Solution

MATHEMATICS

Class 12 - Mathematics

Section A

1.
(d) A is a square matrix

Explanation:

A square matrix is a null matrix if all its entries are zero.

2.
(b) $a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$

Explanation:

By the definition of expansion of determinant, the required relation is

$a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$

3.
(d) 64

Explanation:

$A=\begin{bmatrix}-2 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -2\end{bmatrix}$

|A| = -2[-4 - 0] - 0 + 0

= -8

Now, $|\text{adj } A| = |A|^{n-1}$ ...(where n is the order of matrix n)

$= (-8)^{3-1}$

$= (-8)^2$

= 64

4.
(c) $\frac{5}{x\log\left(x^5\right)\log\left(\log x^5\right)}$

Explanation:

Let $y = \log[\log(\log x^5)]$

$\frac{dy}{dx}=\frac{1}{\log\left(\log x^5\right)}\frac{dy}{dx}\left[\log\left(\log x^5\right)\right]$ (By Chain Rule)

$=\frac{1}{\log\left(\log x^5\right)}\cdot\frac{1}{\log x^5}\frac{d}{dx}\log x^5$

$=\frac{1}{\log\left(x^5\right)\log\left(\log x^5\right)}\frac{1}{x^5}\frac{d}{dx}\left(x^5\right)$

$=\frac{5}{x\log\left(x^5\right)\log\left(\log x^5\right)}$

5.
(d) $3\sqrt{30}$

Explanation:

Use formula for shortest distance between two skew lines.

6. **(a)** $(2, \frac{3}{2})$

Explanation:

The pairs $\left(2,\frac{3}{2}\right)$ is not feasible. Because the degree of any differential equation cannot be rational type. If so, then we use rationalization and convert it into an integer.

7.

(d) Option (c)

Explanation:

If a LPP admits two optimal solutions it has an infinite solution.

8.

(b) $2\sqrt{10}$

Explanation:

We have,

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$...(i)

$\because |\vec{a}+\vec{b}|$ = $10\sqrt{3}$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$ = 300

$\Rightarrow (7)^2 + (11)^2 + 2\vec{a}\cdot\vec{b}$ = 300

$\Rightarrow 2\vec{a}\cdot\vec{b} = 130$

Now, put the value of $|\vec{a}|$, $|\vec{b}|$ and $2\vec{a}\cdot\vec{b}$

in Eq. (i), we get

$\therefore |\vec{a}-\vec{b}|^2$ = $\sqrt{40} = 2\sqrt{10}$

9.

(b) log |sec x + tan x| + C

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

sin (a+b)=sin a cos b+cos a sin b

$\int$ cot x=,log (sin x)+c

Therefore ,

=$\int$ sec $x\frac{\sec x+\tan x}{\sec x+\tan x}dx$

=$\int \frac{\sec^2 x+\sec x \tan x}{\sec x+\tan x}dx$

Put sec x + tan x = t, sec ^{2} x + (sec x tan x)dx = dt

$\Rightarrow \int \frac{dt}{t}$

=log t + c

= log |sec x + tan x| + c

10.

(d) AB = -BA

Explanation:

If A and B anticommute then AB = -BA

11.

(c) not in the region

Explanation:

Since (0, 0) does not satisfy x + y $\geq$ 1

i.e., 0 + 0 $\neq$ 1

$\Rightarrow$ (0, 0) not lie in feasible region represented by x + y $\geq$ 1.

12. **(a)** $[\vec{a}\vec{b}\vec{c}] = 0$

Explanation:

Given that $\vec{a}, \vec{b}, \vec{c}$ are on the same plane.

$\implies \vec{a}, \vec{b}, \vec{c}$ are coplanar

$\implies \vec{a}.(\vec{b}\times\vec{c}) = 0$

$\Rightarrow [\vec{a}\vec{b}\vec{c}] = 0$

13. **(a)** x - 3 y + 7 = 0

Explanation:

Equation of line is given by $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Consider, $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (-1, 2)$

$\therefore \begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ -1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow$ x(3 - 2) - y(2 + 1) + 1(4 + 3) = 0 [expanding along R_1]

$\Rightarrow$ x - 3y + 7 = 0

14. **(a)** $\frac{1}{8}$

Explanation:

Here, s = {(M M M M), (F F F F), ...}

Clearly, n (s) = 16

$\therefore$ Required proability = P [(M M M M) or (F F F F)]

= P [(M M M M) + (F F F F)]

= $\frac{2}{16} + \frac{2}{16} = \frac{4}{16} = \frac{1}{8}$

15.

(d) sin x - cos y

Explanation:

sin x - cos y

16.

(d) $\sqrt{3}$

Explanation:

$\sqrt{3}$ is the correct answer. Area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \hat{b}|$.

17.

(d) $\frac{-2}{(1+x^2)}$

Explanation:

Given that $y = \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$

$\Rightarrow \sec y = \frac{x^2+1}{x^2-1}$

Since $\tan^2 x = \sec^2 x - 1$, thus

$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1 = \frac{4x^2}{(x^2-1)^2}$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we obtain

$y = \tan^{-1}(-\tan 2\theta)$

Uisng -tan x = tan(-x), we obtain

$y = \tan^{-1}(\tan(-2\theta)) = -2\theta = -2\tan^{-1} x$

Differentiating with respect to x, we obtain

$\frac{dy}{dx} = \frac{-2}{1+x^2}$

18.

(c) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$

Explanation:

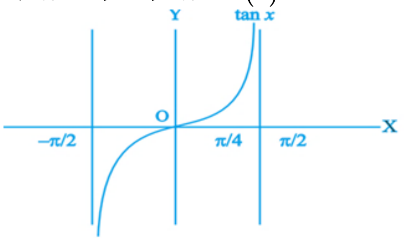
$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{-2}$

19. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
We have, (x - 5)(x - 7)
$\Rightarrow x^2 - 12x + 35$
We know that, $ax^2 + bx + c$ has minimum value $\frac{4ac-b^2}{4a}$.
Here, a = 1, b = -12 and c = 35
$\therefore$ Minimum value of (x - 5)(x - 7) $= \frac{4.1 \cdot 35-(-12)^2}{4.1}$
$= \frac{140-144}{4}$
$= -\frac{4}{4} = -1$

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Assertion: Since, greatest integer [x] gives only integer value.
But f(x) = [x] + x gives all real values and there is no repeated value of f(x) for any value of x.
Hence, f{x) is one-one and onto.

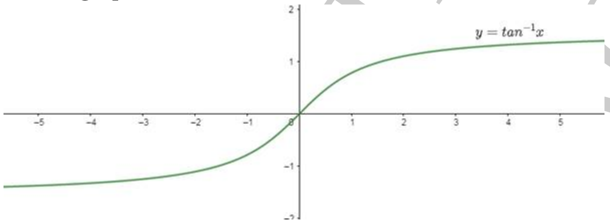
Section B

21. From Fig. we note that tan x is an increasing function in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan\frac{\pi}{4}$. This gives tan 1 > 1
$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$
$\Rightarrow \tan 1 > 1 > \tan^{-1}(1)$



OR

Principal value branch of $\tan^{-1}$ x is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
and its graph is shown below.



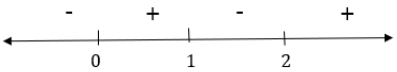
22. Here:
f(x) = a^x
f(x) = a^x log a
Given : F (x) is increasing on R
$\Rightarrow f'(x) > 0$
$\Rightarrow a^x \log a > 0$
The Logarithmic function is defined for positive values, So
$\Rightarrow a > 0$
$\Rightarrow a^x > 0$
We know
$a^x \log a > 0$
It can be possible when $a^x > 0$ and $\log a > 0$
$\Rightarrow a > 1$
So, f(x) is increasing when a > 1

23. Given function is f(x) = $x^4 - 4x^3 + 4x^2 + 15$

$f'(x) = 4x^3 - 12x^2 + 8x$

$= 4x(x^2 - 3x + 2)$

$= 4x(x - 1)(x - 2)$



Function f(x) is decreasing for x$\in[-\infty, 0] \cup [1, 2]$ and increasing in x$\in(0, 1) \cup (2, \infty)$.

OR

Let the radius of circle = r and area of the circle, $A = \pi r^2$

$\therefore \frac{d}{dt}(A) = \frac{d}{dt}\pi r^2$

$\Rightarrow \frac{d}{dt}(A) = 2\pi r. \frac{dr}{dt}$ ...(i)

Since, the area of a circle increases at a uniform rate, then

$\frac{d}{dt}(A) = k$ ...(ii) Where, k is a constant.

From Eqs. (i) and (ii),

$2\pi r. \frac{dr}{dt} = k$

$\Rightarrow \frac{dr}{dt} = \frac{k}{2\pi r} = \frac{k}{2\pi}. \left(\frac{1}{r}\right)$ ...(iii)

Let the perimeter, $P = 2\pi r$

$\therefore \frac{dP}{dt} = \frac{d}{dt}.2\pi r \Rightarrow \frac{dP}{dt} = 2\pi r. \frac{dr}{dt}$

$= 2\pi r. \frac{k}{2\pi}. \frac{1}{r} = \frac{k}{r}$ [Using Eq. (iii)]

$= \frac{dP}{dt} \alpha \frac{1}{r}$

Hence proved

24. Let $I = \int \frac{1}{x(\log x)^m} dx$

$= \int \frac{\frac{1}{x} dx}{(\log x)^m} dx \ldots$(i)

Putting $\log x = t$

$\Rightarrow \frac{1}{x} = \frac{dt}{dx}$

$\Rightarrow \frac{dx}{x} = dt$

$\therefore$ From eq. (i), $I = \int \frac{dt}{t^m} = \int t^{-m} dt$

$= \frac{t^{-m+1}}{-m+1} + c$

$= \frac{(\log x)^{1-m}}{1-m} + c$

25. Here

f(x) = x + cos x + ax +b

$\Rightarrow$ f(x) = 1 - sin x + a

For F (x) to be increasing we must have f '(x) > 0

1 - sinx + a > 0

$\Rightarrow$ sinx < 1+a

We know that the maximum value of sin x is 1

$\Rightarrow$ 1 + a >1

$\Rightarrow$ a > 0

$\Rightarrow$ a $\in (0, \infty)$

Section C

26. Let the given integral be, $I = \int \frac{\sec^2 x}{(\tan^2 x + 4\tan x)} dx$

Putting t = tan x

dt = $\sec^2 x dx$

$I = \int \frac{dt}{(t^3 + 4t)} = \int \frac{dt}{t(t^2+4)}$

Now putting, $\frac{1}{t(t^2+4)} = \frac{A}{t} + \frac{Bt+C}{t^2+4}$ .... (1)

$A(t^2 + 4) + (Bt + C)t = 1$

Putting t = 0,

$A(0 + 4) \times B(0) = 1$

$A = \frac{1}{4}$

By equating the coefficients of t^2 and constant here, we have,

A + B = 0

$\frac{1}{4}+B=0$

$B=-\frac{1}{4}, C=0$

Now From equation (1), we get,

$\int \frac{1}{t\left(t^2+4\right)} dt=\frac{1}{4} \int \frac{dt}{t}-\frac{1}{4} \int \frac{t}{t^2+4} dt$

$=\frac{1}{4} \log t-\frac{1}{4} \times \frac{1}{2} \log \left(t^2+4\right)+c$

$=\frac{1}{4} \log \tan x-\frac{1}{8} \log \left(\tan ^2 x+4\right)+c$

27. Let E_1, E_2 and E_3 be the events of drawing a bolt produced by machine A, B and C respectively. Therefore,we have,

$P(E_1) = \frac{25}{100}=\frac{1}{4}$, $P(E_2) = \frac{35}{100}=\frac{7}{20}$, and $P(E_3) = \frac{40}{100}=\frac{2}{5}$

Let E be the event of drawing a defective bolt. Therefore,

$P(\frac{E}{E_1})$ = probability of drawing a defective bolt, given that it is produced by the machine A = $\frac{5}{100}=\frac{1}{20}$

$P(\frac{E}{E_2})$ = probability of drawing a defective bolt, given that it is produced by the machine B = $\frac{4}{100}=\frac{1}{25}$

$P(\frac{E}{E_3})$ = probability of drawing a defective bolt, given that it is produced by the machine C = $\frac{2}{100}=\frac{1}{50}$

Therefore, we have,

Probability that the bolt drawn is manufactured by C, given that it is defective

$= P(\frac{E_3}{E})$

$= \frac{P\left(\frac{E}{E_3}\right) \cdot P(E_3)}{P\left(\frac{E}{E_1}\right) \cdot P(E_1)+P\left(\frac{E}{E_2}\right) \cdot P(E_2)+P\left(\frac{E}{E_3}\right) \cdot P(E_3)}$ [by Bayes's theorem]

$\frac{\left(\frac{1}{50} \times \frac{2}{5}\right)}{\left(\frac{1}{20} \times \frac{1}{4}\right)+\left(\frac{1}{25} \times \frac{7}{20}\right)+\left(\frac{1}{50} \times \frac{2}{5}\right)} = \left(\frac{1}{125} \times \frac{2000}{69}\right)=\frac{16}{69}$

Hence, the required probability is $\frac{16}{69}$.

28. Let the given integral be, $I=\int_0^{\pi / 2} \frac{\sin ^2 x}{\sin x+\cos x} dx$. ...(i)

Then,

$\Rightarrow \quad I=\int_0^{\pi / 2} \frac{\sin ^2\left(\frac{\pi}{2}-x\right)}{\sin \left(\frac{\pi}{2}-x\right)+\cos \left(\frac{\pi}{2}-x\right)} dx \quad \left[\text{Using : } \int_0^a f(x) dx=\int_0^a f(a-x) dx\right]$

$\Rightarrow \quad I=\int_0^{\pi / 2} \frac{\cos ^2 x}{\cos x+\sin x} dx$ ...(ii)

Adding (i) and (ii), we get

$2 I=\int_0^{\pi / 2} \frac{\sin ^2 x}{\sin x+\cos x}+\frac{\cos ^2 x}{\sin x+\cos x} dx=\int_0^{\pi / 2} \frac{1}{\sin x+\cos x} dx$

$\Rightarrow \quad 2 I=\int_0^{\pi / 2} \frac{1}{\frac{2 \tan x / 2}{1+\tan ^2 x / 2}+\frac{1-\tan ^2 x / 2}{1+\tan ^2 x / 2}} dx$

$\Rightarrow \quad 2 I=\int_0^{\pi / 2} \frac{1+\tan ^2 \frac{x}{2}}{2 \tan \frac{x}{2}+1-\tan ^2 \frac{x}{2}} dx=\int_0^{\pi / 2} \frac{\sec ^2 \frac{x}{2}}{2 \tan \frac{x}{2}+1-\tan ^2 \frac{x}{2}} dx$

Let $\tan \frac{x}{2}=t$. Then, $d\left(\tan \frac{x}{2}\right)=dt \Rightarrow \left(\sec ^2 \frac{x}{2}\right) \frac{1}{2} dx=dt \Rightarrow \sec ^2 \frac{x}{2} dx=2 dt$

Also, $x=0 \Rightarrow t=\tan 0=0$ and $x=\frac{\pi}{2} \Rightarrow t=\tan \frac{\pi}{4}=1$

$\therefore \quad 2 I=\int_0^1 \frac{2 dt}{2 t+1-t^2}=2 \int_0^1 \frac{1}{(\sqrt{2})^2-(t-1)^2} dt$

$\Rightarrow \quad 2 I=2 \times \frac{1}{2 \sqrt{2}}\left[\log \left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right|\right]_0^1$

$\Rightarrow \quad 2 I=\frac{1}{\sqrt{2}}\left\{\log \left(\frac{\sqrt{2}}{\sqrt{2}}\right)-\log \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right\}=\frac{1}{\sqrt{2}}\left\{0-\log \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right\}$

$\Rightarrow \quad 2 I=-\frac{1}{\sqrt{2}} \log \left\{\frac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}\right\}=-\frac{1}{\sqrt{2}} \log (\sqrt{2}-1)^2=-\frac{2}{\sqrt{2}} \log (\sqrt{2}-1)$

$\Rightarrow \quad I=-\frac{1}{\sqrt{2}} \log (\sqrt{2}-1)$

OR

We have from LHS,

$I=\int_0^{2 a} f(x) dx=\int_0^a f(x) dx+\int_a^{2 a} f(x) dx$ ..(i)

Let x = 2a - t, then dx = -dt

$x=a \Rightarrow t=a,$ and $x=2 a \Rightarrow t=0$

$\therefore \int_0^{2 a} f(x) dx=-\int_a^0 f(2 a-t) dt$

$\Rightarrow \int_0^{2a} f(x)dx = \int_0^a f(2a-t)dt$
$\Rightarrow \int_0^{2a} f(x)dx = \int_0^a f(x)dx$
Substituting $\int_0^{2a} f(x)dx = \int_0^a f(x)dx$ in (i)
We get,
$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(x)dx$
$\Rightarrow \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$
Hence proved

29. We have, $\frac{dy}{dx}$ = cos (x + y) + sin (x + y)
Using the hint given and substituting x + y = z
$\Rightarrow \frac{d(z-x)}{dx}$ = cos z + sin z
Differentiating z – x with respect to x
$\Rightarrow \frac{dz}{dx}$ - 1 = cos z + sin z
$\Rightarrow \frac{dz}{dx}$ = 1 + cos z + sin z
$\Rightarrow \frac{dz}{1+\cos z+\sin z}$ = dx
Integrating both sides, we have
$\Rightarrow \int \frac{dz}{1+\cos z+\sin z} = \int dx$
We know that cos 2z = 2 $\cos^2$z – 1 and sin 2z = 2 sin z cos z
$\Rightarrow \int \frac{dz}{1+2\cos^2\frac{z}{2}-1+2\sin\frac{z}{2}\cos\frac{z}{2}} = x$
$\Rightarrow \int \frac{dz}{2\cos^2\frac{z}{2}+2\sin\frac{z}{2}\cos\frac{z}{2}} = x$
$\Rightarrow \int \frac{dz}{2\cos\frac{z}{2}\left(\cos\frac{z}{2}+\sin\frac{z}{2}\right)} = x$
$\Rightarrow \int \frac{dz}{2\cos^2\frac{z}{2}\left(1+\frac{\sin\frac{z}{2}}{\cos\frac{z}{2}}\right)} = x$
$\Rightarrow \int \frac{\sec^2\frac{2}{2}dz}{2\left(1+\tan\frac{z}{2}\right)} = x$
Let 1 + $\tan\frac{z}{2}$ = t
Differentiating with respect to z, we get
$\frac{dt}{dz} = \frac{\sec^2\frac{z}{2}}{2}$
Hence $\frac{\sec^2\frac{z}{2}dz}{2} = dt$
$\Rightarrow \int \frac{\mathrm{dt}}{\mathrm{t}} = \mathrm{x}$
⇒ log t + c = x
Resubstituting t
$\Rightarrow \log\left(1+\tan\frac{z}{2}\right) + c = x$
Resubstitute z
$\Rightarrow \log\left(1+\tan\frac{\mathrm{x+y}}{2}\right) + \mathrm{c} = \mathrm{x}$

OR

It is given that $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$
$\Rightarrow \frac{dy}{2e^{-y}-1} = \frac{dx}{x+1}$
$\Rightarrow \frac{e^y dy}{2-e^y} = \frac{dx}{x+1}$
On integrating both sides, we get,
$\int \frac{e^y dy}{2-e^y} = \log|x+1| + \log C$ .....(i)
Let $2 - e^y = t$
$\therefore \frac{d}{dy}(2-e^y) = \frac{dt}{dy}$
$\Rightarrow -\mathrm{e}^{\mathrm{y}} = \frac{\mathrm{dt}}{\mathrm{dy}}$
⇒ e^ydy = -dt
Substituting value in equation (i), we get,
$\int \frac{-\mathrm{dt}}{\mathrm{t}} = \log|\mathrm{x}+1| + \log \mathrm{c}$

$\Rightarrow$ -log|t| = log|C(x+1)|

$\Rightarrow$ -log|2 - e^y| = log|C(x + 1)|

$\Rightarrow \frac{1}{2-e^y} = C(x+1)$

$\Rightarrow 2 - e^y = \frac{1}{c(x+1)}$ ......(ii)

Now, at x = 0 and y = 0, equation (ii) becomes,

$\Rightarrow 2 - 1 = \frac{1}{C}$

$\Rightarrow$ C = 1

Now, substituting the value of C in equation (ii), we get,

$\Rightarrow 2 - e^y = \frac{1}{(x+1)}$

$\Rightarrow e^y = 2 - \frac{1}{(x+1)}$

$\Rightarrow e^y = \frac{2x+2-1}{(x+1)}$

$\Rightarrow e^y = \frac{2x+1}{(x+1)}$

$\Rightarrow y = \log\left|\frac{2x+1}{x+1}\right|, (x \neq -1)$

Therefore, the required particular solution of the given differential equation is

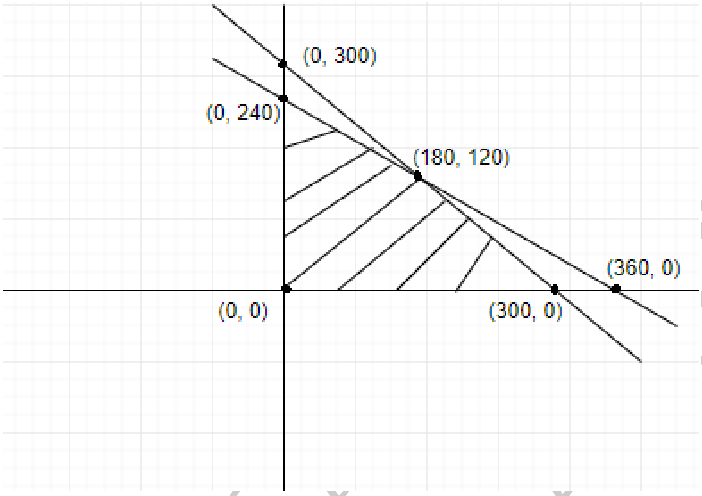
$y = \log\left|\frac{2x+1}{x+1}\right|, (x \neq -1)$

30. The problem is:

Maximise Z = 510x + 675y

subject to the constraints : $\left.\begin{array}{l} x + y \leq 300 \\ 2x + 3y \leq 720 \\ x \geq 0, y \geq 0 \end{array}\right\}$

The shaded region in the Figure is the feasible region ,



2x + 3y = 720

x = 0, y = 240

y = 0, x = 360

Since the feasible region is bounded, therefore maximum of Z must occur at the corner point of OBC.

Corner Point	Value of Z
O(0, 0)	510 (0) + 675 (0) = 0
A(300, 0)	510 (300) + 675 (0) = 153000
B(180, 120)	510 (180) + 675 (120) = **172800** **(Maximum)**
C(0, 240)	510(0) + 675(240) = 162000

Thus, maximum Z is 172800 at the point (180, 120), i.e., the company should produce 180 black and white television sets and 120 coloured television sets to get maximum profit.

OR

We have, maximise Z = 11x + 7y ...... (i)

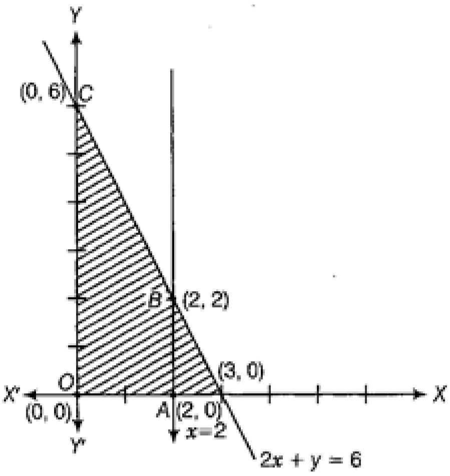
Subject to the constraints

$2x + y \leqslant 6$ ..... (ii)

$x \leqslant 2$ .... (iii)

$x \geqslant 0, y \geqslant 0$ ..... (iv)

We see that, the feasible region as shaded determined by the system of constraint (ii) to (iv) is OABC and is bounded. So, now we shall use corner point method to determine the maximum value of Z.



Corner Points	Corresponding value of Z
(0, 0)	0
(2, 0)	22
(2, 2)	36
(0, 6)	42 (Maximum)

Hence, the maximum value of Z is 42 at (0, 6).

31. $y=\cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$ $\left[\because \sqrt{1\pm\sin x}=\sqrt{\left(\cos\frac{x}{2}\pm\sin\frac{x}{2}\right)^2}\right]$

$=\cot^{-1}\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}$

$=\cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$

$=\cot^{-1}\left(\cot\frac{x}{2}\right)$

$=\frac{x}{2}$

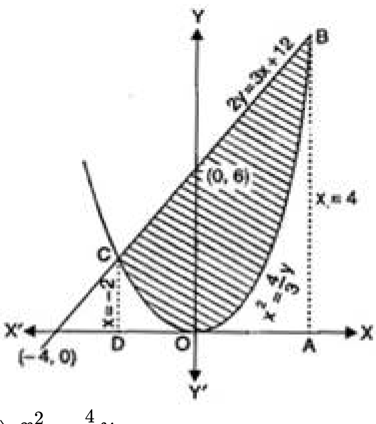
$y=\frac{x}{2}$

$\frac{dy}{dx}=\frac{1}{2}$

Section D

32. Equation of the parabola is

$4y = 3x^2$ …(i)



$\Rightarrow x^2=\frac{4}{3}y$

Equation of the line is 2y = 3x + 12 …(ii)

$\Rightarrow y=\frac{3x+12}{2}=\frac{3x}{2}+6$

In the graph, points of intersection are B (4, 12) and C (-2, 3).

Now, Area ABCD $= \left|\int_{-2}^{4}\left(\frac{3}{2}x+6\right)dx\right|$

$= \left[\frac{3}{4}x^2+6x\right]_{-2}^{4}$

= (12 + 24) - (3 - 12)

= 45 sq units

Again, Area CDO + Area OAB $= \left|\int_{-2}^{4}\left(\frac{3}{4}x^2\right)dx\right|$

$= \left[\frac{3}{4}\cdot\frac{x^3}{3}\right]_{-2}^{4}$

$= \frac{1}{4}[64-(-8)] = 18$ sq. units

∴ Required area = Area ABCD - (Area CDO + Area OAB)

= 45 - 18 = 27 sq. units

33. Let (a_1, b_1) and $(a_2, b_2) \in A \times B$

1. (i) $f(a_1, b_1) = f(a_2, b_2)$

$b_1 = b_2$ and $a_1 = a_2$

$(a_1, b_1) = (a_2, b_2)$

Then $f(a_1, b_1) = f(a_2, b_2)$

$(a_1, b_1) = (a_2, b_2)$ for all

$(a_1, b_1) = (a_2, b_2) \in A \times B$

(ii) f is injective,

Let (b, a) be an arbitrary

Element of $B \times A$. then $b \in B$ and $a \in A$

$\Rightarrow (a, b)) \in (A \times B)$

Thus for all $(b, a) \in B \times A$ their exists $(a, b)) \in (A \times B)$

Hence that

f(a, b) = (b, a)

So $f: A \times B \to B \times A$

f is an onto function.

Hence bijective

OR

f is one-one: For any $x, y \in R$, we have f(x) : f(y)

$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$

⇒ xy + x = xy + y

⇒ x = y

Therefore, f is one-one function.

If f is one-one, let y = R – {1}, then f(x) = y

$\Rightarrow \frac{x}{x+1} = y$

$\Rightarrow x = \frac{y}{1-y}$

It is clear that $x \in R$ for all y = R – {1}, also $x \neq$=-1

Because x = -1

$\Rightarrow \frac{y}{1-y} = -1$

⇒ y = -1 + y which is not possible.

Thus for each R – {1} there exists $x = \frac{y}{1-y} \in$ R – {1} such that

$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$

Therefore f is onto function.

34. Given: Matrix $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$

$\therefore A^{-1} = \frac{1}{|A|} adj. A = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

Matrix $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$\therefore |B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = 54 - 56 = -2 \neq 0$

$\therefore B^{-1} = \frac{1}{|B|} adj. B = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

Now $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

$\therefore |AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix}$ = 67(61) - 87(47) = 4087 - 4089 $= -2 \neq 0$

Now L.H.S. = $(AB)^{-1} = \frac{1}{|AB|} adj. (AB) = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ….(i)

R.H.S. = $B^{-1}A^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ….(ii)

$\therefore$ From eq. (i) and (ii), we get

L.H.S. = R.H.S.

$\Rightarrow (AB)^{-1} = B^{-1}A^{-1}$

35. Given

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$

Here, we have

$\vec{a}_1 = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

$\vec{b}_1 = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{b}_2 = -2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

Thus,

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$

$= \hat{i}(24-18) - \hat{j}(16+12) + \hat{k}(6-6)$

$\vec{b}_1 \times \vec{b}_2 = 6\hat{i} - 28\hat{j} + 0\hat{k}$

$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{6^2 + (-28)^2 + 0^2}$

$= \sqrt{36 + 784 + 9}$

$= \sqrt{820}$

$\vec{a}_2 - \vec{a}_1 = (3-1)\hat{i} + (3-2)\hat{\jmath} + (-5+4)\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

Now, we have

$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (6\hat{\imath} - 28\hat{\jmath} + 0\hat{k}) \cdot (2\hat{\imath} + \hat{\jmath} - \hat{k})$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 - 28 + 0

= -16

Thus,the shortest distance between the given lines is

$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$\Rightarrow d = \left| \frac{-16}{\sqrt{820}} \right|$

$\therefore d = \frac{16}{\sqrt{820}}$ units

OR

$\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}$
$\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$
$\vec{b}_1 = 7\hat{i} - 6\hat{j} + 1\hat{k}$
$\vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$
$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 6\hat{j} + 8\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

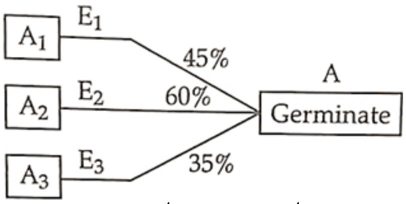
$= -4\hat{i} - 6\hat{j} - 8\hat{k}$
$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (4\hat{i} + 6\hat{j} + 8\hat{k})(-4\hat{i} - 6\hat{j} - 8\hat{k}) = -16 - 36 - 64 = -116$
$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-4)^2 + (-6)^2 + (-8)^2}$
$= \sqrt{116}$
$= 2\sqrt{29}$

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(b_1 \times b_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

$= \left|\frac{-116}{2\sqrt{29}}\right|$
$= 2\sqrt{29}$

Section E

36. i.



Here, P(E$_1$) = $\frac{4}{10}$, P(E$_2$) = $\frac{4}{10}$, P(E$_3$) = $\frac{2}{10}$

$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$

$\therefore$ P(A) = P(E$_1$) $\cdot$ P$\left(\frac{A}{E_1}\right)$ + P(E$_2$) $\cdot$ $P\left(\frac{A}{E_2}\right)$ + P(E$_3$) $\cdot$ $P\left(\frac{A}{E_3}\right)$

= $\frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

= $\frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$

= $\frac{490}{1000}$ = 4.9

ii. Required probability = $P\left(\frac{E_2}{A}\right)$

= $\frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(A)}$

= $\frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$

= $\frac{240}{490}$ = $\frac{24}{49}$

iii. Let,

E$_1$ = Event for getting an even number on die and

E$_2$ = Event that a spade card is selected

$\therefore$ P(E$_1$) = $\frac{3}{6}$

= $\frac{1}{2}$

and P(E$_2$) = $\frac{13}{52} = \frac{1}{4}$

Then, P(E$_1 \cap$ E$_2$) = P(E$_1$) $\cdot$ P(E$_2$)

= $\frac{1}{2}$, $\frac{1}{4}$ = $\frac{1}{8}$

OR

P(A) + P(B) - P(A and B) = P(A)

$\Rightarrow$ P(A) + P(B) - P(A $\cap$ B) = P(A)

$\Rightarrow$ P(B) - P(A $\cap$ B) = 0

$\Rightarrow$ P(A $\cap$ B) = P(B)

$\therefore$ P(A|B) = $\frac{P(A\cap B)}{P(B)}$

= $\frac{P(B)}{P(B)}$

= 1

37. i. Clearly, G be the centroid of $\triangle$BCD, therefore coordinates of G are

$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right)$ = (3, 2, 3)

ii. Since, A $\equiv$ (0, 1, 2) and G = (3, 2, 3)

$\therefore \overrightarrow{AG}$ = (3 - 0)$\hat{i}$ + (2 - 1)$\hat{j}$ + (3 - 2)$\hat{k}$ = 3$\hat{i}$ + $\hat{j}$ + $\hat{k}$

$\Rightarrow |\overrightarrow{AG}|^2$ = $3^2 + 1^2 + 1^2$ = 9 + 1 + 1 = 11

$\Rightarrow |\overrightarrow{AG}| = \sqrt{11}$

iii. Clearly, area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Here, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-0 & 0-1 & 1-2 \\ 4-0 & 3-1 & 6-2 \end{vmatrix}$

= $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix}$

= $\hat{i}$(-4 + 2) - $\hat{j}$(12 + 4) + $\hat{k}$(6 + 4) = -2$\hat{i}$ - 16$\hat{j}$ + 10$\hat{k}$

$\therefore |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-2)^2 + (-16)^2 + 10^2}$

= $\sqrt{4 + 256 + 100} = \sqrt{360} = 6\sqrt{10}$

Hence, area of $\triangle$ABC = $\frac{1}{2} \times 6\sqrt{10} = 3\sqrt{10}$ sq. units

OR

The length of the perpendicular from the vertex D on the opposite face

= |Projection of $\overrightarrow{AD}$ on $\overrightarrow{AB} \times \overrightarrow{AC}$|

= $\left|\frac{(2\hat{i}+2\hat{j})\cdot(-2\hat{i}-16\hat{j}+10\hat{k})}{\sqrt{(-2)^2+(-16)^2+10^2}}\right|$

= $\left|\frac{-4-32}{\sqrt{360}}\right| = \frac{36}{6\sqrt{10}} = \frac{6}{\sqrt{10}}$ units

38. i. f(x) = -0.1x^2 + mx + 98.6, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12).

ii. f(x) = -0.2x + m

At Critical point

0 = -0.2 $\times$ 6 + m

m = 1.2

iii. f(x) = -0.1x^2 + 1.2x + 98.6

f'(x) = -0.2x + 1.2 = -0.2(x - 6)

In the Interval	f'(x)	Conclusion
(0, 6)	+Ve	f is strictly increasing in [0, 6]
(6, 12)	-Ve	f is strictly decreasing in [6, 12]

OR

f(x) = -0.1x^2 + 1.2x + 98.6,

f'(x) = -0.2x + 1.2, f'(6) = 0,

f''(x) = -0.2

f''(6) = -0.2 < 0

Hence, by second derivative test 6 is a point of local maximum. The local maximum value = f(6) = -0.1 $\times$ 6^2 + 1.2 $\times$ 6 + 98.6 = 102.2

We have f(0) = 98.6, f(6) = 102.2, f(12) = 98.6

6 is the point of absolute maximum and the absolute maximum value of the function = 102.2.

0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.