o + \sin 2^o + ... + \sin 359^o + \sin 360^o$ is equal to :   **[4]**

   a) 0   b) $-\sin 1^o$

   c) $\sin 1^o$ d) 1

18. The equation of the transverse and conjugate axes of a hyperbola are respectively $x + 2y - 3 = 0$, $2x - y + 4 = 0$ **[4]**
    and their respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$. The equation of the hyperbola is:

   a) $\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$ b) $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

   c) $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$ d) $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

19. Let A = $\{(x, y) : y = e^x, x \in R\}$, B = $\{(x, y) : y = e^{-x}, x \in R\}$. Then   **[4]**

   a) $A \cup B = R^2$ b) $A \cap B \neq \phi$

   c) $A \cup B \neq R^2$ d) $A \cap B = \phi$

20. The value of $\begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} \frac{1}{2} & \frac{4}{3} \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} \frac{1}{2^2} & \frac{8}{3^2} \\ 3 & 7 \end{vmatrix} + ... \infty$ is equal to: **[4]**

   a) 5   b) $\frac{-13}{2}$

   c) 1   d) 13

MATHS (Section-B)

21. Let f(x) = $\begin{cases} (x+2)^3, & -3 < x \le -1 \\ x^{\frac{2}{3}}, & -1 < x < 2 \end{cases}$ and g(x) = $\int\limits_{-3}^{x}$ f(t) dt, $-3 < x < 2$. Find the number of extremum points   **[4]**
    of g'(x).

22. In a triangle PQR, let $\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}|$ = 3, $|\vec{b}|$ = 4 and $\frac{\vec{a} \cdot (\vec{c} - \vec{b})}{\vec{c} \cdot (\vec{a} - \vec{b})} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|}$, then the value of   **[4]**
    $|\vec{a} \times \vec{b}|^2$ is _____.

23. If A is the area in the first quadrant enclosed by the curve C: $2x^2 - y + 1 = 0$, the tangent to C at the point (1, 3)   **[4]**
    and the line $x + y = 1$, then the value of 60 A is

24. Let $a_1, a_2, a_3,...$ be an A.P. If $\sum\limits_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to _____.   **[4]**

25. Let A = $\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _____.   **[4]**