SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to **[1]**

a) 3A b) I - A

c) A d) I

2. If $A \cdot \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ then A = ? **[1]**

a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

3. If |A| = 2, where A is 2×2 matrix, find $|\text{adj A}|$. **[1]**

a) 0 b) 2

c) -8 d) 10

4. The function f(x) = $\begin{cases} 1, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$ is not continuous at **[1]**

a) x = 0 b) x = -1

c) x = 2 d) x = 1

5. The direction cosines of the line passing through the following points (-2, 4, -5),(1, 2, 3) is: **[1]**

a) $\frac{-3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{-8}{\sqrt{77}}$

b) $\frac{-3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

c) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

d) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{-8}{\sqrt{77}}$

6. The general solution of the differential equation $\frac{dy}{dx}$ + y cot x = cosec x, is **[1]**

a) x + y cos x = C

b) y + x (sin x + cos x) = C

c) x + y sin x = C

d) y sin x = x + C

7. Maximize Z = 50x + 60y , subject to constraints x +2 y ≤ 50 , x +y ≥ 30, x, y ≥ 0. **[1]**

a) 1600

b) 1547

c) 2500

d) 1525

8. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is: **[1]**

a) π

b) $\frac{\pi}{4}$

c) 0

d) $\frac{\pi}{2}$

9. $\int \sqrt{\frac{1+x}{1-x}} dx = ?$ **[1]**

a) $\sin^{-1} x + (1 + x^2) + C$

b) $\sin^{-1} x - \sqrt{1 - x^2} + C$

c) $\sin^{-1} x + \sqrt{1 - x^2} + C$

d) $\sin^{-1} x - (1 + x^2) + C$

10. If S = [s_{ij}] is a scalar matrix such that s_{ii} = k and A is a square matrix of the same order, then AS = SA =? **[1]**

a) kA

b) k + A

c) A^k

d) kS

11. Which of the following is not a convex set? **[1]**

a) [(x, y) : 2x + 5Y < 7]

b) $\{(x, y) : x^2 + y^2 \leq 4\}$

c) {X: |X| = 5}

d) {(x, y) : $3x^2 + 2y^2 \leq 6$)

12. The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$, and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if **[1]**

a) $\lambda = -2$

b) $\lambda = 0$

c) $\lambda = 1$

d) $\lambda = -1$

13. For the system of equations: **[1]**

x + 2y + 3z =1

2x + y + 3z = 2

5x + 5y + 9z = 4

a) there exists infinitely many solution

b) there is only one solution

c) there is no solution

d) there is two solution

14. There are 4 white and 3 black balls in a box. In another box, there are 3 white and 4 black balls. An unbiased die is rolled. If it shows a number less than or equal to 3, then a ball is drawn from the second box, otherwise from the first box. If the ball drawn is black, then the probability that the ball was drawn from the first box, is **[1]**

a) $\frac{1}{2}$

b) $\frac{4}{7}$

c) $\frac{3}{7}$

d) $\frac{6}{7}$

[1]

15. The order and the degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^2 = 4\frac{d^3y}{dx^3}$ respectively are:

a) 3, 3
b) 3, 1
c) 1, 2
d) 1, $\frac{2}{3}$

16. Area of a rectangle having vertices A, B, C and D with position vectors $-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$ and $-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$, respectively is **[1]**

a) 1
b) 2
c) 4
d) 1/2

17. If y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) 1
b) $\frac{1}{2}$
c) -1
d) $\frac{-1}{2}$

18. The direction ratios of the line x - y + z - 5 = 0 = x - 3y - 6 are proportional to **[1]**

a) 3, 1, - 2
b) $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$
c) 2, - 4, 1
d) $\frac{2}{\sqrt{41}}, \frac{-4}{\sqrt{41}}, \frac{1}{\sqrt{41}}$

19. **Assertion (A):** The function f(x) = x^2 - 4x + 6 is strictly increasing in the interval (2, ∞). **[1]**

Reason (R): The function f(x) = x^2 - 4x + 6 is strictly decreasing in the interval ($-\infty$, 2).

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** Let A ={2, 4, 6} and B = {3, 5, 7, 9} and defined a function f = {(2, 3), (4, 5), (6, 7)} from A to B. Then, f is not onto. **[1]**

Reason (R): A function f : A $\rightarrow$ B is said to be onto, if every element of B is the image of some elements of A under f.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the principal value of $\text{cosec}^{-1}(-1)$. **[2]**

OR

For the principal values, evaluate $\sin^{-1}\left[\cos\left\{2cosec^{-1}(-2)\right\}\right]$

22. Find the rate of change of the area of a circle with respect to its radius r when **[2]**

a. r = 3 cm
b. r = 4 cm

23. Find the values of x for which the function , **[2]**

$f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing in R

OR

On which of the following intervals is the function f(x) = x^{100} + sin x - 1 increasing

[2]

24. Evaluate: $\int \frac{1}{x\sqrt{x^4-1}} dx$

25. Find the maximum and minimum value, f(x) = |sin 4x + 3| **[2]**

Section C

26. Evaluate the integral: $\int \frac{1}{\sin(x-a)\sin(x-b)} dx$ **[3]**

27. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly? **[3]**

28. Evaluate: $\int \frac{x^3}{(x^2-4)} dx$. **[3]**

OR

Find the integral: $\int \frac{x+3}{\sqrt{5-4x-x^2}} dx$

29. Show that it is homogeneous and solve it: (x cos $\frac{y}{x}$)$\frac{dy}{dx}$ = (x cos $\frac{y}{x}$) + x **[3]**

OR

Find the particular solution of $e^{dy/dx}$ = x + 1, given that y = 3, when x = 0

30. Solve the Linear Programming Problem graphically: **[3]**

Minimize Z = 3 x_1 + 5x_2

Subject to

$x_1 + 3x_2 \geq 3$

$x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$

OR

Maximize Z = 3x + 2y

Subject to constraints

$x + 2y \leq 10$

$3x + y \leq 15$

$x, y \geq 0$

31. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, when $x \neq 0$. **[3]**

Section D

32. Find the area bounded by the curves y = x and y = x^3 **[5]**

33. Each of the following defines a relation on N: **[5]**

i. x is greater than y, x, y ∈ N
ii. x + y = 10, x, y ∈ N
iii. xy is square of an integer x, y ∈ N
iv. x + 4y = 10x, y ∈ N.

Determine which of the above relations are reflexive, symmetric and transitive.

OR

Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:

i. R = {(x, y): x is wife of y}
ii. R = {(x, y): x is father of y}

34. If A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations y + 2z = 5, x + 2y + 3z = 10, 3x + y + z = 9 **[5]**

35. Find the shortest distance between the lines given below: $\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$ and $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2}$. **[5]**

Hint: Change the given equation is vector form

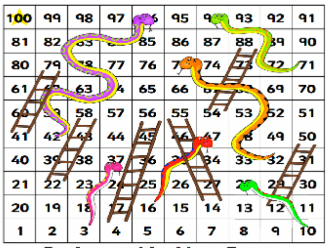
OR

Find the image of the point (5, 9, 3) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.



Aditya rolled down both black and red die together.

First die is black and second is red.

i. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5. (1)

ii. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4. (1)

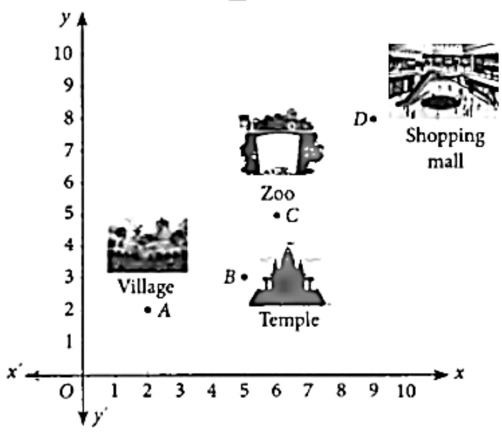
iii. Find the conditional probability of obtaining the sum 10, given that the black die resulted in even number. (2)

OR

Find the conditional probability of obtaining the doublet, given that the red die resulted in a number more than 4. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Girish left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Girish at different places is given in the following graph.



i. Find position vector of B (1)

ii. Find position vector of D (1)

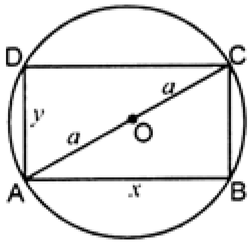
iii. Find the vector $\overrightarrow{BC}$ in terms of $\hat{i}$, $\hat{j}$. (2)

OR

iv. Find the length of vector $\overrightarrow{AD}$. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)

i. Find the perimeter of rectangle in terms of any one side and radius of circle. (1)

ii. Find critical points to maximize the perimeter of rectangle? (1)

iii. Check for maximum or minimum value of perimeter at critical point. (2)

OR

If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region. (2)