SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

Class 12 - Maths & Stats (Gen)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

The question paper is divided into FOUR sections.

1. **Section A:** Q. 1 contains Eight multiple-choice questions, each carrying Two marks.
   Q. 2 contains Four very short answer-type questions, each carrying One mark.
2. **Section B:** Q. 3 to Q. 14 contain Twelve short answer type questions, each carrying Two marks. (Attempt any Eight)
3. **Section C:** Q. 15 to Q. 26 contain Twelve short answer type questions, each carrying Three marks. (Attempt any Eight)
4. **Section D:** Q. 27 to Q. 34 contain Eight long answer-type questions, each carrying Four marks. (Attempt any Five)
5. The use of a log table is allowed. The use of a calculator is not allowed.
6. The figures to the right indicate full marks.
7. The use of graph paper is not necessary. Only a rough sketch of the graph is expected.
8. For each multiple-choice type of question, only the first attempt will be considered for evaluation.
9. Start answering each section on a new page.

Section A

1. **Select and write the correct answer for the following multiple-choice type of questions :** **[16]**

(a) The negation of $p \wedge (q \rightarrow r)$ is ________. **[2]**

a) $p \vee (\sim q \vee r)$ b) $\sim p \wedge (\sim q \rightarrow \sim r)$

c) $p \rightarrow (q \wedge \sim r)$ d) $\sim p \wedge (\sim q \rightarrow r)$

(b) If in $\triangle ABC$ with usual notations $a = 18,\ b = 24,\ c = 30$, then $\sin \frac{A}{2}$ is equal to ________. **[2]**

a) $\frac{1}{\sqrt{10}}$ b) $\frac{1}{2\sqrt{5}}$

c) $\frac{1}{\sqrt{15}}$ d) $\frac{1}{\sqrt{5}}$

(c) The slopes of the lines given by $12x^2 + bxy - y^2 = 0$ differ by 7. Then the value of b is: **[2]**

a) 2 b) 1

c) ± 2 d) ± 1

(d) The negation of $(p \vee \sim q) \wedge r$ is _______. **[2]**

a) $(\sim p \wedge q) \vee \sim r$ b) $(\sim p \vee q) \wedge \sim r$

c) $(\sim p \wedge q) \vee r$

d) $(\sim p \wedge q) \wedge r$

(e) If $\sec\left(\frac{x+y}{x-y}\right) = a^2$, then $\frac{d^2y}{dx^2} =$ ________. **[2]**

a) x

b) y

c) 0

d) $\frac{y}{x}$

(f) $\int \frac{1}{1+\cos x}\, dx =$ ________. **[2]**

a) $2\tan\left(\frac{x}{2}\right) + c$

b) $-\cot\left(\frac{x}{2}\right) + c$

c) $-2\cot\left(\frac{x}{2}\right) + c$

d) $\tan\left(\frac{x}{2}\right) + c$

(g) The integrating factor of linear differential equation $\frac{dy}{dx} + y\sec x = \tan x$ is **[2]**

a) $\sec x \cdot \tan x$

b) $\sec x \cdot \cot x$

c) $\sec x - \tan x$

d) $\sec x + \tan x$

(h) Given $X \sim B(n, p)$. If $p = 0.6, E(X) = 6$, then the value of $\mathrm{Var}(X)$ is ________. **[2]**

a) 2.5

b) 2.4

c) 2.3

d) 2.6

2. **Answer the following questions :** **[4]**

(a) If $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines and $h^2 = ab \neq 0$ then find the ratio of their slopes. **[1]**

(b) If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} - 3\bar{b} - 2\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment BA. **[1]**

(c) Evaluate: $\int \frac{dx}{x^2+4x+8}$ **[1]**

(d) Write the degree of the differential equation $(y''')^2 + 3(y'') + 3xy' + 5y = 0$ **[1]**

Section B

Attempt any 8 questions

3. Examine whether the statement pattern $(p \to q) \leftrightarrow (\sim p \vee q)$ is a tautology, contradiction or contingency. **[2]**

4. Find the inverse of matrix $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$ by using elementary row transformations. **[2]**

5. Find the principal value of $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$. **[2]**

6. Find the direction cosines of the vector $2\hat{i} + 2\hat{j} - \hat{k}$. **[2]**

7. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $5\bar{a} + 3\bar{b} - 8\bar{c} = \bar{0}$ then find the ratio in which the point C divides the line segment AB. **[2]**

8. Without using truth table prove that **[2]**

$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$

9. Find the approximate value of $\cos(60^\circ 30')$. **[2]**

(Given: $1^o = 0.0175^c$, $\sin 60^o = 0.8660$)

10. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x}\, dx$ **[2]**

11. Find the area of the region bounded by the curve $y = x^2$ and the lines $x = 1, x = 2$ and $y = 0$. **[2]**

12. A particle is moving along X-axis. Its acceleration at time t is proportional to its velocity at that time. Find the differential equation of the motion of the particle. **[2]**

13. Find the expected value, variance and standard deviation of random variable X whose probability mass function (p.m.f.) is given below: **[2]**

$X=x$	1	2	3
$P(X=x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

14. Evaluate: $\int_0^{\frac{\pi}{2}} \sqrt{1-\cos 4x}\, dx$ **[2]**

Section C

Attempt any 8 questions

15. If $\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}$, then find the value of **x**. **[3]**

16. In $\triangle ABC$, if $a+b+c=2s$, then prove that $\sin\left(\frac{A}{2}\right)=\sqrt{\frac{(s-b)(s-c)}{bc}}$, with usual notations. **[3]**

17. If one of the lines given by $ax^2+2hxy+by^2=0$ bisects an angle between the co-ordinate axes then show that $(a+b)^2=4h^2$. **[3]**

18. Find the cartesian equation of the line passing through the points $A(3,4,-7)$ and $B(6,-1,1)$. **[3]**

19. If $\bar{a},\bar{b},\bar{c}$ are the position vectors of the points A,B,C respectively and $2\bar{a}+3\,\bar{b}-5\bar{c}=\bar{0}$, then find the ratio in which the point C divides the line segment AB. **[3]**

20. Find the equation of the plane passing through the intersection of the planes $3x+2y-z+1=0$ and $x+y+z-2=0$ and the point $(2,2,1)$. **[3]**

21. If $x^p y^q=(x+y)^{p+q}$, then prove that $\frac{dy}{dx}=\frac{y}{x}$. **[3]**

22. Solve: $\int \sec^3(2x)dx$ **[3]**

23. Solve the differential equation: **[3]**

$\frac{dy}{dx}+y\sec x=\tan x$

24. Find the equation of tangent to the curve $y=x^2+2e^x+2$ at the point $(0,4)$. **[3]**

25. The probability that a person who undergoes a kidney operation will be recovered is 0.5. Find the probability that of the 6 patients who undergo similar operations: **[3]**

i. none will recover

ii. half of them will recover.

26. The probability distribution of X is as follows: **[3]**

X	0	1	2	3	4
$P(X=x)$	0.1	K	2 K	2 K	K

Find:

i. K

ii. $P(X<2)$

iii. $P(X\geq 3)$

Section D

Attempt any 5 questions

27. Solve the following linear programming problem: **[4]**

Maximise: $z=150x+250y$

Subject to: $4x+y\leq 40, 3x+2y\leq 60, x\geq 0, y\geq 0$

28. The sum of three numbers is 9. If we multiply the third number by 3 and add to the second number, we get 16. By adding the first and the third number and then subtracting twice the second number from this sum, we get 6. Use this information and find the system of linear equations. Hence, find the three numbers using matrices. **[4]**

29. If $A(5,1,p), B(1,q,p)$ and $C(1,-2,3)$ are vertices of a triangle and $G\left(r, \frac{-4}{3}, \frac{1}{3}\right)$ is its centroid, then find the values of p, q, r by vector method. **[4]**

30. Find the shortest distance between the lines **[4]**
$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

31. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t so that y is differentiable function of x and $\frac{dx}{dt} \neq 0$, then prove that: **[4]**

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Hence find $\frac{dy}{dx}$ if $x = \sin t$ and $y = \cos t$.

32. Verify Rolle's theorem for the following function: $f(x) = x^2 - 4x + 10$ on $[0,4]$. **[4]**

33. Evaluate: $\int \frac{d\theta}{\sin\theta + \sin 2\theta}$ **[4]**

34. Prove that: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. **[4]**