SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

Class 12 - Maths & Stats (Gen)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

The question paper is divided into FOUR sections.

1. **Section A:** Q. 1 contains Eight multiple-choice questions, each carrying Two marks.
   Q. 2 contains Four very short answer-type questions, each carrying One mark.
2. **Section B:** Q. 3 to Q. 14 contain Twelve short answer type questions, each carrying Two marks. (Attempt any Eight)
3. **Section C:** Q. 15 to Q. 26 contain Twelve short answer type questions, each carrying Three marks. (Attempt any Eight)
4. **Section D:** Q. 27 to Q. 34 contain Eight long answer-type questions, each carrying Four marks. (Attempt any Five)
5. The use of a log table is allowed. The use of a calculator is not allowed.
6. The figures to the right indicate full marks.
7. The use of graph paper is not necessary. Only a rough sketch of the graph is expected.
8. For each multiple-choice type of question, only the first attempt will be considered for evaluation.
9. Start answering each section on a new page.

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Section A

1. **Select and write the correct answer for the following multiple-choice type of questions :** **[16]**

(a) The negation of $(p \vee \sim q) \wedge r$ is _____. **[2]**

a) $(\sim p \wedge q) \vee \sim r$ b) $(\sim p \vee q) \wedge \sim r$

c) $(\sim p \wedge q) \vee r$ d) $(\sim p \wedge q) \wedge r$

(b) In $\triangle ABC$, if $a = 2,\ b = 3$ and $\sin A = \frac{2}{3}$, then $\angle B =$ _____. **[2]**

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

(c) If an equation $hxy + gx + fy + c = 0$ represents a pair of lines, then **[2]**

a) $gh = cf$ b) $fg = ch$

c) $hf = -cg$ d) $fh = cg$

(d) The dual of statement $t \vee (p \vee q)$ is _____. **[2]**

a) $c \wedge (p \vee q)$ b) $t \wedge (p \vee q)$

c) $c \wedge (p \wedge q)$ d) $t \wedge (p \wedge q)$

(e) If $f(x) = x^5 + 2x - 3$, then $\left(f^{-1}\right)'(-3) =$ _____. **[2]**

a) $-\frac{1}{3}$ b) $\frac{1}{2}$

c) -3 d) 0

(f) $\int \cos^3 x \, dx =$ _____. **[2]**

a) $\frac{1}{12}\sin 3x - \frac{3}{4}\sin x + c$ b) $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x + c$

c) $\frac{1}{12}\sin 3x - \frac{1}{4}\sin x + c$ d) $\frac{1}{12}\sin 3x + \frac{1}{4}\sin x + c$

(g) The solution of the differential equation $\frac{dy}{dx} = \sec x - y\tan x$ is **[2]**

a) $y\sec x + \tan x = c$ b) $y\sec x = \tan x + c$

c) $\sec x = y\tan x + c$ d) $\sec x + y\tan x = c$

(h) Let the probability mass function (p.m.f.) of a random variable X be **[2]**

$P(X = x) = {}^4C_x \left(\frac{5}{9}\right)^x \times \left(\frac{4}{9}\right)^{4-x}$, for x = 0, 1, 2, 3, 4, then $E(X)$ is equal to _____.

a) $\frac{12}{9}$ b) $\frac{9}{20}$

c) $\frac{9}{25}$ d) $\frac{20}{9}$

2. **Answer the following questions :** **[4]**

(a) Find the separate equations of the lines represented by the equation $3x^2 - 10xy - 8y^2 = 0$. **[1]**

(b) If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} - 3\bar{b} - 2\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment BA. **[1]**

(c) Evaluate: $\int \frac{1}{x \log x \log(\log x)} dx$ **[1]**

(d) Write the degree of the differential equation $e^{\frac{dy}{dx}} + \frac{dy}{dx} = x$ **[1]**

Section B

Attempt any 8 questions

3. Write the following compound statements symbolically: **[2]**

i. Nagpur is in Maharashtra and Chennai is in Tamilnadu.

ii. If ΔABC is right angled at B, then $m\angle A + m\angle C = 90^\circ$.

4. Find the matrix of co-factors for the matrix $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$. **[2]**

5. Find the cartesian co-ordinates of the point whose polar co-ordinates are $\left(\frac{1}{2}, \frac{\pi}{3}\right)$. **[2]**

6. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively such that $3\bar{a} + 5\bar{b} = 8\bar{c}$, then find the ratio in which C divides AB. **[2]**

7. Find the volume of the parallelopiped whose coterminous edges are given by vectors **[2]**
$2\hat{i} + 3\hat{j} - 4\hat{k}, 5\hat{i} + 7\hat{j} + 5\hat{k}$ and $4\hat{i} + 5\hat{j} - 2\hat{k}$.

8. Without using truth table prove that **[2]**
$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$

9. Show that the function $f(x) = x^3 + 10x + 7$, $x \in R$ is strictly increasing. **[2]**

10. Evaluate: $\int_0^{\frac{\pi}{2}} \sin^2 x dx$ **[2]**

11. Find the area of the region bounded by the curve $y = x^2$ and the lines $x = 1, x = 2$ and $y = 0$. **[2]**

12. Find the integrating factor (I.F.) of the differential equation: $\frac{dy}{dx} + y = e^{-x}$ **[2]**

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13. A bakerman sells 5 types of cakes. Profits due to the sale of each type of cake is respectively ₹ 3, ₹ 2.5, ₹ 2, ₹ 1.5, ₹ 1 . The demands for these cakes are $10\%, 5\%, 25\%, 45\%$ and 15% respectively. What is the expected profit per cake? **[2]**

14. Evaluate: $\int_0^{\frac{\pi}{2}} \cos^2 x dx$ **[2]**

Section C

Attempt any 8 questions

15. With usual notations, in $\triangle ABC$, prove that $a(b\cos C - c\cos B) = b^2 - c^2$. **[3]**

16. Find the general solution of the equation $\sin 2x + \sin 4x + \sin 6x = 0$. **[3]**

17. Find the joint equation of pair of lines passing through the origin and perpendicular to the lines represented by $ax^2 + 2hxy + by^2 = 0$. **[3]**

18. Find the angle between the lines $\frac{x-1}{4} = \frac{y-3}{1} = \frac{z}{8}$ and $\frac{x-2}{2} = \frac{y+1}{2} = \frac{z-4}{1}$. **[3]**

19. Find the direction cosines of the line perpendicular to the lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$. **[3]**

20. Find the cartesian equation of the line passing through the points $A(3, 4, -7)$ and $B(6, -1, 1)$. **[3]**

21. If $y = \tan^{-1}\left(\frac{8x}{1-15x^2}\right)$ then find $\frac{dy}{dx}$. **[3]**

22. Evaluate: $\int \frac{(x+1)}{(x+2)(x+3)} dx$ **[3]**

23. Solve the differential equation $\frac{dy}{dx} - y = e^x$ **[3]**

Hence find the particular solution for $x = 0$ and $y = 1$.

24. If $f(x) = 3x + \frac{1}{3x}$ find the values of x for which function f(x) is decreasing. **[3]**

25. A fair coin is tossed 8 times. Find the probability that: **[3]**

i. it shows no head

ii. it shows head at least once.

26. For the following probability density function (p.d.f.) of X, find: **[3]**

i. P(X < 1)

if f(x) = $\frac{x^2}{18}$,

= 0,

ii. P(|X| < 1)

-3 < x < 3

otherwise

Section D

Attempt any 5 questions

27. Solve the following L.P.P. using graphical method: **[4]**

Minimize: $Z = 8x + 10y$

Subject to: $2x + y \geq 7, 2x + 3y \geq 15, y \geq 2, x \geq 0, y \geq 0$

28. If three numbers are added, their sum is 2. If two times the second number is subtracted from the sum of first and third numbers we get 8 and if three times the first number is added to the sum of second and third numbers we get 4. Find the numbers using matrices. **[4]**

29. By vector method show that the quadrilateral with vertices $A(1, 2, -1)$, B $(8, -3, -4)$, C $(5, -4, 1), D(-2, 1, 4)$ is a parallelogram. **[4]**

30. Find the equations of the planes parallel to the plane $x + 2y + 2z + 8 = 0$ which are at the distance of 2 units from the point $(1, 1, 2)$. **[4]**

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31. If $x = f(t), y = g(t)$ are differentiable functions of parameter **t** then prove that y is a differentiable function of **x** and **[4]**

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{dx}{dt} \neq 0$

32. A wire of length 36 meters is bent to form a rectangle. Find its dimensions if the area of the rectangle is maximum. **[4]**

33. Evaluate: $\int \frac{5e^x}{(e^x+1)(e^{2x}+9)} dx$ **[4]**

34. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. **[4]**

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