SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

Class 12 - Maths & Stats (Gen)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

The question paper is divided into FOUR sections.

1. **Section A:** Q. 1 contains Eight multiple-choice questions, each carrying Two marks.
   Q. 2 contains Four very short answer-type questions, each carrying One mark.
2. **Section B:** Q. 3 to Q. 14 contain Twelve short answer type questions, each carrying Two marks. (Attempt any Eight)
3. **Section C:** Q. 15 to Q. 26 contain Twelve short answer type questions, each carrying Three marks. (Attempt any Eight)
4. **Section D:** Q. 27 to Q. 34 contain Eight long answer-type questions, each carrying Four marks. (Attempt any Five)
5. The use of a log table is allowed. The use of a calculator is not allowed.
6. The figures to the right indicate full marks.
7. The use of graph paper is not necessary. Only a rough sketch of the graph is expected.
8. For each multiple-choice type of question, only the first attempt will be considered for evaluation.
9. Start answering each section on a new page.

Section A

1. **Select and write the correct answer for the following multiple-choice type of questions :** **[16]**

(a) The dual of statement $t \vee (p \vee q)$ is _____. **[2]**

a) $c \wedge (p \vee q)$ b) $t \wedge (p \vee q)$

c) $c \wedge (p \wedge q)$ d) $t \wedge (p \wedge q)$

(b) The principal solutions of $\sec x = \frac{2}{\sqrt{3}}$ are **[2]**

a) $\frac{\pi}{4}, \frac{11\pi}{4}$ b) $\frac{\pi}{6}, \frac{11\pi}{4}$

c) $\frac{\pi}{3}, \frac{11\pi}{6}$ d) $\frac{\pi}{6}, \frac{11\pi}{6}$

(c) If $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$, then the value of k is **[2]**

a) $\frac{1}{2}$ b) $\frac{-11}{2}$

c) $\frac{5}{2}$ d) $\frac{11}{2}$

(d) If $A = \{2, 3, 4, 5, 6\}$, then which of the following is not true? **[2]**

a) $\exists x \in$ A such that $x + 2 < 5$ b) $\exists x \in$ A such that $x + 2 < 9$

c) $\exists x \in$ A such that $x + 3 = 8$ d) $\forall x \in A$ such that $x + 6 \geq 9$

(e) If $x = at^4, y = 2at^2$, then $\frac{dy}{dx} =$ _____. **[2]**

a) $-\frac{1}{t^2}$ b) $2t^2$

c) t^2 d) $\frac{1}{t^2}$

(f) $\int \frac{dx}{9x^2+1} =$ _____. **[2]**

a) $\frac{1}{3}\tan^{-1}(6x) + c$ b) $\frac{1}{3}\tan^{-1} x + c$

c) $\frac{1}{3}\tan^{-1}(2x) + c$ d) $\frac{1}{3}\tan^{-1}(3x) + c$

(g) The solution of the differential equation $\frac{dx}{dt} = \frac{x \log x}{t}$ is _____. **[2]**

a) $x = e^{ct}$ b) $x = e^t + t$

c) $xe^{ct} = 0$ d) $x + e^{ct} = 0$

(h) If $X \sim B(n, p)$ and $E(X) = 12, \text{Var}(X) = 4$, then the value of n is _____. **[2]**

a) 36 b) 18

c) 3 d) 48

2. **Answer the following questions :** **[4]**

(a) Find the value of k, if lines represented by $kx^2 + 4xy - 4y^2 = 0$ are perpendicular to each other. **[1]**

(b) If the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $p\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear, then find the value of p. **[1]**

(c) Evaluate: $\int \frac{1}{\sin x \cdot \cos^2 x}\, dx$ **[1]**

(d) Write the degree of the differential equation $e^{\frac{dy}{dx}} + \frac{dy}{dx} = x$ **[1]**

Section B

Attempt any 8 questions

3. Using truth table verify that: **[2]**

$(p \wedge q) \vee \sim q \equiv p \vee \sim q$

4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $AX = I$, then find X by using elementary transformations. **[2]**

5. In $\triangle ABC$, prove that **[2]**

$ac\cos B - bc\cos A = a^2 - b^2$.

6. If the vectors $-3\hat{i} + 4\hat{j} - 2\hat{k}, \hat{i} + 2\hat{k}$ and $\hat{i} - p\hat{j}$ are coplanar, then find the value of p. **[2]**

7. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} + 3\bar{b} - 8\bar{c} = \bar{0}$ then find the ratio in which the point C divides the line segment AB. **[2]**

8. Discuss the statement pattern, using truth table: $\sim (\sim p \wedge \sim q) \vee q$ **[2]**

9. Find the approximate value of $\sqrt{8.95}$. **[2]**

10. Evaluate: $\int_0^{\frac{\pi}{2}} \sin^2 x dx$ **[2]**

11. Find the area of the region bounded by the curve $y = x^2$ and the lines $x = 1, x = 2$ and $y = 0$. **[2]**

12. Solve the differential equation $\frac{dy}{dx} + y = e^{-x}$ **[2]**

13. If $f(x) = kx, 0 < x < 2$ **[2]**

$= 0$, otherwise,

is a probability density function of a random variable X, then find:

i. value of k.

ii. $P(1 < X < 2)$

14. Evaluate: $\int_0^{\frac{\pi}{2}} \sqrt{1-\cos 4x}\,dx$ **[2]**

Section C

Attempt any 8 questions

15. Find the general solution of the equation $\sin x = \tan x$. **[3]**
16. In any $\triangle ABC$, if a^2, b^2, c^2 are in arithmetic progression, then prove that, cot A, cot B, cot C are in arithmetic progression. **[3]**
17. Find k, if the equation $kxy + 10x + 6y + 4 = 0$ represents a pair of straight lines. **[3]**
18. Find the angle between the planes $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = 3$ and $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 1$. **[3]**
19. If a line makes angles $90^\circ, 135^\circ, 45^\circ$ with X, Y and Z axes respectively, then find its direction cosines. **[3]**
20. The cartesian equations of line are $3x - 1 = 6y + 2 = 1 - z$. Find the vector equation of line. **[3]**
21. Differentiate $\cos^{-1}\left(\frac{3\cos x - 2\sin x}{\sqrt{13}}\right)$ w. r. t. x. **[3]**
22. Evaluate: $\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} dx$. **[3]**
23. Obtain the differential equation by eliminating the arbitrary constants A, B from the equation: **[3]**
    $y = A\cos(\log x) + B\sin(\log x)$
24. Find the approximate value of $\sin(30^\circ 30')$. **[3]**
    Given that $1^\circ = 0.0175^c$ and cos 30° = 0.866
25. The probability that a bomb will hit a target is 0.8. Find the probability that out of 10 bombs dropped, exactly 4 will hit the target. **[3]**
26. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X. **[3]**

Section D

Attempt any 5 questions

27. A company manufactures bicycles and tricycles each of which must be processed through machines A and B. Machine A has maximum of 120 hours available and machine B has maximum of 180 hours available. Manufacturing a bicycle requires 6 hours on machine A and 3 hours on machine B. Manufacturing a tricycle requires 4 hours on machine A and 10 hours on machine B. If profits are ₹ 180 for a bicycle and ₹ 220 for a tricycle, formulate and solve the L.P.P. to determine the number of bicycles and tricycles that should be manufactured in order to maximize the profit. **[4]**
28. Express the following equations in matrix form and solve them by the method of reduction: **[4]**
    $x + y + z = 6, 3x - y + 3z = 6$ and $5x + 5y - 4z = 3$
29. Show that the points $A(2, 1, -1), B(0, -1, 0)$, $C(4, 0, 4)$ and $D(2, 0, 1)$ are coplanar. **[4]**
30. Find the vector equation of the plane passing through the points $A(1, 0, 1), B(1, -1, 1)$ and $C(4, -3, 2)$. **[4]**
31. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t, so that y is function of x and $\frac{dx}{dt} \neq 0$, then prove that **[4]**
    $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.
    Hence find $\frac{dy}{dx}$, if $x = at^2, y = 2at$.
32. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 9$ on $[1, 4]$. **[4]**
33. Prove that: **[4]**
    $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$
34. Evaluate: $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x}\,dx$ **[4]**