SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

Class 12 - Mathematics

Time Allowed: 2 hours and 58 minutes **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. Two matrices A = $[a_{ij}]$ and B = $[b_{ij}]$ are said to be equal, if they are of same order and for all i and j, their elements are **[1]**

   a) $a_{ij} = -b_{ij}$
   b) $a_{ij} = b_{ij}$
   c) $a_{ij} = b_{ji}$
   d) $a_{ij} + b_{ij} = 0$

2. Let a, b, c be positive real numbers.What type of solutions do the following system of equations in x, y and z has? **[1]**

   $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

   a) finitely many solutions
   b) no solution
   c) unique solution
   d) infinitely many solutions

3. The system of equations **[1]**

   x + y + z = 2,
   3 x - y + 2z = 6
   3x + y + z = -18
   has:

   a) zero solution as the only solution
   b) an infinite number of solutions
   c) a unique solution
   d) no solution

4. If $y = x\sqrt{1-x^2} + \sin^{-1}x$, then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{1}{\sqrt{1-x^2}}$
b) $\sqrt{1-x^2}$
c) $2\sqrt{1-x^2}$
d) $4\sqrt{1-x^2}$

5. If O is the origin, OP = 3 with direction ratios proportional to - 1, 2, - 2 then the coordinates of P are **[1]**

a) (3, 6 , - 9)
b) (1 , 2 , 2)
c) (- 1 , 2 , - 2)
d) ($\frac{-1}{9}, \frac{2}{9}, \frac{-2}{9}$)

6. The order of the differential equation whose general solution is given by y = $c_1 \cos(2x + c_2)$ + $(c_3 + c_4)\, a^{x+c_5}$ + $c_6 \sin(x - c_7)$ is **[1]**

a) 6
b) 3
c) 4
d) 5

7. By graphical method, the solution of linear programming problem **[1]**

Maximize Z = $3x_1 + 5x_2$

Subject to $3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1 \geq 0$, $x_2 \geq 0$, is

a) $x_1 = 2$, $x_2 = 0$, Z = 6
b) $x_1 = 4$, $x_2 = 6$, Z = 42
c) $x_1 = 2$, $x_2 = 6$, Z = 36
d) $x_1 = 4$, $x_2 = 3$, Z = 27

8. Two adjacent sides of a parallelogram are represented by the vectors $\vec{a} = (3\hat{i} + \hat{j} + 4\hat{k})$ and $\vec{b} = (\hat{i} - \hat{j} + \hat{k})$ the area of the parallelogram is **[1]**

a) 6 sq units
b) $\sqrt{42}$
c) $\sqrt{40}$
d) $\sqrt{35}$

9. $\int \frac{\cos(\log x)}{x} dx$ is equal to **[1]**

a) – sin (log x)+C
b) $\log(\sin x) + C$
c) sin(log x)+C
d) $\frac{\sin(\log x)}{x} + C$

10. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ is equal to **[1]**

a) A + B
b) 2 BA
c) AB
d) 2 AB

11. Maximize Z = – x + 2y, subject to the constraints: x ≥ 3, x + y ≥ 5, x + 2y ≥ 6, y ≥ 0. **[1]**

a) Z has no maximum value
b) Maximum Z = 14 at (2, 6)
c) Maximum Z = 12 at (2, 6)
d) Maximum Z = 10 at (2, 6)

12. Which one of the following is the unit vector perpendicular to both $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$? **[1]**

a) $\frac{\hat{j}+\hat{k}}{\sqrt{2}}$
b) $\hat{k}$
c) $\pm\frac{\hat{i}+\hat{j}}{\sqrt{2}}$
d) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

13. The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$ is **[1]**

a) $\frac{-1}{2}$

b) 4

c) $\frac{1}{2}$

d) 0

14. If A and B are such events that P(A) > 0 and P(B) $\neq$ 1, then P(A'/B') equals. **[1]**

a) $\frac{1-P(A\cup B)}{P(B')}$

b) $P(A')/P(B')$

c) $1-P(A/B)$

d) $1-P(A'/B)$

15. The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}$ = 0 is: **[1]**

a) $\frac{1}{x}+\frac{1}{y}$ = C

b) log x - log y = C

c) xy = C

d) x + y = C

16. Consider the vectors $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and b = $4\hat{i}-4\hat{j}+7\hat{k}$. **[1]**

What is the scalar projection of $\vec{a}$ on $\vec{b}$?

a) $\frac{23}{9}$

b) $\frac{17}{9}$

c) 1

d) $\frac{19}{9}$

17. If $y=\tan^{-1}$x and $z=\cot^{-1}x$ then $\frac{dy}{dz}$is equal to **[1]**

a) 1

b) $\frac{\pi}{2}$

c) – 1

d) $\frac{\pi}{4}$

18. Find the shortest distance between the lines $\vec{r}=\hat{i}+2\hat{j}+\hat{k}+\lambda\left(\hat{i}-\hat{j}+\hat{k}.\right)$ $and\ \vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu\left(2\hat{i}+\hat{j}+\widehat{2k}.\right)$, $\lambda,\mu\in R$ **[1]**

a) $\frac{4\sqrt{2}}{2}$

b) $\frac{3\sqrt{2}}{5}$

c) $\frac{3\sqrt{2}}{2}$

d) $\frac{5\sqrt{2}}{2}$

19. **Assertion (A):** If the circumference of the circle is changing at the rate of 10 cm/s, then the area of the circle changes at the rate 30 cm^2/s, if radius is 3 cm. **[1]**

Reason (R): If A and r are the area and radius of the circle, respectively, then rate of change of area of the circle is given by $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. Let R be the relation in the set of integers Z given by R = {(a, a) : 2 divides (a - a)} **[1]**

Assertion (A): R is a reflexive relation.

Reason (R): A relation is said to be reflexive. if x R x, $\forall$ x $\in$ z

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = ?$ **[2]**

OR

Write the interval for the principal value of function and draw its graph: $\cot^{-1} x$.

22. Find the points of local maxima or local minima and corresponding local maximum and local minimum values of the function. Also, find the points of inflection, if any: f(x) = x + $\frac{a^2}{x}, a > 0, x \neq 0$. **[2]**

23. The amount of pollution content added in air in a city due to x diesel vehicles is given by P(x) = $0.005x^3 + 0.02x^2 + 30x$. **[2]**

Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question?

OR

Find the maximum and minimum values of the function f(x) = $\frac{4}{x+2} + x$.

24. Evaluate the definite integral: $\int_0^1 \frac{x}{x+1} dx$ **[2]**

25. Show that f(x) sin x is increasing function on the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ **[2]**

Section C

26. Integrate the function $x \cos^{-1} x$ **[3]**

27. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01 and that of a motorcycle is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle. **[3]**

28. Evaluate: $\int \frac{x^2}{(a^6 - x^6)} dx$ **[3]**

OR

Evaluate: $\int \frac{4x^4+3}{(x^2+2)(x^2+3)(x^2+4)} dx$

29. Find a particular solution of the differential equation (x - y) (dx + dy) = dx - dy, given that y = -1, when x = 0. **[3]**

OR

Solve the initial value problem: $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$, y(0) = 0

30. Maximise the function Z = 11x + 7y, subject to the constraints: $x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0$. **[3]**

OR

Solve the Linear Programming Problem graphically:

Minimize Z = l8x + 10y

Subject to

4x + y > 20

2x + 3 y > 30

x, y > 0

31. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$. **[3]**

Section D

32. If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m. **[5]**

33. If A = {1, 2, 3, 4}, define relations on A which have properties of being: **[5]**

   a. reflexive, transitive but not symmetric
   b. symmetric but neither reflexive nor transitive
   c. reflexive, symmetric and transitive.

OR

Show the relation R in the set A = {x ∈ Z : 0 ≤ x ≤12}, given by R = {(a, b) : a = b}, is an equivalence relation. Find the set of all elements related to 1 in each case.

34. Solve the following system of equations by using determinants: **[5]**

x + y + z = 1

ax + by + cz = k

$a^2 x + b^2 y + c^2 z = k^2$

35. If $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are the direction cosines of three mutually perpendicular lines, prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3, m_1 + m_2 + m_3, n_1 + n_2 + n_3$ makes equal angles with them. **[5]**

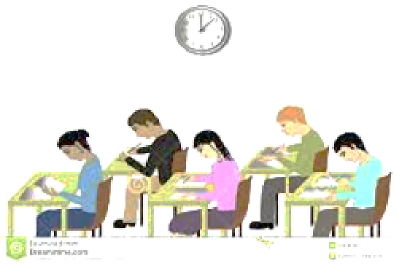
OR

Find the shortest distance between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and $\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.



i. Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics? (1)

ii. Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics? (1)

iii. Find the probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics? (2)

OR

Find the probability that the selected student has passed in Economics, if it is known that he has failed in Mathematics? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

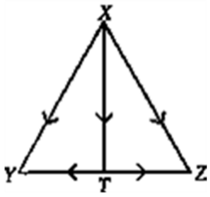
i. If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then find $\vec{q} + \vec{r}$. (1)

ii. If ABCD is a parallelogram and AC and BD are its diagonals, then find the value of $\overrightarrow{AC} + \overrightarrow{BD}$. (1)

iii. If ABCD is a parallelogram, where $\overrightarrow{AB} = 2\vec{a}$ and $\overrightarrow{BC} = 2\vec{b}$, then find the value of $\overrightarrow{AC} - \overrightarrow{BD}$. (2)

OR

If T is the mid point of side YZ of △XYZ, then what is the value of $\overrightarrow{XY} + \overrightarrow{XZ}$. (2)



38. **Read the following text carefully and answer the questions that follow:** **[4]**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

i. If l(x) denotes the combined light intensity, then find the value of x so that I(x) is minimum. (1)
ii. Find the darkest spot between the two lights. (1)
iii. If you are in between the lamp posts, at distance x feet from the stronger light, then write the combined light intensity coming from both lamp posts as function of x. (2)

OR

Find the minimum combined light intensity? (2)