SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

Class 12 - Maths & Stats (Gen)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

The question paper is divided into FOUR sections.

1. **Section A:** Q. 1 contains Eight multiple-choice questions, each carrying Two marks.
   Q. 2 contains Four very short answer-type questions, each carrying One mark.
2. **Section B:** Q. 3 to Q. 14 contain Twelve short answer type questions, each carrying Two marks. (Attempt any Eight)
3. **Section C:** Q. 15 to Q. 26 contain Twelve short answer type questions, each carrying Three marks. (Attempt any Eight)
4. **Section D:** Q. 27 to Q. 34 contain Eight long answer-type questions, each carrying Four marks. (Attempt any Five)
5. The use of a log table is allowed. The use of a calculator is not allowed.
6. The figures to the right indicate full marks.
7. The use of graph paper is not necessary. Only a rough sketch of the graph is expected.
8. For each multiple-choice type of question, only the first attempt will be considered for evaluation.
9. Start answering each section on a new page.

Section A

1. **Select and write the correct answer for the following multiple-choice type of questions :** **[16]**

(a) The negation of $p \wedge (q \rightarrow r)$ is **[2]**

a) $\sim p \wedge (q \rightarrow r)$ b) $\sim p \vee (q \wedge \sim r)$

c) $p \vee (\sim q \vee r)$ d) $\sim p \wedge (\sim q \rightarrow \sim r)$

(b) If $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$, then x is **[2]**

a) $-\frac{1}{2}$ b) 0

c) 1   d) $\frac{1}{2}$

(c) The joint equation of the pair of lines passing through $(2, 3)$ and parallel to the coordinate axes is **[2]**

a) $xy = 0$ b) $xy - 3x - 2y + 6 = 0$

c) $xy + 3x + 2y + 6 = 0$ d) $xy - 3x - 2y - 6 = 0$

(d) If $p \wedge q = F, p \rightarrow q = F$, then the truth values of p and q are: **[2]**

a) T, T b) F, F

c) F, T		d) T, F

(e) If y is a function of x and $\log(x+y)=2xy$, then the value of $y'(0)=$ ________. **[2]**

a) -1		b) 2

c) 0		d) 1

(f) If $\int \frac{dx}{4x^2-1}=A\log\left(\frac{2x-1}{2x+1}\right)+c$, then $A=$ ________. **[2]**

a) $\frac{1}{4}$		b) $\frac{1}{3}$

c) 1		d) $\frac{1}{2}$

(g) The integrating factor of linear differential equation $x\frac{dy}{dx}+2y=x^2\log x$ is ________. **[2]**

a) x^2		b) x

c) $\frac{1}{x^2}$		d) $\frac{1}{x}$

(h) Given $X\sim B(n,p)$. If $p=0.6, E(X)=6$, then the value of $\mathrm{Var}(X)$ is ________. **[2]**

a) 2.5		b) 2.4

c) 2.3		d) 2.6

2. **Answer the following questions :** **[4]**

(a) Write the separate equations of lines represented by the equation $5x^2-9y^2=0$. **[1]**

(b) If the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $p\hat{i}+6\hat{j}-8\hat{k}$ are collinear, then find the value of p. **[1]**

(c) Evaluate: $\int \sec^n x\cdot\tan x\,dx$. **[1]**

(d) Write the degree of the differential equation $(y''')^2+3(y'')+3xy'+5y=0$ **[1]**

Section B

Attempt any 8 questions

3. Using truth tables, examine whether the statement pattern $(p\wedge q)\vee(p\wedge r)$ is a tautology, contradiction or contingency. **[2]**

4. Find the inverse of matrix $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 0\end{bmatrix}$ by using elementary row transformations. **[2]**

5. Find the general solution of $\tan 2x=0$. **[2]**

6. If $\bar{p}=\hat{i}-2\hat{j}+\hat{k}$ and $\bar{q}=\hat{i}+4\hat{j}-2\hat{k}$ are position vector (P.V.) of points P and Q, find the position vector of the point R which divides segment PQ internally in the ratio 2:1. **[2]**

7. Find the direction cosines of the vector $\hat{i}+2\hat{j}-2\hat{k}$. **[2]**

8. Write the truth values of the following statements: **[2]**

i. 2 is a rational number and $\sqrt{2}$ is an irrational number.

ii. $2+3=5$ or $\sqrt{2}+\sqrt{3}=\sqrt{5}$.

9. Check whether the conditions of Rolle's theorem are satisfied by the function $f(x)=(x-1)(x-2)(x-3), x\in[1,3]$. **[2]**

10. Evaluate: $\int_0^{\frac{\pi}{2}}\cos^2 x dx$ **[2]**

11. Find the area of the region bounded by the curve $y=x^2$ and the lines $x=1, x=2$ and $y=0$. **[2]**

12. Solve the differential equation $y\frac{dy}{dx}+x=0$. **[2]**

13. The probability distribution of X, the number of defects per 10 metres of a fabric is given by **[2]**

x	0	1	2	3	4

$P(X = x)$	0.45	0.35	0.15	0.03	0.02

Find the variance of X.

14. Evaluate: $\int_0^{\frac{\pi}{2}} \sqrt{1 - \cos 4x}\, dx$ **[2]**

Section C

Attempt any 8 questions

15. Show that: **[3]**

$\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$.

16. If $-1 \leq x \leq 1$, then prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ **[3]**

17. If one of the lines given by $ax^2 + 2hxy + by^2 = 0$ bisects an angle between the co-ordinate axes then show that $(a + b)^2 = 4h^2$. **[3]**

18. If points $A(5, 5, \lambda), B(-1, 3, 2)$ and $C(-4, 2, -2)$ are collinear, then find the value of λ. **[3]**

19. Prove that the volume of a tetrahedron with coterminus edges $\bar{a}, \bar{b}$ and $\bar{c}$ is $\frac{1}{6}[\bar{a}\, \bar{b}\bar{c}]$. Hence, find the volume of tetrahedron whose coterminus edges are $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \bar{b} = -\hat{i} + \hat{j} + 2\hat{k}$ and $\bar{c} = 2\hat{i} + \hat{j} + 4\hat{k}$. **[3]**

20. Find the vector equation of the plane passing through the point $A(-1, 2, -5)$ and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$. **[3]**

21. If $\log_{10}\left(\frac{x^3 - y^3}{x^3 + y^3}\right) = 2$, then show that $\frac{dy}{dx} = -\frac{99x^2}{101y^2}$. **[3]**

22. Evaluate: $\int \frac{(x+1)}{(x+2)(x+3)} dx$ **[3]**

23. Find the particular solution of the differential equation $\frac{dy}{dx} = e^{2y} \cos x$, when $x = \frac{\pi}{6}, y = 0$. **[3]**

24. Find the approximate value of $\tan^{-1}(1.002)$. **[3]**

[Given:π = 3.1416]

25. Given is $X \sim B(n, p)$. If $E(X) = 6$, and $\text{Var}(X) = 4.2$, find the value of **[3]**

    i. n

    ii. n or p

26. Find k if the function $f(x)$ is defined by **[3]**

$f(x) = kx(1 - x),$ for $0 < x < 1$

$= 0,$ otherwise,

is the probability density function (p.d.f.) of a random variable (r.v.) X. Also find $P\left(X < \frac{1}{2}\right)$.

Section D

Attempt any 5 questions

27. Solve the following L. P. P. graphically: **[4]**

Minimize: $Z \doteq 6x + 2y$

Subject to: $5x + 9y \leq 90, x + y \geq 4,\ y \leq 8, x \geq 0, y \geq 0$

28. Find the inverse of the matrix, $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by using column transformations. **[4]**

29. Using vector method, find incentre of the triangle whose vertices are $P(0, 4, 0)$, $Q(0, 0, 3)$ and $R(0, 4, 3)$. **[4]**

30. Find the shortest distance between the lines $\bar{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$ where λ and μ are parameters. **[4]**

31. If $x = f(t), y = g(t)$ are differentiable functions of parameter **t** then prove that y is a differentiable function of **x** and **[4]**

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{dx}{dt} \neq 0$

32. A box with a square base is to have an open top. The surface area of box is $147 sq. cm$. What should be its dimensions in order that the volume is largest? **[4]**

33. Evaluate: $\int \frac{dx}{2+\cos x-\sin x}$ **[4]**

34. Evaluate: $\int\limits_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\ dx$ **[4]**