SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

Class 12 - Maths & Stats (Gen)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

The question paper is divided into FOUR sections.

1. **Section A:** Q. 1 contains Eight multiple-choice questions, each carrying Two marks.
   Q. 2 contains Four very short answer-type questions, each carrying One mark.
2. **Section B:** Q. 3 to Q. 14 contain Twelve short answer type questions, each carrying Two marks. (Attempt any Eight)
3. **Section C:** Q. 15 to Q. 26 contain Twelve short answer type questions, each carrying Three marks. (Attempt any Eight)
4. **Section D:** Q. 27 to Q. 34 contain Eight long answer-type questions, each carrying Four marks. (Attempt any Five)
5. The use of a log table is allowed. The use of a calculator is not allowed.
6. The figures to the right indicate full marks.
7. The use of graph paper is not necessary. Only a rough sketch of the graph is expected.
8. For each multiple-choice type of question, only the first attempt will be considered for evaluation.
9. Start answering each section on a new page.

Section A

1. **Select and write the correct answer for the following multiple-choice type of questions :** **[16]**

(a) The dual of statement $t \vee (p \vee q)$ is _____. **[2]**

a) $c \wedge (p \vee q)$ b) $t \wedge (p \vee q)$

c) $c \wedge (p \wedge q)$ d) $t \wedge (p \wedge q)$

(b) The principal solutions of the equation $\cos\theta = \frac{1}{2}$ are _____. **[2]**

a) $\frac{\pi}{3}, \frac{2\pi}{3}$ b) $\frac{\pi}{6}, \frac{5\pi}{6}$

c) $\frac{\pi}{3}, \frac{5\pi}{3}$ d) $\frac{\pi}{6}, \frac{7\pi}{6}$

(c) If the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product, then the value of k is _____. **[2]**

a) -1 b) 1

c) 2 d) -2

(d) If $p \wedge q = F, p \to q = F$, then the truth values of p and q are: **[2]**

a) T, T b) F, F

c) F, T d) T, F

(e) If $x = at^4, y = 2at^2$, then $\frac{dy}{dx} =$ _____. **[2]**

a) $-\frac{1}{t^2}$ b) $2t^2$

c) t^2 d) $\frac{1}{t^2}$

(f) $\int \frac{(x+3)}{(x+4)^2} \cdot e^x \, dx$ is equal to _____. **[2]**

a) $\frac{e^x}{x+4} + c$ b) $\frac{e^x}{(x+4)^2} + c$

c) $\frac{1}{(x+4)^2} + c$ d) $\frac{e^x}{x+3} + c$

(g) The differential equation whose general solution is $y = \log x + c$ is _____. **[2]**

a) $\frac{dy}{dx} + x = 0$ b) $x \cdot \frac{dy}{dx} + 1 = 0$

c) $\frac{1}{x} \cdot \frac{dy}{dx} = 0$ d) $x \cdot \frac{dy}{dx} = 1$

(h) If $X \sim B(n, p)$ and $E(X) = 12, \text{Var}(X) = 4$, then the value of n is _____. **[2]**

a) 36 b) 18

c) 3 d) 48

2. **Answer the following questions :** **[4]**

(a) Write the joint equation of co-ordinate axes. **[1]**

(b) If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} - 3\bar{b} - 2\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment BA. **[1]**

(c) If $f'(x) = x^{-1}$, then find $f(x)$. **[1]**

(d) Write the degree of the differential equation $e^{\frac{dy}{dx}} + \frac{dy}{dx} = x$ **[1]**

Section B

Attempt any 8 questions

3. Examine whether the following logical statement pattern is tautology, contradiction or contingency. **[2]**

$[(p \to q) \wedge q] \to p$

4. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$, then find A^{-1} by adjoint method. **[2]**

5. Find the general solution of the equation $4\cos^2 x = 1$. **[2]**

6. If $\bar{a} = 3\hat{i} - 2\hat{j} + 7\hat{k}, \bar{b} = 5\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{c} = \hat{i} + \hat{j} - \hat{k}$, then find $\bar{a} \cdot (\bar{b} \times \bar{c})$. **[2]**

7. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} + 3\bar{b} - 8\bar{c} = \bar{0}$ then find the ratio in which the point C divides the line segment AB. **[2]**

8. Examine whether the statement pattern $(p \to q) \leftrightarrow (\sim p \vee q)$ is a tautology, contradiction or contingency. **[2]**

9. Find the approximate value of $\cos(60^\circ 30')$. **[2]**

(Given: $1^o = 0.0175^c$, $\sin 60^o = 0.8660$)

10. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x} \, dx$ **[2]**

11. Find the area of the region bounded by the curve $y = x^2$ and the lines $x = 1, x = 2$ and $y = 0$. **[2]**

12. Solve the differential equation $\frac{dy}{dx} = x^2 y + y$. **[2]**

13. In a meeting 70% of the members favour and 30% oppose a certain proposal. A member is selected at random **[2]**

and we take $X = 0$ if he opposes, and $X = 1$ if he is in favour. Find $E(X)$ and $\text{Var}(X)$.

14. Evaluate: $\int_0^{\frac{\pi}{2}} \cos^3 x dx$ **[2]**

Section C

Attempt any 8 questions

15. The angles of the $\triangle ABC$ are in A.P. and b : $c = \sqrt{3} : \sqrt{2}$ then find $\angle A, \angle B, \angle C$. **[3]**

16. Show that: **[3]**

$\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{77}{85}\right)$.

17. Find the joint equation of pair of lines passing through the origin and perpendicular to the lines represented by $ax^2 + 2hxy + by^2 = 0$. **[3]**

18. The co-ordinates of the foot of a perpendicular drawn from the origin to the plane are $(2, 3, 1)$. Find the equation of the plane in vector form. **[3]**

19. Let $A(\bar{a})$ and $B(\bar{b})$ be any two points in the space and $R(\bar{r})$ be a third point on the line AB dividing the segment AB externally in the ratio m : n. Then $\bar{r} = \frac{m\bar{b} - n\bar{a}}{m-n}$. **[3]**

20. The cartesian equations of line are $3x - 1 = 6y + 2 = 1 - z$. Find the vector equation of line. **[3]**

21. If $y = \sin^{-1} x$, then show that: **[3]**

$(1 - x^2)\frac{d^2y}{dx^2} - x \times \frac{dy}{dx} = 0$.

22. Solve: $\int \sec^3(2x) dx$ **[3]**

23. Solve the differential equation $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$. **[3]**

24. The displacement **s** of a moving particle at time **t** is given by $s = 5 + 20t - 2t^2$. Find its acceleration when the velocity is zero. **[3]**

25. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles 9 articles are defective? **[3]**

26. Given the p.d.f. (probability density function) of a continuous random variable X as: **[3]**

$f(x) = \frac{x^2}{3}, -1 < x < 2$

$= 0$, otherwise

Determine the c.d.f. (cumulative distribution function) of X and hence find

i. $P(X < 1), P(X \leq -2), P(X > 0)$,
$P(1 < X < 2)$

ii. $P(X < 1), P(X > 0), P(1 < X < 2)$.

Section D

Attempt any 5 questions

27. Solve the following LPP by using graphical method. **[4]**

Maximize: $Z = 6x + 4y$,

Subject to $x \leq 2, x + y \leq 3, -2x + y \leq 1, x \geq 0, y \geq 0$.

28. Solve the following system of equations by method of inversion. **[4]**

$x + y + z = -1, y + z = 2, x + y - z = 3$

29. Find the volume of the parallelopiped whose vertices are A(3, 2, -1), B(-2, 2, -3), $C(3, 5, -2)$ and $D(-2, 5, 4)$. **[4]**

30. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$; and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar. Find the equation of the plane containing them. **[4]**

31. If $y = e^{\tan x} + (\log x)^{\tan x}$, then find $\frac{dy}{dx}$. **[4]**

32. Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x + 4}$ on the interval $[0, 5]$. **[4]**

33. Prove that: $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + c$ **[4]**

34. Prove that: **[4]**

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx\,,$$

if $f(x)$ is an even function. = 0, if $f(x)$ is an odd function.