Solution

MATHEMATICS

Class 12 - Mathematics

Section A

1.

(d) I

Explanation:

Given that $A^2 = A$

Calculating value of $(I + A)^3 - 7A$:

$(I+A)^3-7A=I^3+A^3+3I^2A+3IA^2-7A$

$=\mathrm{I}+\mathrm{A}^2\cdot\mathrm{A}+3\mathrm{A}+3\mathrm{A}^2-7\mathrm{A}\left(\mathrm{I}^{\mathrm{n}}=\mathrm{I}\text{ and }\mathrm{I}\cdot\mathrm{A}=\mathrm{A}\right)$

$=I+A\cdot A+3A+3A-7A\left(A^2=A\right)$

$=I+A^2+3A+3A-7A$

$= I+7A-7A$

Hence, $(I + A)^3 - 7A = I$

2. **(a)** $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

Explanation:

The matrix on the R.H.S of the given matrix is of order 2×2 and the one given on left side is 2×2 . Therefore A has to be a 2×2 matrix.

Let $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}3 & 2\\ 1 & -1\end{pmatrix}=\begin{pmatrix}4 & 1\\ 2 & 3\end{pmatrix}$

$\begin{pmatrix}3a+b & 2a-b\\ 3c+d & 2c-d\end{pmatrix}=\begin{pmatrix}4 & 1\\ 2 & 3\end{pmatrix}$

3a + b = 4 ...(i)

2a - b = 1 ...(ii)

3c + d = 2 ...(iii)

2c - d = 3 ...(iv)

Using (i) and (ii)

a = 1

b = 1

Using (iii) and (iv)

c = 1

d = -1

So A becomes $\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$

3.

(b) 2

Explanation:

$|A| = 2$, $|\text{adj } A| = |A|^{2-1} = |A| = 2$

4. **(a)** x = 0

Explanation:

Given, function f(x) = $\begin{cases}1, & \text{if } x\neq0\\ 2, & \text{if } x=0\end{cases}$

At x = 0,

Value of function, f(0) = 2

And $\lim_{x\to 0}$ f(x) = 1

$\because \lim_{x\to 0}$ f(x) $\neq$ f(0)

or $\lim_{x\to a}$ f(x) $\neq$ value of function

$\therefore$ Function f(x) is not continuous at x = 0.

5.

(c) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

Explanation:

DR's are 1 + 2, 2 - 4, 3 + 5, i.e. 3, -2, 8.

Dividing by $|\sqrt{9+4+64}| = \sqrt{77}$

DC's are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

6.

(d) y sin x = x + C

Explanation:

We have,

$\frac{dy}{dx}$ + y cot x = cosec x

Comparing with $\frac{dy}{dx}$ + Py = Q of the above equation then, we get

$\Rightarrow$ P = cot x, Q = cosec x

I.F. = $e^{\int Pdx} = e^{\int \cot x dx} = e^{\log \sin x}$ = sin x

Multiplying on both sides by sin x

sin x $\frac{dy}{dx}$ + y cos x = 1

$\Rightarrow \frac{d}{dx}(y \sin x)$ = 1

$\Rightarrow$ y sin x = $\int 1dx$

$\Rightarrow$ y sin x = x + C

7.

(c) 2500

Explanation:

Here , Maximize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

Corner points	Z = 50x +60 y
P(50 ,0)	2500
Q(0 , 30)	1800
R(10, 20)	1700

Hence, the maximum value is 2500

8.

(d) $\frac{\pi}{2}$

Explanation:

$\frac{\pi}{2}$

9.

(b) $\sin^{-1} x - \sqrt{1-x^2} + C$

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \sec^2 x dx = \tan x$

Therefore ,

$= \int \sqrt{\frac{(1+x)^2}{(1+x)(1-x)}} dx$

$= \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx$

Put 1 - x^2 = t

-2x dx = dt

$= \sin^{-1}x - \frac{1}{2}\int \frac{1}{\sqrt{t}}dt + c$

$= \sin^{-1}x - \frac{1}{2}\frac{\sqrt{t}}{\frac{1}{2}} + c$

$= \sin^{-1}x - \sqrt{t} + c = \sin^{-1}x - \sqrt{1-x^2} + c$

10. **(a)** kA

Explanation:

A = $[S_{ij}]$

S = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

As, S_{ij} = k

Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ {Square Matrix}

AS = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

= $\begin{bmatrix} \mathrm{ka}_{11} & \mathrm{ka}_{12} \\ \mathrm{ka}_{21} & \mathrm{ka}_{22} \end{bmatrix}$

= k $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

= kA

SA = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

= $\begin{bmatrix} \mathrm{ka}_{11} & \mathrm{ka}_{12} \\ \mathrm{ka}_{21} & \mathrm{ka}_{22} \end{bmatrix}$

= k $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

= kA

Hence, AS = SA = kA

11.

(c) {X: |X| = 5}

Explanation:

|x| = 5 is not a convex set as any two points from negative and positive x -axis if joined will not lie in set.

12. **(a)** $\lambda = -2$

Explanation:

Given that, $\lambda\hat{\imath} + \hat{j} + 2\hat{k}$, $\hat{\imath} + \lambda\hat{j} - \hat{k}$, and $2\hat{\imath} - \hat{j} + \lambda\hat{k}$ are coplanar.

Let $\vec{a} = \lambda\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + \lambda\hat{k}$

Now, $\vec{a}, \vec{b}, \vec{c}$ are coplanar

If, $\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda\left(\lambda^2 - 1\right) - 1(\lambda + 2) + 2(-1 - 2\lambda) = 0$

$\Rightarrow \lambda^3 - \lambda - \lambda - 2 - 2 - 4\lambda = 0$

$\Rightarrow \lambda^3 - 6\lambda - 4 = 0$

$\Rightarrow (\lambda + 2)\left(\lambda^2 - 2\lambda - 2\right) = 0$

$\Rightarrow \lambda = -2$ and $\lambda = \frac{2 \pm \sqrt{(-2)^2 - 4\times1\times-2}}{2} = \frac{2\pm\sqrt{12}}{2}$

$\Rightarrow \lambda = -2$ and $\lambda = 1 \pm \sqrt{3}$.

13.

(b) there is only one solution

Explanation:

For the system of equations:

x + 2y + 3z = 1

2x + y + 3z = 2

5x + 5y + 9z = 4

The matrix equation corresponding to the above system is

$$\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 5 & 5 & 9\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 4\end{bmatrix}$$

Let A = $\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 5 & 5 & 9\end{bmatrix}$

$\therefore$ |A| $\neq$ 0

Hence, a unique solution exists of the above system.

14.

(c) $\frac{3}{7}$

Explanation:

Box I $\rightarrow$ 4 W, 3 B, Box II $\rightarrow$ 3 W, 4 B

Probability for choosing first box = $\frac{3}{6}=\frac{1}{2}$

Probability for choosing the second box = $\frac{1}{2}$

$\therefore$ Required probability = $\frac{\frac{1}{2}\times\frac{3}{7}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{4}{7}}=\frac{\frac{3}{14}}{\frac{3}{14}+\frac{4}{14}}$

= $\frac{3/14}{7/14}=\frac{3}{7}$

15.

(b) 3, 1

Explanation:

3, 1

16.

(b) 2

Explanation:

We have:

$\overrightarrow{OA}=-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$ (position vector of A) similarly , $\overrightarrow{OB}=\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}$, $\overrightarrow{OC}=\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$,

$\overrightarrow{OD}=-\hat{i}-\frac{1}{2}\hat{j}+4\hat{k}$:

,where $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\left(\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right)-\left(-\hat{i}+\frac{1}{2}\hat{j}+4\hat{k}\right)=2\hat{i}+0\hat{j}+0\hat{k}$ (by triangle law of vector addition),similarly $\overrightarrow{AD}=0\hat{i}-\hat{j}+0\hat{k}$, Therefore , area of rectangle ABCD is given by $\left|\overrightarrow{AB}X\,\overrightarrow{AD}\right|$,where$\overrightarrow{AB}X\overrightarrow{AD}=$

$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 0\\ 0 & -1 & 0\end{vmatrix}=\hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(-2-0)$=$-2\hat{k}$,$\left|\overrightarrow{AB}X\,\overrightarrow{AD}\right|$=$\sqrt{0^2+0^2+(-2)^2}=$ 2 sq. units.

17.

(b) $\frac{1}{2}$

Explanation:

Given that y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

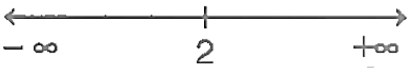
Using 1 - cos x = 2sin^2 $\frac{x}{2}$ and Using sin x = 2 sin x $\frac{x}{2}$cos $\frac{x}{2}$, we obtain

y = $\tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$ or y = $\tan^{-1}\tan\frac{x}{2}$

$y = \frac{x}{2}$
Differentiating with respect to x, we obtain
$\frac{dy}{dx} = \frac{1}{2}$

18. **(a)** 3, 1, - 2
Explanation:
We have,
x - y + z - 5 = 0 = x - 3y - 6
⇒ x - 3y - 6 = 0
&, x - y + z - 5 = 0
⇒ x = 3y + 6 ....(i)
x - y + z - 5 = 0 ...(ii)
From (i) and (ii)
We get, 3y + 6 - y + z - 5 = 0
⇒ 2y + z + 1 = 0
⇒ y = $\frac{-z-1}{2}$
y = $\frac{x-6}{3}$ [from (i)]
$\therefore \frac{x-6}{3} = y = \frac{-z-1}{2}$
So, the given equation can be re-written as
$\frac{x-6}{3} = \frac{y}{1} = \frac{z+1}{-2}$
Hence, the direction ratios of the given line are proportional to 3, 1, -2.

19.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
We have, f(x) = x^2 - 4x + 6
or f'(x) = 2x - 4 = 2 (x - 2)



Therefore, f'(x) = 0 gives x = 2.
Now, the point x = 2 divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.
In the interval $(-\infty, 2)$, f'(x) = 2x - 4 < 0.
Therefore, f is strictly decreasing in this interval.
Also, in the interval $(2, \infty)$, f'(x) > 0 and so the function f is strictly increasing in this interval.
Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Assertion: Given that,
A = {2, 4, 6},
R = {3, 5, 7, 9}
and R = {(2,3), (4, 5), (6, 7)}
Here, f(2) = 3, f(4) = 5 and f(6) = 7
It can be seen that the images of distinct elements of A under f are distinct.
Hence, function f is one-one but f is not onto as 9 $\in$ B does not have a pre-image in A.
Hence, both Assertion and Reason are true, but Reason is not a correct explanation of Assertion.

Section B

21. We know that the range of principal value of cosec^{-1} is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - [0]
Let cosec^{-1} (-1) = θ. Then we have, cosec θ = -1
cosec θ = -1 = -cosec $\frac{\pi}{2}$ = cosec $\left(\frac{-\pi}{2}\right)$
$\therefore \theta = \frac{-\pi}{2} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - [0]
Hence, the principal value of cosec^{-1} (-1) is equal to $\frac{-\pi}{2}$

OR

First of all we need to find the principal value for $\text{cosec}^{-1}(-2)$

Let,

$\text{cosec}^{-1}(-2) = Y$

$\Rightarrow$ cosec y = –2

$\Rightarrow$ –cosec y = 2

$\Rightarrow$ –cosec $\frac{\pi}{6}$ = 2

As we know that cosec($-\theta$) = –cosecθ

$\therefore$ –cosec $\frac{\pi}{6}$ = cosec $\left(\frac{-\pi}{6}\right)$

The range of principal value of cosec^{-1} (-2) is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]-\{0\}$ and

cosec $\left(\frac{-\pi}{6}\right)$ = –2

Thus,the princi value of $\text{cosec}^{-1}(-2)$ is $\frac{-\pi}{6}$.

$\therefore$ Now, the question changes to

$\text{Sin}^{-1}[\cos\frac{-\pi}{6}]$

Cos($-\theta$) = cos(θ)

$\therefore$ we can write the above expression as

$\text{Sin}^{-1}[\cos\frac{\pi}{6}]$

Let,

$\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ = Y

$\Rightarrow$ sin y = $\frac{\sqrt{3}}{2}$

$\Rightarrow$ Y = $\frac{\pi}{3}$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and sin $\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{3}$

Hence, the principal value of the given equation is $\frac{\pi}{3}$

22. Let x denote the area of the circle of variable radius r.

$\because$Area of circle $(x) = \pi r^2$

$\therefore$ Rate of change of area x w.r.t. r

$\frac{dx}{dr} = \pi(2r) = 2\pi r$

a. When r = 3 cm, then

$\frac{dx}{dr} = 2\pi(3) = 6\pi cm^2$/ sec

b. When r = 4 cm, then

$\frac{dx}{dr} = 2\pi(4) = 8\pi cm^2$/ sec

23. we have, f(x)=$kx^3 - 9x^2 + 9x + 3$

$\Rightarrow f'(x) = 3kx^2 - 18x + 9$

Since f(x) is increasing on R, therefore , $f'(x) > 0 \; \forall \; x \in R$

$\Rightarrow \; 3kx^2 - 18x + 9 > 0, \forall \; x \in \; R$

$\Rightarrow \; kx^2 - 6x + 3 > 0, \forall \; x \in \; R$

$\Rightarrow$ k>0 and 36-12k<0 [$\because \; ax^2 + bx + c > 0, \forall \; x \in \; R \Rightarrow \; a > 0 \; and \; discriminant < 0$]

$\Rightarrow \; k > 3$

Hence,f(x) is increasing on R, if k>3.

OR

We have seen that the function, f'(x) > 0 for 0 < x < 1. But, f'(x) can be positive as well as negative when - 1 < x < 0. So, f'(x) can be positive as well as negative for x ∈ (-1, 1). Hence, f(x) is neither increasing nor decreasing on (- 1, 1).

24. Let I = $\int \frac{1}{x\sqrt{x^4-1}}dx$ ...(i)

Also et x^2 = t then, we have

d(x^2) = dt

2x dx = dt

$\Rightarrow dx = \frac{dt}{2x}$

Putting x = t and $dx = \frac{dt}{2x}$ in equation (i), we get

$I = \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x}$

$= \frac{1}{2} \int \frac{1}{x^2\sqrt{t^2-1}} dt$

$= \frac{1}{2} \int \frac{1}{t\sqrt{t^2-1}} dt$

$= \frac{1}{2}$ $\sec^{-1} t + c$

$\therefore I = \frac{1}{2}$ $\sec^{-1} t + c$

25. It is given that f(x) = |sin 4x + 3|

Now, we can see that $-1 \leq \sin 4x \leq 1$

$\Rightarrow 2 \leq \sin 4x + 3 \leq 4$

$\Rightarrow 2 \leq |\sin 4x + 3| \leq 4$

Therefore, the maximum and minimum value of function h are 4 and 2 respectively.

Section C

26. let the given integral be,

l = $\int \frac{1}{\sin(x-a)\cdot\sin(x-b)} dx$

$= \frac{1}{\sin(b-a)} \int \frac{\sin(b-a)}{\sin(x-a)\cdot\sin(x-b)}$dx

$= \frac{1}{\sin(b-a)} \int \frac{\sin[(x-a)-(x-b)]}{\sin(x-a)\cdot\sin(x-b)} dx$

$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)\cdot\cos(x-b)-\cos(x-a)\sin(x-b)}{\sin(x-a)\cdot\sin(x-b)} dx$

$= \frac{1}{\sin(b-a)} \int \left[\frac{\sin(x-a)\cdot\cos(x-b)}{\sin(x-a)\cdot\sin(x-b)} - \frac{\cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}\right]$ dx

$= \frac{1}{\sin(b-a)} \int [\cot(x-b) - \cot(x-a)]\mathrm{dx}$

$= \frac{1}{\sin(b-a)} \int \cot(x-b) dx - \int \cot(x-a) dx$

$= \frac{1}{\sin(b-a)}$ [ln |sin(x - b)| - ln |sin(x - a)|] + C

$= \frac{1}{\sin(b-a)} \left[\ln\left|\frac{\sin(x-b)}{\sin(x-a)}\right|\right] + C$

$= \frac{-1}{\sin(a-b)} \left[\ln\left|\frac{\sin(x-b)}{\sin(x-a)}\right|\right] + C$

$= \frac{1}{\sin(a-b)} \ln\left|\frac{\sin(x-a)}{\sin(x-b)}\right| + C$

27. Let E_1: Event that the student knows the answer

E_2: Event that the student guesses the answer

E: Event that the answer is correct

Here, E_1 and E_2 are mutually exclusive and exhaustive events.

$\therefore \quad P(E_1) = \frac{3}{5}$ and $P(E_2) = \frac{2}{5}$

Now,$P\left(\frac{E}{E_1}\right) = P$ (the student answered correctly, given the knows the answer)=1

$P\left(\frac{E}{E_2}\right) = P$ (the student answered correctly, given he gusses)= $\frac{1}{3}$

The probability that the student knows the answer given that he answered it correctly is given by $P\left(\frac{E_1}{E}\right)$

By using Baye's theorem, we get

$P\left(\frac{E_1}{E}\right) = \frac{P\left(\frac{E}{E_1}\right)\cdot P(E_1)}{P\left(\frac{E}{E_1}\right)P(E_1)+P\left(\frac{E}{E_2}\right)P(E_2)}$

$= \frac{1\times\frac{3}{5}}{1\times\frac{3}{5}+\frac{1}{3}\times\frac{2}{5}} = \frac{\frac{3}{5}}{\frac{3}{5}+\frac{2}{15}}$

$= \frac{\frac{3}{5}}{\frac{15+10}{15}} = \frac{3\times3}{25} = \frac{9}{25}$

28. Let the given integral be,

$I = \int \frac{x^2}{x^2-4} dx$

Therefore by long division.
$I=\int x+\frac{4x}{x^2-4}dx$
$I=\int xdx+\int\frac{4x}{x^2-4}dx$
$=\frac{x^2}{2}+\int\frac{4x}{(x-2)(x+2)}dx$
Let $I_1=\int\frac{4x}{(x-2)(x+2)}dx$
So
$I=\frac{x^2}{2}+I_1$
Therefore $I_1=\int\frac{4x}{x^2-4}dx$
Putting x^2 - 4 = t
2xdx = dt
$I_1=2\int\frac{dt}{t}$
$I_1=2\log|x^2-4|+c$
Putting the value of I_1 in I,
$I=\frac{x^2}{2}+2\log|x^2-4|+c$

OR

Given integral is: $\int\frac{x+3}{\sqrt{5-4x-x^2}}dx$
Let us express
$x+3=A\frac{d}{dx}(5-4x-x^2)+B$ = A (- 4 - 2x) + B
Equating the coefficients of x and the constant terms from both sides, we get
- 2A = 1 and -4 A + B = 3, i.e., $A=-\frac{1}{2}$ and B = 1
Therefore, $\int\frac{x+3}{\sqrt{5-4x-x^2}}dx=-\frac{1}{2}\int\frac{(-4-2x)dx}{\sqrt{5-4x-x^2}}+\int\frac{dx}{\sqrt{5-4x-x^2}}$
= $-\frac{1}{2}I_1+I_2$ ....(i)
In I_1, put 5 - 4x - x^2 = t, so that (- 4 - 2x) dx = dt.
Therefore, $I_1=\int\frac{(-4-2x)dx}{\sqrt{5-4x-x^2}}=\int\frac{dt}{\sqrt{t}}=2\sqrt{t}+C_1$
= $2\sqrt{5-4x-x^2}+\mathrm{C}_1$ ....(ii)
Now consider $I_2=\int\frac{dx}{\sqrt{5-4x-x^2}}=\int\frac{dx}{\sqrt{9-(x+2)^2}}$
Put x + 2 = t, so that dx = dt.
Therefore, $I_2=\int\frac{dt}{\sqrt{3^2-t^2}}=\sin^{-1}\frac{t}{3}+C_2$
= $\sin^{-1}\frac{x+2}{3}+C_2$ ....(iii)
Substituting (ii) and (iii) in (i), we obtain
$\int\frac{x+3}{\sqrt{5-4x-x^2}}=-\sqrt{5-4x-x^2}+\sin^{-1}\frac{x+2}{3}+\mathrm{C},\text{ where }\mathrm{C}=\mathrm{C}_2-\frac{\mathrm{C}_1}{2}$

29. $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$
$\frac{dy}{dx}=\frac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{y}{x}\right)}$ ...(i)
let y = vx
$\frac{dy}{dx}=v.1+x.\frac{dv}{dx}$ ...(ii)
Put $\frac{dy}{dx}$ in eq ...(i)
$v+x\frac{dv}{dx}=\frac{vx\cos v+x}{x\cos v}$
$v+x\frac{dv}{dx}=\frac{v\cos v+1}{\cos v}$
$x\frac{dv}{dx}=\frac{v\cos v-1}{\cos v}-v$
$x\frac{dv}{dx}=\frac{v\cos v+1-v\cos v}{\cos v}$
$x\frac{dv}{dx}=\frac{1}{\cos v}$
$\int\cos vdv=\int\frac{dx}{x}$
$\sin v=\log x+\log c$ sin v = log x + log c
$\sin v=\log|cx|$ sin v = log|cx|$[\because y=vx]$ [y = vx]
$\sin\left(\frac{y}{x}\right)=\log|cx|$

OR

The given differential equation is,

$e^{\frac{dy}{dx}}$ = x + 1

$\Rightarrow \frac{dy}{dx}$ = log (x + 1)

$\Rightarrow$dy = log (x + 1) dx

Integrating both sides, we get

$\int$ dy = $\int$ log (x + 1) dx

$\Rightarrow$ y = log (x + 1) $\int$ 1 dx - $\int \left[\frac{d}{dx}\{\log(x+1)\} \int 1 dx\right]$ dx

$\Rightarrow$ y = x log (x + 1) $- \int \frac{1}{x+1} \times x dx$

$\Rightarrow$ y = x log (x + 1) $- \int \left(1 - \frac{1}{x+1}\right)$ dx

$\Rightarrow$ y = x log (x + 1) $- \int dx + \int \frac{1}{x+1} dx$

$\Rightarrow$ y = x log (x + 1) - x + log |x + 1| + C

$\Rightarrow$y = (x + 1) log |x + 1| - x + C ...(i)

It is given that at x = 0 and y = 3

Substituting the values of x and y in (i), we get

C = 3

Therefore, substituting the value of C in (i), we get

y = (x + 1) log |x + 1| - x + 3

Hence, y = (x + 1) log |x + 1| - x + 3 is the required solution.

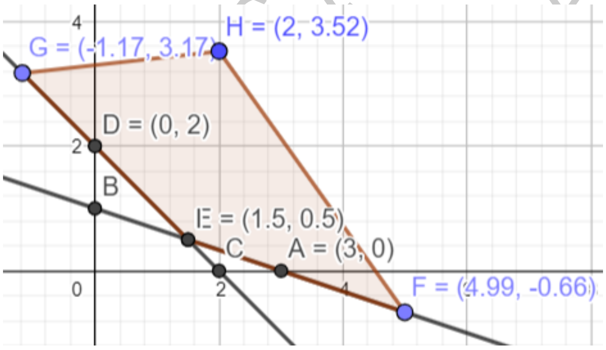
30. First, we will convert the given inequations into equations, we obtain the following equations:

$x_1 + 3x_2 = 3$, $x_1 + x_2 = 2$, $x_1 = 0$ and $x_2 = 0$

Region represented by $x_1 + 3x_2 \geq 3$:

The line $x_1 + 3x_2 = 3$ meets the coordinate axes at A(3,0) and B(0,1) respectively. By joining these points we obtain the line $x_1 + 3x_2 = 3$

Clearly (0,0) does not satisfies the inequation $x_1 + 3x_2 \geq 3$. So, the region in the plane which does not contain the origin represents the solution set of the inequation $x_1 + 3x_2 \geq 3$ Region represented by $x_1 + x_2 \geq 2$ The line $x_1 + x_2 = 2$ meets the coordinate axes at C(2,0) and D(0,2) respectively. By joining these points we obtain the line $x_1 + x_2 = 2$ Clearly (0,0) does not satisfies the inequation $x_1 + x_2 \geq 2$. So, the region containing the origin represents the solution set of the inequation $x_1 + x_2 \geq 2$ Region represented by $x_1 \geq 0$ and $x_2 \geq 0$ since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \geq 0$ and $x_2 \geq 0$ The feasible region determined by subject to the constraints are, $x_1 + 3x_2 \geq 3$, $x_1 + x_2 \geq 2$,and the non-negative restrictions $x_1 \geq 0$, and $x_2 \geq 0$, are as follows



The corner points of the feasible region are O(0,0), B(0,1), $E\left(\frac{3}{2}, \frac{1}{2}\right)$ and C(2,0)

The values of objective function at the corner points are as follows:

Corner point : $z = 3x_1 + 5x_2$

O(0, 0) : $3 \times 0 + 5 \times 0 = 0$

B(0, 1) : $3 \times 0 + 5 \times 1 = 5$

$E\left(\frac{3}{2}, \frac{1}{2}\right)$: $\frac{3}{2} + 5 \times \frac{1}{2} = 7$

C(2, 0) : $3 \times 2 + 5 \times 0 = 6$

Therefore, the minimum value of objective function Z is 0 at the point O(0,0). Hence, $x_1 = 0$ and $x_2 = 0$ is the optimal solution of

the given LPP.
Thus, the optimal value of objective function Z is 0.

OR

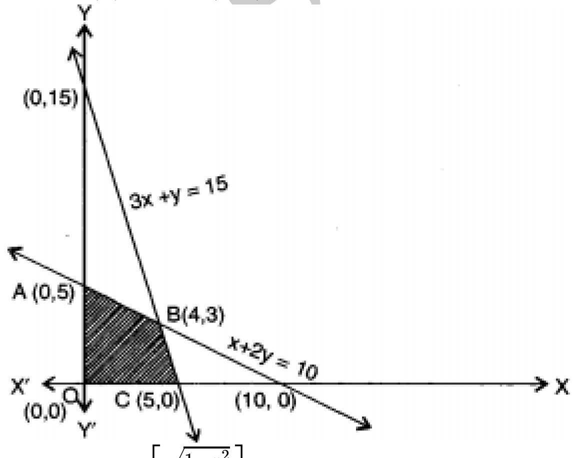
Linear constraints
$x + 2y \leq 10$
$3x + y \leq 15$
$x, y \geq 0$
and objective function is max $(Z) = 3x + 2y$.
Reducing the above inequations into equations and finding their point of intersections, i.e.,
$x + 2y = 10$ ... (i)
$3x + y = 15$ ... (ii)
$x = 0, y = 0$ ... (iii)

Equations	**Point of intersection**
(i) and (ii)	$x = 4 \; and \; y = 3 \Rightarrow (4, 3)$
(i) and (iii)	at $x = 0 \Rightarrow y = 5 \Rightarrow (0, 5)$
	at y = 0 ⇒ x = 10 ⇒ (10, 0)
(ii) and (iii)	at x = 0 ⇒ y = 15 ⇒ (0, 15)
	at y = 0 ⇒ x = 5 ⇒ (5, 0)

Now for feasible region, using origin testing method for each constraint
$x + 2y \leq 10$, let x = 0, y = 0
$\Rightarrow 0 \leq 10$ i.e., true
$\Rightarrow$ The shaded region will be toward the origin.
Non negative restrictions $x \geq 0, y \geq 0$ indicates that the feasible region will exist in first quadrant.
Now, corner points are $A(0,5), B(4,3) C(5,0) \; and \; D(0,0)$.
For optimal solution substituting the value of all corner points in $Z = 3x + 2y$,

Corner points	**Z**
$A(0,5)$	10
$B(4,3)$	18 maximum
$C(5,0)$	15
$D(0,0)$	0

Hence, the maximum value of Z exist when x = 4 and y = 3
$\Rightarrow$ max (Z) = 18 at (4, 3)



31. Let u = $\tan^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right]$

Put x = $\cos\theta \Rightarrow \theta = \cos^{-1} x$

Then, u = $\tan^{-1}\left[\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right]$

$= \tan^{-1}\left[\frac{\sqrt{\sin^2\theta}}{\cos\theta}\right]\left[\because \cos^2\theta+\sin^2\theta=1 \Rightarrow \sin^2\theta=1-\cos^2\theta\right]$

$= \tan^{-1}\left[\frac{\sin\theta}{\cos\theta}\right]$

$= \tan^{-1}[\tan\theta]=\theta$

$\Rightarrow \quad u=\cos^{-1}x$

On differentiating both sides w.r.t x, we get

$\frac{du}{dx}=-\frac{1}{\sqrt{1-x^2}}$

Again , let v $=\cos^{-1}\left(2x\sqrt{1-x^2}\right)$

Put x $=\cos\theta \Rightarrow \theta=\cos^{-1}x$

Then, v $=\cos^{-1}\left[2\cos\theta\sqrt{1-\cos^2\theta}\right]$

$=\cos^{-1}[2\cos\theta\sin\theta]\left[\begin{array}{l}\because \sin\theta=\sqrt{1-\cos^2\theta}\\ \Rightarrow \sin^2\theta=1-\cos^2\theta\end{array}\right]$

$=\cos^{-1}[\sin 2\theta]$

$=\cos^{-1}\left[\cos\left(\frac{\pi}{2}-2\theta\right)\right]=\frac{\pi}{2}-2\theta$

$\Rightarrow v=\frac{\pi}{2}-2\cos^{-1}x\ \left[\because \theta=\cos^{-1}x\right]$

On differentiating both sides w.r.t x, we get

$\frac{dv}{dx}=\frac{2}{\sqrt{1-x^2}}$

Now, $\frac{du}{dv}=\frac{du}{dx}\times\frac{dx}{dv}=-\frac{1}{\sqrt{1-x^2}}\times\frac{\sqrt{1-x^2}}{2}$

$=-\frac{1}{2}$

Section D

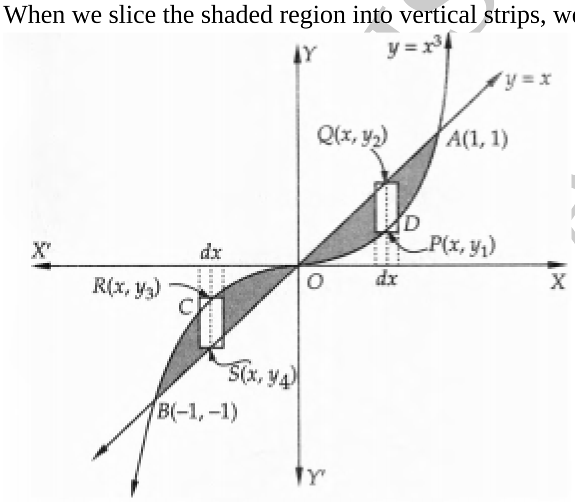
32. The given curves are ,

y = x ...(i)

and y = x^3 ...(ii)

The sketch of the curve y = x^3 is shown in Fig. Clearly, y =x is a line passing through the origin and making an angle of 45° with x-axis. The shaded portion shown in Fig. is the region bounded by the curves y = x and y = x^3. Solving y = x and y = x^3 simultaneously, we find that the two curves intersect at O (0, 0), A (1,1) and B (- 1, - 1).

When we slice the shaded region into vertical strips, we observe that the vertical strips change their character at O.



Therefore, the required area is given by,

Required area = Area BCOB + Area ODAO

Area BCOB: Each vertical strip in this region has its lower end on y = x and the upper end on y = x^3. Therefore, the approximating rectangle shown in this region has length $=|y_4-y_3|$, width = dx and area $=|y_4-y_3|$ dx. Since the approximating rectangle can move from x = - 1 to x = 0.

$\therefore$ Area BCOB $=\int_0^0|y_4-y_3|\,dx=\int_{-1}^0-(y_4-y_3)\,dx$ [$\because$ $y_4 < y_3 \therefore y_4 - y_3 < 0$]

$=\int_{-1}^0-\left(x-x^3\right)dx$ [$\because$ R (x, y_3) and S (x, y_4) lie on (ii) and (i) respectively $\therefore$ $y_3 = x_3$ and $y_4 = x$]

$=\int_{-1}^0\left(x^3-x\right)dx=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^0=0-\left(\frac{1}{4}-\frac{1}{2}\right)=\frac{1}{4}$ sq. units

Area ODAO: Each vertical strip in this region has its two ends on (ii) and (i) respectively. So, the approximating rectangle shown in this region has length $= |y_2 - y_1|$, width = dx and therefore,we have,

Area ODAO $= \int_0^1 |y_2 - y_1|\,dx = \int_0^1 (y_2 - y_1)\,dx$ [$\because$ $y_2 > y_1 \therefore y_2 - y_1 > 0$]

$= \int_0^1 \left(x - x^3\right) dx$ [$\because$ P (x, y_1) and Q (x, y_2) lie on (ii) and (i) respectively $\therefore$ $y_1 = x^3$ and $y_2 = x$]

$= \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ sq. units

$\therefore$ Required area = Area BCOB + Area ODAO $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ sq. units

33. i. x is greater than y, x, y $\in$ N

For xRx x > x is not true for any x $\in$ N.

Therefore, R is not reflexive.

Let (x, y) $\in$ R $\Rightarrow$ xRy

x > y

but y > x is not true for any x, y $\in$ N

Thus, R is not symmetric.

Let xRy and yRz

x > y and y > z $\Rightarrow$ x > z

$\Rightarrow$ xRz

So, R is transitive.

ii. x + y = 10, x, y $\in$ N

R={(x,y) : x+y = 10 ,x,y $\in$ R}

R = {(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)} (1, 1) $\notin$ R

So, R is not reflexive.

(x, y) $\in$ R $\Rightarrow$ (y, x) $\in$ R

Therefore, R is symmetric.

(1, 9) $\in$ R, (9, 1) $\in$ R $\Rightarrow$ (1, 1) $\notin$ R

Hence, R is not transitive.

iii. Given xy, is square of an integer x, y $\in$ N

$\Rightarrow$ R = {(x, y) : xy is a square of an integer x, y $\in$ N}

(x, x) $\in$ R, $\forall$x $\in$ N

As x^2 is square of an integer for any x $\in$ N

Hence, R is reflexive.

If (x, y) $\in$ R $\Rightarrow$ (y, x) $\in$ R

Therefore, R is symmetric.

If (x, y) $\in$ R (y, z) $\in$ R

So, xy is square of an integer and yz is square of an integer.

Let xy = m^2 and yz = n^2 for some m, n $\in$ Z

$x = \frac{m^2}{y}$ and $z = \frac{x^2}{y}$

$xz = \frac{m^2 n^2}{y^2}$, Which is square of an integer.

So, R is transitive.

iv. x + 4y = 10, x, y $\in$ N

R = {(x, y) : x + 4y = 10, x, y $\in$ N}

R{(2, 2), (6, 1)}

(1, 1), (3, 3) ..... $\notin$ R

Thus, R is not reflexive.

(6, 1) $\in$ R but (1, 6) $\notin$ R

Hence, R is not symmetric.

(x, y) $\in$ R $\Rightarrow$ x + 4y = 10 but (y, z) $\in$ R

y + 4z = 10 $\Rightarrow$ (x, z) $\in$ R

So, R is transitive.

OR

i. R = {(x, y) : x is wife of y}

Reflexive: since x can not be wife of x.

$\therefore$ (x, x) $\notin$R

⇒ R is not reflexive

Symmetric: Let (x, y) ∈R

⇒ x is wife of y

⇒ Y is husband of x

⇒(y, x) ∉R

⇒ R is not symmetric

Transitive: Let (x, y) ∈ R and (y, z) ∈R

⇒ x is wife of y and y is husband of z which is a contradiction

⇒ (x, z) ∉R

⇒ R is not transitive

ii. A be the set of human beings

R = {(x, y) : x is father of y}

Reflexive: since x can not be father of x

∴ (x, x) ∉ R

⇒R is not reflexive

Symmetric: Let (x, y) ∈R

⇒ x is father of y

⇒ y can not be father of x

⇒ (y, x) ∉R

⇒ R is not symmetric

Transitive:

Now, let (x,y), (y,z) ϵ R

⇒ x is the father of y and y is the father of z.

⇒ x is not the father of z.

⇒ Indeed x is the grandfather of z.

⇒ (x,z) ∉ R

⇒ R is not transitive.

34. $|A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$ = -1(1-9)+2(1-6) = 8-10

$|A|$ = -2 $\neq 0 \Rightarrow A^{-1}$ exists

Now, A_{11} = -1, A_{12} = 8, A_{13} = -5

A_{21} = 1, A_{22} = -6, A_{23} = 3

A_{31} = -1 , A_{32} = 2, A_{33} = -1

adj A = $\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}$ adjA = $\frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

Given system of equations are

y + 2z = 5, x + 2y + 3z = 10, 3x + y + z = 9

It can be written as $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$ or AX = B,

where A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$

Now AX = B ⇒ X = $A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -4 \\ -2 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ x = 2, y = 1, z = 2

35. Here,it is given that the equation of line

$L_1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$

Lines L_1 is passing through point (1, -2, 3) and has direction (-1, 1, -2)

Thus, vector equation of line L_1 is

$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$

And

$L_2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$

Line L_2 is passing through point (1, -1, -1) and has direction ratios (2, 2, -3)

Thus, vector equation of line L_2 is

$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$

Now, to calculate distance between the lines,

$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$

$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$

Here,

$\vec{a}_1 = i - 2\hat{j} + 3\hat{k}$

$\overrightarrow{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

$\overrightarrow{a_2} = \hat{1} - \hat{j} - \hat{k}$

$\overrightarrow{b_2} = 2\hat{i} + 2\hat{j} - 2\hat{k}$

Thus,

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 2 & 2 & -2 \end{vmatrix}$$

$= \hat{i}(-2 + 4) - j(2 + 4) + \hat{k}(-2 - 2)$

$\therefore \overrightarrow{b_1} \times \overrightarrow{b_2} = 2\hat{\imath} - 6\hat{\jmath} - 4\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{2^2 + (-6)^2 + (-4)^2}$

$= \sqrt{4 + 36 + 16}$

$= \sqrt{56}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (1 - 1)\hat{i} + (-1 + 2)\hat{j} + (-1 - 3)\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 0\hat{\imath} + \hat{\jmath} - 4\hat{k}$

Now,we have

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (2\hat{i} - 6\hat{j} - 4\hat{k}) \cdot (0\hat{\imath} + \hat{\jmath} - 4\hat{k})$

$= (2 \times 0) + ((-6) \times 1) + ((-4) \times (-4))$

= 0 - 6 + 16

= 10

Therefore, the shortest distance between the given line is

$d = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

$\Rightarrow d = \left|\frac{10}{\sqrt{56}}\right|$

$\Rightarrow d = \frac{10}{\sqrt{56}}$

$\therefore d = \frac{10}{\sqrt{56}}$ units

OR

Suppose,

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

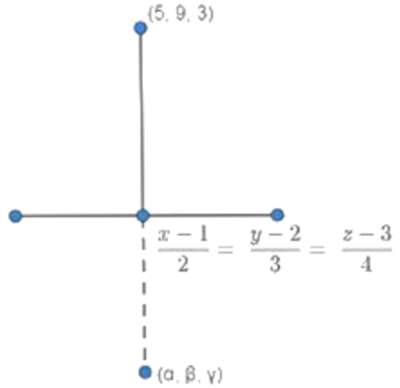
So the foot of the perpendicular is $(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

The direction ratios of the perpendicular is

$(2\lambda + 1 - 5) : (3\lambda + 2 - 9) : (4\lambda + 3 - 3)$

$\Rightarrow (2\lambda - 4) : (3\lambda - 7) : (4\lambda)$

Direction ratio of the line is 2 : 3 : 4



From the direction ratio of the line and the direct ratio of its perpendicular, we have

$2(2\lambda - 4) + 3(3\lambda - 7) + 4(4\lambda) = 0$

$\Rightarrow 4\lambda - 8 + 9\lambda - 21 + 16\lambda = 0$

$\Rightarrow 29\lambda = 29$

$\Rightarrow \lambda = 1$

Therefore, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and (α, β, γ)

Therefore, we have

$\frac{\alpha+5}{2} = 3 \Rightarrow \alpha = 1$

$\frac{\beta+9}{2} = 5 \Rightarrow \beta = 1$

$\frac{y+3}{2} = 7 \Rightarrow \gamma = 11$

Therefore, the image is (1, 1, 11)

Section E

36. i. Let A represents obtaining a sum greater than 9 and B represents black die resulted in a 5.

n(S) = 36

n(A) = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)} = 6

n(B) = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5)} = 6

n(A ∩ B) = {(5, 5), (5, 6)} = 2

P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$

ii. Let A represents obtaining a sum 8 and B represents red die resulted in number less than 4.

n(S) = 36

n(A) = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) = 5

n(B) = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)} = 18

n(A ∩ B) = {(5, 3), (6, 2)} = 2

P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$

iii. Let A represents obtaining a sum 10 and B represents black die resulted in even number.

n(S) = 36

n(A) = {(4, 6), (6, 4), (5, 5)} = 3

n(B) = {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} = 18

n(A ∩ B) = {(4, 6), (6, 4)} = 2

P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$

OR

Let A represents getting doublet and B represents red die resulted in number greater than 4.

n(S) = 36

n(A) = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} = 6

n(B) = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6), (5, 5), (5, 6), (6, 5), (6, 6)} = 12

n(A ∩ B) = {(4, 4), (5, 5), (6, 6)} = 3

P(A/B) = $\frac{P(A\cap B)}{P(B)} = \frac{\frac{n(A\cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$ = $\frac{\frac{3}{36}}{\frac{12}{36}} = \frac{1}{4}$

37. i. Here (5, 3) are the coordinates of B.

$\therefore$ Position vector of B = $5\hat{i} + 3\hat{j}$

ii. Here (9, 8) are the coordinates of D.

$\therefore$ Position vector of D = $9\hat{i} + 8\hat{j}$

iii. Position vector of B = $5\hat{i} + 3\hat{j}$ and Position vecto of C = $6\hat{i} + 5\hat{j}$

$\therefore \overrightarrow{BC}$ = (6 - 5)$\hat{i}$ + (5 - 3)$\hat{j}$ = $\hat{i} + 2\hat{j}$

OR

Since P.V. of A = $2\hat{i} + 2\hat{j}$, P.V. of D = $9\hat{i} + 8\hat{j}$

$\therefore \overrightarrow{AD}$ = (9 - 2)$\hat{i}$ + (8 - 2)$\hat{j}$ = $7\hat{i} + 6\hat{j}$

$|\overrightarrow{AD}|^2 = 7^2 + 6^2 = 49 + 36 = 85$

$\Rightarrow |\overrightarrow{AD}| = \sqrt{85}$ units

38. i. Let ‘y’ be the breadth and ‘x’ be the length of rectangle and ‘a’ is radius of given circle.

From fig $4a^2 = x^2 + y^2$

$\Rightarrow y^2 = 4a^2 - x^2$

$\Rightarrow$ y = $\sqrt{4a^2 - x^2}$

Perimeter (P) = 2x + 2y = $2\left(x + \sqrt{4a^2 - x^2}\right)$

ii. We know that P = $2\left(x + \sqrt{4a^2 - x^2}\right)$

Critical points to maximize perimeter $\frac{dP}{dx} = 0$

$\Rightarrow \frac{dp}{dx} = 2\left(1 + \frac{1}{2\sqrt{4a^2-x^2}}(-2x)\right)$ = 0

$2\left(\frac{\sqrt{4a^2-x^2}-x}{\sqrt{4a^2-x^2}}\right)$ = 0

$\Rightarrow \sqrt{4a^2 - x^2}$ = x

$\Rightarrow 4a^2 - x^2 = x^2$

$\Rightarrow 2a^2 = x^2$

$\Rightarrow$ x = $\pm\sqrt{2}a$

when x = $\sqrt{2}a$, y = $\sqrt{2}a$

when x = $-\sqrt{2}a$ not possible as 'x' is length critical point is ($\sqrt{2}a$, $\sqrt{2}a$)

iii. $\frac{dp}{dx} = 2\left(1 + \frac{1}{2\sqrt{4a^2-x^2}}(-2x)\right)$

$\frac{d^2P}{dx^2} = -2\left(\frac{\sqrt{4a^2-x^2}-(x)\left(\frac{-2x}{2\sqrt{4^2-x^2}}\right)}{(4a^2-x^2)}\right)$

= $-2\left(\frac{(4a^2-x^2)+x^2}{(4a^2-x^2)^{3/2}}\right)$

$\Rightarrow \left.\frac{d^2P}{dx^2}\right]_{x=a\sqrt{2}}$ = $-2\left(\frac{4a^2}{(4a^2-2a^2)^{3/2}}\right) = \frac{-2}{(2\sqrt{2})a} < 0$

Perimeter is maximum at a critical point.

OR

From the above results know that x = y = $\sqrt{2}a$

a = radius

Here, x = y = $10\sqrt{2}$

Perimeter = P = 4 $\times$ side = $40\sqrt{2}$ cm