#### Solution

## PHYSICS

## **Class 12 - Physics**

### Section A

1. Select and write the correct answers for the following multiple choice type of questions:

(b) 
$$6 \times 10^{-19} J$$
  
Explanation: {  
 $\phi_0 = \frac{hc}{\lambda_0} = \frac{6.63 \times 10^{34} \times 3 \times 10^8}{3315 \times 10^{-10}} = 6 \times 10^{-19} J$ 

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(ii) (d) isobaric process Explanation: { isobaric process

(i)

(iii) (c) the band gap of the material of semiconductorExplanation: {

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(iv) (b) increases with increase in wavelength.Explanation: {

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- (v) (a) once maximum and once minimum
   Explanation: {
   once maximum and once minimum
- (vi) **(b)** 5 cm

**Explanation:** { Rise of the liquid column,  $h \propto \frac{1}{r} \Rightarrow h_2 = h_1 \times \frac{r_1}{r_2} = 2.5 \times 2 = 5 \ cm$ 

(vii) (a) 20

Explanation: {

$$egin{aligned} &r_n \propto n^2 \ \therefore rac{r_n}{r_1} &= \left(rac{n_n}{n_1}
ight)^2 \ \therefore &n_n &= \sqrt{rac{r_n}{r_1}} imes n_1 &= \sqrt{rac{212}{0.53}} imes 1 = 20 \end{aligned}$$

(viii) (d) low coercivity and low retentivity

# Explanation: {

Coercivity is the measure of a material's resistance to changes in its magnetization state. Since soft iron can be easily magnetized and demagnetized with a relatively small applied magnetic field, it has low coercivity. Retentivity is the ability of a material to retain its magnetization once the external magnetic field is removed. Since soft iron does not hold on to its magnetization when the external magnetic field is removed, it has low retentivity. Thus, soft iron's low coercivity allows it to easily switch between magnetized and demagnetized states with minimal energy loss, while its low retentivity ensures that it does not retain magnetization between alternating cycles. These properties make soft iron an ideal material for transformer cores, where rapid and reversible changes in

magnetization are essential for the efficient transfer of electrical energy.

(ix) **(b)** 1:4

# Explanation: {

Rate of loss of heat  $Q \propto A$ 

 $\therefore \frac{Q_1}{Q_2} = \frac{A_1}{A_2} = \frac{4\pi (r_1^2)}{4\pi (r_2)^2} = \frac{6^2}{12^2} = \frac{1}{4}$ 

(x) (d) both base-emitter and base-collector junctions are forward biased.

#### Explanation: {

both base-emitter and base-collector junctions are forward biased.

- 2. Answer the following questions:
  - (i) Einstein's photoelectric equation:

 $K \cdot E_{ ext{max}} = (\mathrm{h} v - \phi_0)$ 

- (ii) Periodic oscillations of gradually decreasing amplitude are called damped harmonic oscillations.
- (iii)In parallel resonant circuit, current is minimum and impedance is maximum.
- (iv)Moving coil galvanometer is converted into an ammeter by connecting a low resistance in parallel with the galvanometer, which effectively reduces the resistance of the galvanometer. This low resistance connected in parallel is called as shunt (S).
- (v) **Statement:** The moment of inertia  $(I_0)$  of an object about any axis is the sum of its moment of inertia  $(I_c)$  about an axis parallel to the given axis, and passing through the centre of mass and the product of the mass of the object and the square of the distance between the two axes.

Mathematically,  $I_o = I_c + Mh^2$ 

#### **Proof:**

- i. Consider an object of mass M. Axis MOP is any axis passing through point *O*.
- ii. Axis ACB is passing through the centre of mass C of the object, parallel to the axis MOP, and at a distance h from it (: h = CO).



- iii. Consider a mass element 'dm' located at point D. Perpendicular on OC (produced) from point D is DN.
- iv. Moment of inertia of the object about the axis ACB is  $I_c = \int (DC)^2 dm$ , and about the axis MOP it is  $I_o = \int (DO)^2 dm$ .

$$\therefore I_o = \int (DO)^2 dm = \int \left[ (DN)^2 + (NO)^2 \right] dm$$
  
= 
$$\int \left[ (DN)^2 + (NC)^2 + 2 \cdot NC \cdot CO + (CO)^2 \right] dm$$
  
= 
$$\int \left[ (DC)^2 + 2NC \cdot h + h^2 \right] dm$$
  
...(using Pythagoras theorem in  $\triangle DNC$ )  
= 
$$\int (DC)^2 dm + 2h \int NC \cdot dm + h^2 \int dm$$

- Now,  $\int (DC)^2 dm = I_c$  and  $\int dm = M$
- v. NC is the distance of a point from the centre of mass. Any mass distribution is symmetric about the centre of mass. Thus, from the definition of the centre of mass,  $\int NC \cdot dm = 0$

$$\therefore I_o = I_c + Mh^2$$

This is the mathematical form of the theorem of parallel axes.

(vi)Expression for electric field intensity at a point outside an infinitely long charged conducing cylinder:

$$E = \frac{\lambda}{2\pi k \varepsilon_0 r}$$

(viiForce on a closed circuit in a magnetic field is zero.

(vii) the ratio of magnetic dipole moment with angular momentum of revolving electron is called the gyromagnetic ratio.

## Section B

- 3. i. Coefficient of viscosity: The coefficient of viscosity is defined as the viscous force per unit area per unit velocity gradient. ii. Formula:  $\eta = \frac{F}{dv}$ 
  - iii. Unit of  $\eta$  is  $Ns/m^2$  or decapoise in S.I. system



#### 5. Centripetal force:

i. The force providing centripetal or radial acceleration is called as centripetal or radial force.

 $F_{CPF}=-m\omega^2\stackrel{
ightarrow}{r}$ 

where, r = radius of circular path.

- ii. In magnitude, $F_{CPF}=mr\omega^2=rac{mv^2}{r}=mv\omega$
- iii. The direction of this force is along the radius and towards centre (centre seeking).
- 6. i. The ability of a conductor to store the electric charge is called capacitance of conductor.
  - ii. SI unit of capacity of conductor is farad (F).

$$\therefore 1 F = \frac{1C}{1V}$$

Thus, the capacity of a conductor is said to be 1 farad if the potential difference across it rises by 1 volt, when 1C charge is given to it.

## 7. Definition: Phase in S.H.M. (or for any motion) is the state of oscillation.

# Particle performing S.H.M., starting from the positive extreme position:

**Equations:** As the particle starts from the positive extreme position,  $\phi = \frac{\pi}{2}$ 

$$\therefore$$
 Phase,  $heta=\omega t+\phi=\omega t+rac{\pi}{2}$ 

$$\therefore$$
 Displacement,  $x = A\sin(\omega t + \frac{\pi}{2}) = A\cos\omega t$ 

Velocity, 
$$v = \frac{dx}{dt} = \frac{d(A\cos\omega t)}{dt} = -A\omega \cdot \sin(\omega t)$$

dt

Acceleration, 
$$a = \frac{dv}{dt} = \frac{dv}{dt}$$
  
=  $-A\omega^2 \cos(\omega t)$ 

Table:

(t)	0	T/4	T/2	3T/4	Т	$5 \ T/4$
(θ)	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$2\pi$	$\frac{5\pi}{2}$	$3\pi$
(x)	А	0	-A	0	А	0
(v)	0	$-A\omega$	0	$A\omega$	0	$-A\omega$
(a)	$-A\omega^2$	0	$A\omega^2$	0	$-A\omega^2$	0

Graph:



- a. Variation of displacement with time
- b. Variation of velocity with time
- c. Variation of acceleration with time
- 8. i. Mechanical equilibrium:
  - a. For a system to be in mechanical equilibrium, there should not be any unbalanced forces acting within the system and between the system and its surrounding.
  - b. Also, the pressure in the system should be same throughout the system and should not change with time.

#### ii. Thermal equilibrium:

For a system to be in thermal equilibrium, the temperature of the system should be uniform throughout and it should not change with time. A system when in thermal equilibrium is described in terms of state variables.

9. Given: 
$$L = 0.1H, C = 25 \times 10^{-6} F, R = 15\Omega$$

 $e_{rms} = 120 V$ To find: Resonant frequency ( $f_r$ ) Formula:  $f_r = \frac{1}{2\pi\sqrt{LC}}$ Calculation: From formula,  $f_r = \frac{1}{2\times 3.142 \times \sqrt{0.1 \times 25 \times 10^{-6}}}$   $= \frac{1}{9.9356 \times 10^{-3}}$   $f_r = 100.6 Hz$ The value of resonant frequency is 100.6Hz. 10. Given that,  $x = 5 \sin(\frac{\pi t}{3})m$ 

Velocity of a particle performing S.H.M. is given by,

$$v = rac{dx}{dt} = 5 imes rac{d}{dt} \left[ \sin\left(rac{\pi t}{3}
ight) 
ight] = 5 imes \cos\left(rac{\pi t}{3}
ight) imes rac{\pi}{3}$$
  
In  $t = 1$  second,

v = 5 × 1/2 × π/3 = 2.62 m/s
 11. When two or more waves, travelling through a medium, pass through a common point, each wave produces its own displacement at that point, independent of the presence of the other wave. The resultant displacement at that point is equal to the vector sum of the displacements due to the individual wave at that point.

12. Kinetic energy of gas  $=\frac{3}{2}RT$ 

 $\begin{array}{l} \therefore \text{ K.E. } \propto T \\ \therefore \frac{(\text{ K.E. })_2}{(\text{ K. })_1} = \frac{T_2}{T_1} \\ \therefore \frac{1}{2} = \frac{T_2}{273} \\ \therefore T_2 = 136.5K \end{array}$ 

At 136.5 K, the average kinetic energy of the gas will be exactly half of its value at N.T.P.

13. Given:  $\sigma = 0.1 \Omega/cm$ =  $0.1 imes 100 \Omega/m = 10 \Omega/m$ ,  $l_1 = 300 \ cm = 3 \ m$  $E_1 = 1.5 \ V, E_2 = 1.4 \ V$  To find: Current (I), balancing length (l<sub>2</sub>).

Formulae:

i.  $I = \frac{E_1}{\sigma l_1}$ ii.  $\frac{E_1}{E_2} = \frac{l_1}{l_2}$ Calculation: Using formula (i),  $I = rac{1.5}{10 imes 3}$  $= 0.05 \ A$ Using formula (ii),  $l_2 = rac{E_2 l_1}{E_1} = rac{1.4 imes 3}{1.5} = 2.8 \ m$ i. The flow of current is 0.05 A. ii. The balancing length for second cell is 2.8m. 14. Given: Series limit of Lyman Series,  $(\lambda_L) = 912 \mathring{A}$ To find: Shortest wavelength of i. Balmer series ii. Paschen series Formula:  $\frac{1}{\lambda} = R \left| \frac{1}{n^2} - \frac{1}{m^2} \right|$ Calculation: From formula, For shortest wavelength,  $m = \infty$ .  $\therefore \lambda = \frac{n^2}{R}$ For Lyman series, n = 1 $\lambda_L = rac{n^2}{R} = rac{1^2}{R} = rac{1}{R} = 912 \overset{\circ}{A}$ i. For Balmer series, n = 2 $\lambda_B=rac{2^2}{R}=rac{4}{R}=4 imes 912=3648 \overset{\circ}{A}$ ii. For Paschen series, n=3 $\lambda_P=rac{3^2}{R}=rac{9}{R}=9 imes 912=8208 \overset{\circ}{A}$ The shortest wavelength of i. Balmer series is 3648 Aii. Paschen series is 8208A

Section C

15. Conditions for obtaining sharp and steady interference pattern are:

- i. The two sources of light must be coherent.
- ii. The two sources of light must be monochromatic.
- iii. The two interfering waves must have the same amplitude.
- iv. The separation between the two slits (d) must be small in comparison to the distance between the plane containing the slits and the observing screen (D).
- v. The two slits should be narrow.
- vi. The two waves should be in the same state of polarization if polarized light is used for the experiment.



17. Definition: The ratio of magnetic moment to the volume of the material is called magnetization.

It is denoted by  $\vec{M}$ . Unit:  $Am^{-1}$  in SI system. Dimensions:  $\left[M^0L^{-1}T^0I^1\right]$ 

- 18. The phenomenon of emission of electrons from a metal surface, when radiation of appropriate frequency is incident on it, is known as photoelectric effect.
  - i. If increasingly negative potentials were applied to the collector in experiment of photoelectric effect, the photocurrent decreases and for some typical value  $(-V_0)$ , photocurrent becomes zero. This value of  $V_0$  is termed as cut-off or stopping potential.
  - ii. The minimum amount of energy required to be provided to an electron to pull it out of the metal from the surface is called the work function of the metal.
- 19. Sources of errors:
  - i. The cross section of the wire may not be uniform.
  - ii. The ends of the wire are soldered to the metallic strip where contact resistance is developed, which is not taken into account.
  - iii. The measurements of  $l_x$  and  $l_R$  may not be accurate.

To minimize the errors:

- i. The value of R is so adjusted that the null point is obtained around middle one third of the wire (between 34 cm and 66 cm) so that percentage error in the measurement of  $l_x$  and  $l_R$  are minimum and nearly the same.
- ii. The experiment is repeated by interchanging the positions of unknown resistance X and known resistance box R.
- iii. The jockey should be tapped on the wire and not slide. The jockey is used to detect whether there is a current through the central branch. This is possible only by tapping the jockey.

20.	Sr. No.	Progresslive waves	Stationary waves	
	i.	The disturbance travels from one region to the other with definite velocity.	Disturbance remains in the region where it is produced, velocity of the wave is zero.	
ii. A		Amplitudes of all particles are same.	Amplitudes of particles are different.	
	iii.	Particles do not cross each other.	All the particles cross their mean positions simultaneously.	
	iv.	All the particles are moving.	Particles at the position of nodes are always at rest.	
v. There is		There is no transfer of energy.	Energy is transmitted from one region to another.	
	vi.	Phases of adjacent particles are different.	All particles between two consecutive nodes are moving in the same direction and are in phase while those in adjacent loops are moving in opposite directions and differ in phase by 180°	

- 21. i. For parallel combination of two coils, the current through each parallel inductor is a fraction of the total current and the voltage across each parallel inductor is same.
  - ii. As a result, a change in total current will result in less voltage dropped across the parallel array than for any one of the individual inductors.
  - iii. There will be less voltage drop across parallel inductors for a given rate of change in current than for any of the individual inductors.
  - iv. Less voltage for the same rate of change in current results in less inductance.
  - v. Thus, the total inductance of two coils is less than the inductance of either coil.

22. Given 
$$Y = 0.1 \sin 4\pi (50t - 0.1x)$$

To find: Amplitude (*A*), frequency (*n*), wavelength ( $\lambda$ ) and velocity (*v*) Formula:

i. 
$$y = A \sin 2\pi \left( nt - rac{x}{\lambda} \right)$$

ii. 
$$v=n\lambda$$

Calculation: Comparing with formula (i),

 $Y = 0.1 \sin 2\pi (100t - 0.2x)$ 

A=0.1~m=10~cm $n=100Hz, oldsymbol{\lambda}=5~m$ Using formula (ii), v=100 imes5=500~m/s

For the given wave amplitude, frequency, wavelength and velocity are respectively, 10 cm , 100 Hz, 5 m and 500 m/s. 23. Given:  $_an_g = 1.5$ ,  $v = 3.5 \times 10^{14} Hz$ ,  $c = 3 \times 10^8 m/s$ 

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To find:
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i. Change in wavelength of light  $(\Delta \lambda)$ 

ii. Wave number of light  $(ar{v})$ 

Formulae:

i. 
$$\bar{v} = \frac{1}{\lambda}$$
  
ii.  $a n_a = \frac{\lambda}{2}$ 

iii. 
$$c = v\lambda$$

Calculation:

i. Using formula (iii),

$$\lambda_{a} = \frac{c}{v} = \frac{3 \times 10^{8}}{3.5 \times 10^{14}} = \frac{6}{7} \times 10^{-6} = 0.8571 \times 10^{-6} m = 8571 \overset{o}{A}$$
Using formula (ii),  

$$\lambda_{g} = \frac{\lambda_{a}}{n_{g}} = \frac{8571 \overset{o}{A}}{1.5}$$

$$\therefore \lambda_{g} = 5714 \overset{o}{A}$$

$$\therefore \Delta \lambda = \lambda_a - \lambda_a = 8571 - 5714 = 2857 \overset{o}{A}$$

ii. Now, using formula (i), $ar{v}=rac{1}{5.714 imes10^{-7}}=1.75 imes10^6~m^{-7}$ 

i. The change in wavelength of light is 2857A.

ii. The wave number of light is  $1.\,75 imes10^6m^{-1}$ 

24. Given: 
$$N=250,\; d=18\;cm,\; r=9\;cm=9 imes 10^{-2}\;m, I=12\;A$$

To find: Magnetic moment of the coil (m)

Formula: m = NIA

Calculation: For the coil,

 $A=\pi r^2$ 

Using formula,

$$m=NI\pi r^2$$

 $=250 imes12 imes3.142 imes\left(9 imes10^{-2}
ight)^2$ 

$$=250 imes12 imes3.142 imes81 imes10^{-4}$$
 .

$$= \{ \mathrm{antilog}[\log(250) + \log(12) + \log(3.142) + \log(81)] \} imes 10^{-4}$$

 $= \{ \text{ antilog } [2.3979 + 1.0792 + 0.4972 + 1.9085] \} \times 10^{-4}$ 

- $= \{ antilog[5.8828] \} \times 10^{-4}$
- $= 0.7635 imes 10^6 imes 10^{-4}$

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∴ m = 76.35 Am<sup>2</sup>
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Magnetic moment of the coil is **76.35** Am<sup>2</sup>.

25. Given:  $R = 100\Omega$ , V = 220 V, f = 50 Hz

To find:

i. rms current  $(i_{rms})$ 

ii. Net power consumed 
$$(P_{av})$$

Formulae:

i.  $i_{rms} = \frac{e_{rms}}{R}$ ii.  $P_{av} = e_{rms}i_{rms}$ Calculation: From formula (i),  $i_{rms} = \frac{220}{100} = 2.2 A$ From formula (ii),

 $P_{av}=220\times 2.2=484~W$ 

i. The rms current in the circuit is 2.2 A. ii. Net power consumed over a full cycle is 484 W. 26. Given:  $A_1(t_1) = 10^{10}/hr$  for  $t_1 = 20hrs$ ,  $A_2(t_2) = 5 \times 10^9/hr$  for  $t_2 = 30hrs$ . To find: Decay constant ( $\lambda$ ). Formula:  $A(t) = A_0 e^{-\lambda t}$ . Calculation: From formula (i),  $A\left(t_{1}
ight)=A_{0}e^{-\lambda_{1}}$  ,  $10^{10} = A_0 e^{-20\lambda}$  .  $A\left(t_{2}
ight)=A_{0}e^{-\lambda t_{2}}$  ,  $5 imes 10^9 = A_0 e^{-30\lambda}$  . Dividing equation (1) by (2),  $\therefore rac{10^{10}}{5 imes 10^9} = e^{-20\lambda + 30\lambda} = e^{10\lambda} \; .$ Taking natural logarithm on both the sides,  $\ln\left(\frac{10^{10}}{5 \times 10^9}\right) = 10\lambda$  .  $\therefore 2.303 \log(2) = 10\lambda$ .  $0.6932 = 10\lambda$  .  $\therefore \lambda = 0.06932$  per hour. Decay constant is 0.06932 per hour. Section **D** 

## 27. Expression for terminal velocity:

i. Consider a sphere of radius (r) and density ( $\rho$ ) falling under gravity through a liquid of density ( $\sigma$ ) and coefficient of viscosity ( $\eta$ ) as shown in figure.



ii. Forces acting on the sphere during downward motion are,

- a. Viscous force  $=F_v=6\pi\eta rv$  (directed upwards)
- b. Weight of the sphere,  $(F_g) mg = \frac{4}{3}\pi r^3 \rho g$  (directed downwards)
- c. Upward thrust as Buoyant force (  $F_u$  )  $F_u = rac{4}{3}\pi r^3\sigma \,g$  (directed upwards)
- iii. As the downward velocity increases, the viscous force increases. A stage is reached, when sphere attains terminal velocity.
- iv. When the sphere attains the terminal velocity, the total downward force acting on the sphere is balanced by the total upward force acting on the sphere.
  - $\therefore$  Total downward force = Total upward force
  - : Weight of sphere (mg) = Viscous Force + Buoyant force due to medium

$$\therefore \frac{4}{3}\pi r^{3}\rho g = 6\pi\eta rv + \frac{4}{3}\pi r^{3}\sigma g$$
  
$$\therefore 6\pi\eta rv = \left(\frac{4}{3}\pi r^{3}\rho g\right) - \left(\frac{4}{3}\pi r^{3}\sigma g\right)$$
  
$$\therefore 6\pi\eta rv = \left(\frac{4}{3}\right)\pi r^{3}g(\rho - \sigma)$$
  
$$\therefore v = \left(\frac{4}{3}\right)\pi rr^{3}g(\rho - \sigma) \times \frac{1}{6\pi\eta r}$$
  
$$\therefore v = \frac{2}{9}\frac{r^{2}g(\rho - \sigma)}{\eta} \dots (1)$$

This is the expression for terminal velocity of the sphere.

# 28. Answer the following questions:

(i)	Step-up transformer	Step-down transformer	
	(1) The number of turns in its secondary is more than that in its primary $(N_{\rm S}>N_{\rm P})$ .	The number of turns in primary is greater than secondary $(N_{\mathrm{P}}>N_{\mathrm{S}}).$	
	(2) Alternating voltage across the ends of its secondary is more than that across its primary i.e., $e_S > e_P$	Alternating voltage across the ends of the primary is more than that across its secondary i.e., $e_P > e_S$	

(3) Transformer ratio K > 1.	Transformer ratio K < 1.	
(4) Primary coil made of thick wire.	Secondary coil made of thick wire.	
(5) Secondary coil is made of thin wire.	Primary coil is made of thin wire.	
(6) Current through secondary is less than primary.	Current through primary is less than secondary.	

(ii) Given:  $e = 96mV = 96 \times 10^{-3} V$ ,  $\frac{dl}{dt} = 1.20 A/s$ 

To find: Mutual Inductance (M)

Formula: 
$$M = \frac{|e|}{|\frac{dt}{dt}|}$$

Calculation: From formula

$$M = rac{96 imes 10^{-3}}{1.2} = 80 imes 10^{-3} H = 80 m H$$

Mutual Inductance of the two coils is 80 mH.

29. Answer the following questions:

(i) **Thermodynamic process:** A process by which two or more of state variables of a system can be changed is called a thermodynamic process or a thermodynamic change.

Types of thermodynamic processes:

- i. Quasi-static process
- ii. isothermal process
- iii. adiabatic process
- iv. isochoric process
- v. isobaric process
- vi. reversible process
- vii. irreversible process
- viii. cyclic process

(Any two types)

(ii) Given:  $W = -104 \; J, Q = -125 \; kJ = -125000 \; J$ 

To find: Change in internal energy  $(\Delta U)$ 

Formula:  $\Delta \mathrm{U} = |\mathrm{Q}| - |\mathrm{W}|$ 

Calculation: From formula,

$$\Delta U = |Q| - |W|$$

$$\therefore \Delta U = (125000 - 104)J = 124896 J$$

Change in internal energy is 124.896 kJ.

#### 30. Answer:

(i) **Definition:** A body, which absorbs the entire radiant energy incident on it, is called an ideal or perfect blackbody.

(ii) Given: 
$$m_1 = 1 \ kg, \ T_1 = 300 \ K, \ K_1 = 2.5 \times 10^6 \ J$$

 $m_2 = 4 \; kg, \; T_2 = 600 \; K, M_1 = 28, M_2 = 32$ 

To find: Kinetic energy (K<sub>2</sub>) Formula:  $K = \frac{3}{2} \frac{mRT}{M}$ Calculation: From formula,

$$egin{aligned} K_1 &= rac{3}{2} \left( rac{1 imes R imes 300}{28} 
ight) \ K_2 &= rac{3}{2} \left( rac{4 imes R imes 600}{32} 
ight) \ dots rac{K_1}{K_2} &= rac{4 imes 600}{32} imes rac{28}{300} \ K_2 &= rac{224}{32} imes 2.5 imes 10^6 \ K_2 &= 7 imes 2.5 imes 10^6 \end{aligned}$$

 $K_2^{}=17.5 imes 10^6 J$ 

The kinetic energy of 4 kg oxygen at 600 K is  $17.5 \times 10^6~J.$ 

31. **Statement:** The moment of inertia ( $L_o$ ) of an object about any axis is the sum of its moment of inertia ( $I_c$ ) about an axis parallel to the given axis, and passing through the centre of mass and the product of the mass of the object and the square of the distance between the two axes. Mathematically,  $I_o = I_c + Mh^2$ **Proof:** 

- i. Consider an object of mass M. Axis MOP is any axis passing through point O.
- ii. Axis ACB is passing through the centre of mass C of the object, parallel to the axis MOP, and at a distance h from it (: h = CO).



iii. Consider a mass element 'dm' located at point D. Perpendicular on OC (produced) from point D is DN.

iv. Moment of inertia of the object about the axis ACB is  $I_c = \int (DC)^2 dm$ , and about the axis MOP it is  $I_o = \int (DO)^2 dm$ .

 $\therefore I_o = \int (DO)^2 dm = \int \left[ (DN)^2 + (NO)^2 \right] dm$   $= \int \left[ (DN)^2 + (NC)^2 + 2 \cdot NC \cdot CO + (CO)^2 \right] dm$   $= \int \left[ (DC)^2 + 2NC \cdot h + h^2 \right] dm$   $\dots (\text{using Pythagoras theorem in } \triangle DNC )$   $= \int (DC)^2 dm + 2h \int NC \cdot dm + h^2 \int dm$   $\text{Now, } \int (DC)^2 dm = I_C \text{, and } \int dm = M$ 

v. NC is the distance of a point from the centre of mass. Any mass distribution is symmetric about the centre of mass. Thus, from the definition of the centre of mass,  $\int NC \cdot dm = 0$ 

$$I_o = I_c + Mh^2$$

This is the mathematical form of the theorem of parallel axes.