Solution

MATHEMATICS

Class 10 - Mathematics

Section A

1.

(b) (18, 25)

Explanation:

The numbers that do not share any common factor other than 1 are called co-primes.
factors of 18 are: 1, 2, 3, 6, 9 and 18
factors of 25 are: 1, 5, 25
The two numbers do not share any common factor other than 1.
They are co-primes to each other.

2.

(b) 0

Explanation:

There is no zero as the graph does not intersect the x-axis at any point.

3.

(b) one or many solutions

Explanation:

A system of linear equations is said to be consistent if it has at least one solution or can have many solutions. If a consistent system has an infinite number of solutions, it is dependent. When you graph the equations, both equations represent the same line. If a system has no solution, it is said to be inconsistent. The graphs of the lines do not intersect, so the graphs are parallel and there is no solution.

4.

(c) b^2 - 4ac

Explanation:

Discriminant of the quadratic equation $ax^2 + bx + c = 0$ is $D = b^2 - 4ac$

5.

(d) n (n + 2)

Explanation:

$a_n = 2n + 1$
a or $a_1 = 2 \times 1 + 2 = 2 + 1 = 3$
$a_2 = 2 \times 2 + 1 = 4 + 1 = 5$
$\therefore d = a_2 - a_1 = 5 - 3 = 2$
$\therefore \mathrm{S}_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{n}{2}[2 \times 3 + (n-1) \times 2]$
$= \frac{n}{2}[6 + 2n - 2] = \frac{n}{2}[2n + 4]$
= n(n + 2)

6.

(b) $\sqrt{2a^2 + 2b^2}$

Explanation:

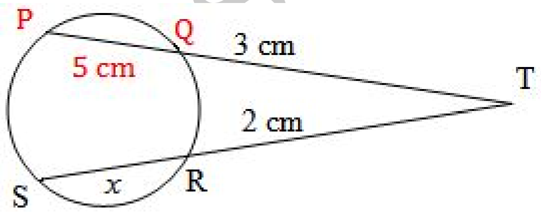
distance between the point. (0, 0) and (a - b, a + b) is
= $\sqrt{(a-b-0)^2 + (a+b-0)^2}$
= $\sqrt{(a-b)^2 + (a+b)^2}$

= $\sqrt{a^2+b^2-2ab+a^2+b^2+2ab}$
= $\sqrt{2\left(a^2+b^2\right)}$ = $\sqrt{2a^2+2b^2}$ units.

7. **(a)** (2, 3)
Explanation:
We are given three vertices (0, 0), (2, 0) and (0, 3) of a rectangle.
We have to find the coordinates of the fourth vertex.
By plotting the given vertices on an XY plane, C (0, 3) are the consecutive vertices.
Consider D to represent the fourth vertex.
Since, AB = 2 units and BC = 3 units.
Thus, point D is at a horizontal distance of 3 units and a vertical distance of 2 units from the origin.
Thus, the coordinates of the fourth vertex of the rectangle are (2, 3).

8.
(c) 1.75 cm
Explanation:
$\angle \mathrm{BAD}=180^{\circ}-(\angle \mathrm{EAB}+\angle \mathrm{ADC})$ = {180^o - 110^o - 35^o = 35^o
Since AD bisect $\angle$A, then
$\frac{\mathrm{AB}}{\mathrm{AC}}=\frac{\mathrm{BD}}{\mathrm{CD}}$ [Internal bisector of an angle divides opposite sides in the ratio of the sides containing the angle]
$\Rightarrow \frac{5}{7}=\frac{3-\mathrm{CD}}{\mathrm{CD}}$
$\Rightarrow$ 5CD = 21 - 7CD
$\Rightarrow$ 12CD = 21
$\Rightarrow$ CD = 1.75 cm

9. **(a)** 10 cm
Explanation:
Let point of contact of tangent AB be P, point of contact of tangent BC be Q and point of contact of tangent AC be R.
Since, Tangents from an external points are equal.
$\therefore$ BP = BQ = 3cm
PA = AR = 4 cm
$\Rightarrow$ CR = 11 - 4 = 7 cm
CR = QC = 7 cm
$\therefore$ BC = CQ + BQ = 7 + 3 = 10 cm

10.
(d) 10 cm
Explanation:


We know that if two chords intersect each other at T outside the circle, then TP $\times$ TQ = TS $\times$ TR Let SR = $x$cm
$\Rightarrow$(5 + 3) $\times$ 3 = (x + 2) $\times$ 2
$\Rightarrow$x + 2 = 12
$\Rightarrow$$x=10$ cm x = 10 cm
$\therefore$ SR = 10 cm

11.
(b) 16
Explanation:
x = 4 sinθ
y = 4 cosθ

now,
$x^2 + y^2 = (4\sin\theta)^2 + (4\cos\theta)^2$
$= 16\sin^2\theta + 16\cos^2\theta$
$= 16\left(\sin^2\theta + \cos^2\theta\right)$
= 16(1) = 6

12.

(c) Only (i)

Explanation:

i. We know, 3sinθ - cosθ)^4 = 3((sinθ - cosθ)^2)^2
= 3(1^2 + 4 sin$^2\theta$ cos^2 θ - 4 sinθcosθ) ...(i)
and, 6(sinθ + cosθ)^2 = 6 + 12 sinθcosθ ...(ii)
Also, 4(sin$^6\theta$ + cos$^6\theta$) = 4((sin$^2\theta$)^3 + (cos$^2\theta$)^3)
= 4(1) - 12sin$^2\theta$cos$^2\theta$ ...(iii)
Adding (i), (ii) and (iii), we get
3 + 12sin$^2\theta$cos$^2\theta$ - 12sinθcosθ + 6 + 12sinθcosθ + 4 - 12sin$^2\theta$cos$^2\theta$
= 3 + 6 + 4 = 13, which is independent of θ.

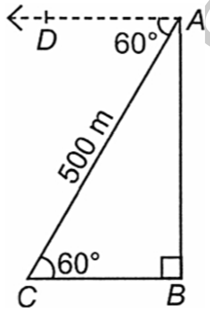
ii. We have,
cosecθ - sinθ = $\frac{1}{\sin\theta}$ - sinθ = $\frac{1-\sin^2\theta}{\sin\theta}$
$\Rightarrow$ a^3 = $\frac{\cos^2\theta}{\sin\theta}$ $\Rightarrow$ a^2 = $\left(\frac{\cos^2\theta}{\sin\theta}\right)^{\frac{2}{3}}$
Similarly, secθ - cosθ = $\frac{1}{\cos\theta}$ - cosθ = $\frac{1-\cos^2\theta}{\cos\theta}$
$\Rightarrow$ b^3 = $\frac{\sin^2\theta}{\cos\theta}$ $\Rightarrow$ b^2 = $\left(\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}}$
$\therefore$ a^{2}b^2(a^2 + b^2) = $\left(\frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} \left(\left(\frac{\cos^2\theta}{\sin\theta}\right)^{\frac{2}{3}} + \left(\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}}\right)$
= (sinθ·cosθ)$^{\frac{2}{3}}$ $\left(\frac{\cos^2\theta+\sin^2\theta}{(\sin\theta\cdot\cos\theta)^{\frac{2}{3}}}\right)$ = 1

13.

(b) 250 m

Explanation:

Let AB be the hill and C be the point on the horizontal.



$\angle$DAC = $\angle$ACB = 60^o (Alternate angles)
In $\triangle$ABC, cos60^o = $\frac{BC}{AC}$
$\Rightarrow$ $\frac{1}{2}$ = $\frac{BC}{500}$
$\Rightarrow$ 2BC = 500
$\Rightarrow$ BC = 250 m

14.

(c) 40882.8 m^2

Explanation:

The area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$
$= \frac{123^\circ}{360^\circ} \times \frac{22}{7} \times 138^2$
= 20441.4 m^2
Area covered by the man of the walking track in a day = 20441.4 + 20441.4
= 40882.8 m^2

15. **(a)** 3π cm^2
Explanation:
In quadrilateral ABCD
$\angle$A + $\angle$B + $\angle$C + $\angle$D = 360^o
$\angle$A + 90^o + 90^o + 60^o = 360^o $\Rightarrow$ $\angle$A = 120^o
Now, area of shaded region = $\frac{120^\circ}{360^\circ} \times \pi \times 3^2$
= $\frac{1}{3} \times \pi \times 3 \times 3 = 3\pi$ cm^2

16. **(c)** $\frac{1}{26}$
Explanation:
Probability = $\frac{Number\ of\ favorable\ outcomes}{Total\ number\ of\ possible\ outcomes}$
Number of favorable outcomes = 2 (red kings)
Total number of possible outcomes = 52 (total cards in the deck)
Probability = $\frac{2}{52}$ = $\frac{1}{26}$
Therefore, the probability of drawing a red king is $\frac{1}{26}$.

17. **(d)** $\frac{17}{90}$
Explanation:
a and b are two number to be selected from the integers = 1 to 10 without replacement of a and b
i.e., 1 to 10 = 10
and 2 to 10 = 9
No. of ways = 10 $\times$ 9 = 90
Probability of $\frac{a}{b}$ where it is an integer
$\therefore$ Possible event will be
= $\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{6}{1}, \frac{7}{1}, \frac{8}{1}, \frac{9}{1}, \frac{10}{1}, \frac{4}{2}, \frac{6}{2}, \frac{8}{2}, \frac{10}{2}, \frac{6}{3}, \frac{9}{3}, \frac{8}{4}, \frac{10}{5}$ = 17
$P(E) = \frac{m}{n} = \frac{17}{90}$

18. **(a)** 11.2
Explanation:
The first 10 composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18
$\therefore$ Mean = $\frac{\text{Sum of first 10 composite numbers}}{10}$
= $\frac{4+6+8+9+10+12+14+15+16+18}{10}$
= $\frac{112}{10}$
= 11.2

19. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Both A and R are true and R is the correct explanation of A.

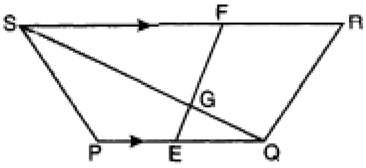
Section B

21. In order to get the result we have to find the HCF of 2002 and 2618. There prime factors are,

$2002 = 2 \times 7 \times 11 \times 13$ and

$2618 = 2 \times 7 \times 11 \times 17$

Hence HCF = $2 \times 7 \times 11 = 154$

22. 

In $\triangle$GEQ and $\triangle$GFS

$\angle$EGQ $\cong$ $\angle$FGS ($\because$ Vertically opposite angles)

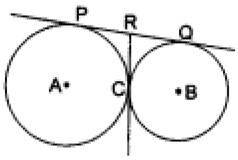
$\angle$EQG $\cong$ $\angle$FSG ($\because$ Alternate angles)

$\therefore$ $\triangle$GEQ $\sim$ $\triangle$GFS ($\because$ AA similarity)

or, $\frac{EQ}{FS} = \frac{GQ}{GS}$

or, $EQ \times GS = GQ \times FS$

Hence proved.

23. 

In the given figure, PR and CR are both tangents drawn to the same circle from an external point R.

$\therefore$ PR = CR. ...(i)

Also, QR and CR are both tangents drawn to the same circle (second circle) from an external point R

QR = CR ... (ii)

From (i) and (ii), we get

PR = QR [each equal to CR].

R is the midpoint of PQ,

i.e., the common tangent to the circles at C, bisects the common tangent at P and Q.

24. $2cot^2 A - 1 = 2(cosec^2 A - 1) - 1 \left(\because \cot^2\theta = -1 + \operatorname{cosec}^2\theta\right)$

$= 2cosec^2 A - 2 - 1$

$= \frac{2}{\sin^2 A} - 3 \ (\because cosec\theta = \frac{1}{sin\theta})$

$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3$

$2cot^2 A - 1 = \frac{8}{3} - 3 = \frac{8-9}{3} = \frac{-1}{3}$

OR

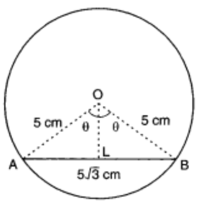
We have

LHS = $\frac{\tan A + \sin A}{\tan A - \sin A}$

$= \frac{\frac{\sin A}{\cos A} + \sin A}{\frac{\sin A}{\cos A} - \sin A} = \frac{\sin A\left(\frac{1}{\cos A} + 1\right)}{\sin A\left(\frac{1}{\cos A} - 1\right)}$

$= \frac{\left(\frac{1}{\cos A} + 1\right)}{\left(\frac{1}{\cos A} - 1\right)} = \frac{\sec A + 1}{\sec A - 1} = \text{RHS}$

$\therefore$ LHS = RHS

25. 

It is given that AB = $5\sqrt{3}$ cm.

$\Rightarrow$ AL = BL = $\frac{5\sqrt{3}}{2}$ cm

Let $\angle AOB = 2\theta$. Then, $\angle$AOL = $\angle$BOL = θ

In $\triangle$OLA, we have

$\sin\theta = \frac{AL}{OA} = \frac{\frac{5\sqrt{3}}{2}}{5} = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 60^\circ$

$\Rightarrow \angle AOB = 120^\circ$

$\therefore$ Area of sector AOB = $\frac{120}{360} \times \pi \times 5^2 \text{cm}^2 = \frac{25\pi}{3} \text{cm}^2$

OR

$\theta = 56^\circ$ and let r be the radius of the circle

Area of sector $= \frac{\pi r^2 \theta}{360^\circ}$

$= 17.6 cm^2$

$\Rightarrow \frac{22}{7} \times r^2 \times \frac{56^\circ}{360^\circ}$ = 17.6

$r^2 = \left(\frac{17.6 \times 360 \times 7}{22 \times 56}\right) \text{cm}^2$

$r^2 = 36 cm^2$

$\Rightarrow r = \sqrt{36}\text{cm}$

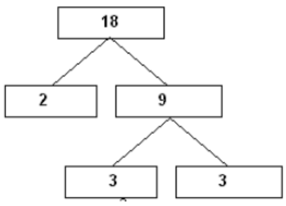
r = 6 cm

Hence radius = 6cm

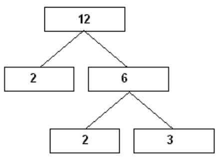
Section C

26. By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.

$18 = 2 \times 3 \times 3 = 2 \times 3^2$



12 = 2 $\times$ 2 $\times$ 3 = $2^2 \times 3$



LCM $(18, 12) = 2^2 \times 3^2 = 36$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

27. f(x) = x^2 - 2x - 8

= x^2 - 4x + 2x - 8

= x(x - 4) + 2(x - 4)

= (x + 2) (x - 4)

f(x) = 0 if x+2 = 0 or x-4 = 0

x = -2 or 4

So the zeroes of the polynomials are -2 and 4.

For the Polynomial f(x)=x^2 - 2x - 8

a=1,b=-2, c=-8

Sum of the zeroes = - 2 + 4 = 2 = -$\frac{b}{a}$

Product of zeros =(-2)(4) = -8 = $\frac{c}{a}$

Hence, the relationship between the zeros and coefficients is verified.

28. Let the present age of the father be x years and the sum of the present age of his two children be y years. Then, according to the question,

x = 3y

$\Rightarrow$ x - 3y = 0 .....(1)

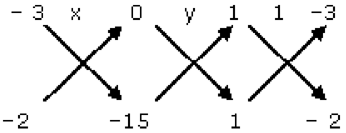
And, x + 5 = 2(y + 5 + 5)

$\Rightarrow$ x + 5 = 2(y + 10)

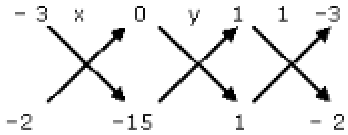
$\Rightarrow$ x + 5 = 2y + 20

$\Rightarrow$ x - 2y - 15 = 0 ....(2)

to solve the equations (1) and (2) by cross multiplication method, we draw the diagram below:



To solve the equation (1) and (2) by cross multiplication method, we draw the diagram below:



Then,

$\Rightarrow \frac{x}{(-3)(-15)-(-2)(0)} = \frac{y}{(0)(1)-(-15)(1)} = \frac{1}{(1)(-2)-(1)(-3)}$

$\Rightarrow \frac{x}{45} = \frac{y}{15} = \frac{1}{1}$

$\Rightarrow$ x = 45 and y = 15

Hence, the age of the father is 45 years.

Verification, Substituting x = 45, y = 15,

We find that both the equations (1) and (2) are satisfied as shown below:

x - 3y = 45 - 3(15) = 45 - 45 = 0

x - 2y - 15 = 45 - 2(15) - 15 = 0

Hence, the solution we have got is correct.

OR

Given pair of equations is

x + y = 3 …(i)

and 3x + 3y = 9 …(ii)

On comparing with standard form we get

$a_1 = 1, b_1 = 1, c_1 = - 3;$

And $a_2 = 3, b_2 = 3, c_2 = - 9;$

$a_1 / a_2 = 1/3$

$b_1 / b_2 = 1/3$

$c_1 / c_2 = 1/3$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e. coincident lines

Hence, the given pair of linear equations is coincident and having infinitely many solutions.

The given pair of linear equations is consistent.

Now, x + y = 3 or y = 3 - x

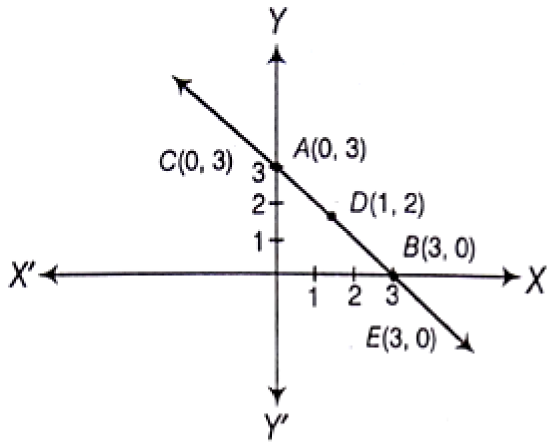
If x = 0 then y = 3, If x = 3, then y = 0.

x	0	3
y	3	0
Points	A	B

and 3x + 3y = 9 or $y = \frac{9-3x}{3}$

If x = 0 then y = 3, if x = 1, then y = 2, and if x = 3, then y = 0.

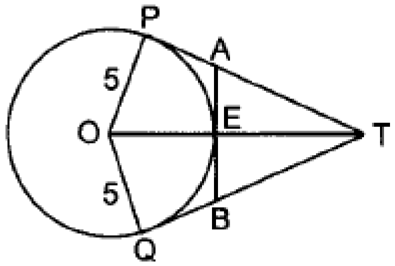
x	0	1	3
y	3	2	0
Points	C	D	E



Plotting the points A(0, 3) and B(3, 0), we get the line AB. Again, plotting the points C(0, 3) and D(1, 2) and E(3, 0), we get the line CDE.

29. According to the question,

O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E.



$\because$ OP $\perp$ TP [Radius from point of contact of the tangent]

$\therefore \angle$OPT = 90°

In right $\triangle$OPT *

OT^2 = OP^2 + PT^2

$\Rightarrow$ (13)^2 = (5)^2 + PT^2 $\Rightarrow$ PT = 12 cm

Let AP = x cm AE = AP $\Rightarrow$ AE = x cm

and AT = (12 - x)cm

TE = OT - OE = 13 - 5 = 8 cm

$\because$OE $\perp$ AB [Radius from the point of contact]

$\therefore \angle$AEO = 90° $\Rightarrow \angle$AET = 90°

In right $\triangle$AET,

$\mathrm{AT}^2 = \mathrm{AE}^2 + \mathrm{ET}^2$

$(12 - x)^2 = x^2 + 8^2$

$\Rightarrow 144 + x^2 - 24x = x^2 + 64$

$\Rightarrow \quad 24x = 80 \Rightarrow x = \frac{80}{24} = \frac{10}{3}$ cm

Also BE = AE = $\frac{10}{3}$ cm

$\Rightarrow \quad AB = \frac{10}{3} + \frac{10}{3} = \frac{20}{3}$ cm

OR

The lengths of tangents drawn from an external point to a circle are equal.

$\therefore AR = AQ$ ...(i)

$BR = BP$ ...(ii)

$CQ = CP$ ...(iii)

Given that ABC is an isosceles triangle AB = AC.

Subtract AP on both sides, we obtain

$AB - AR = AC - AR$

$\Rightarrow AB - AR = AC - AQ$ (from (i)

$\therefore BR = CQ.$

$\Rightarrow BP = CQ$ (from (ii)

$\Rightarrow BP = CP$ (from (iii))

$\therefore$ BC is bisected at the point of contact R.

30. To prove-

$\frac{1}{(\cot A)(\sec A)-\cot A} - \operatorname{cosec} A = \operatorname{cosec} A - \frac{1}{(\cot A)(\sec A)+\cot A}$

Taking LHS

$= \frac{1}{(\cot A)(\sec A)-\cot A} - \operatorname{cosec} A$

$= \frac{1}{(\frac{\cos A}{\sin A})(\frac{1}{\cos A})-\left(\frac{\cos A}{\sin A}\right)} - \frac{1}{\sin A}$

$\frac{1}{(\frac{1}{\sin A})-\left(\frac{\cos A}{\sin A}\right)} - \frac{1}{\sin A} = \frac{1}{\frac{1-\cos A}{\sin A}} - \frac{1}{\sin A} = \frac{\sin A}{1-\cos A} - \frac{1}{\sin A} = \frac{\sin^2 A-1+\cos A}{(1-\cos A)\sin A}$

$= \frac{-\cos^2 A+\cos A}{(1-\cos A)\sin A} = \frac{\cos A(1-\cos A)}{(1-\cos A)\sin A}$ $\{\because \sin^2 A + \cos^2 A = 1\}$

$= \frac{\cos A}{\sin A} = cotA$

Now, taking RHS

$= \operatorname{cosec} A - \frac{1}{(\cot A)(\sec A)+\cot A}$

$= \frac{1}{\sin A} - \frac{1}{(\frac{\cos A}{\sin A})(\frac{1}{\cos A})+\frac{\cos A}{\sin A}}$

$= \frac{1}{\sin A} - \frac{1}{(\frac{1}{\sin A})+\frac{\cos A}{\sin A}} = \frac{1}{\sin A} - \frac{\sin A}{(1+\cos A)}$

$= \frac{1+\cos A-\sin^2 A}{(1+\cos A)\sin A} = \frac{\cos^2 A+\cos A}{(1+\cos A)\sin A}$

$= \frac{\cos A(\cos A+1)}{(1+\cos A)\sin A} = \frac{\cos A}{\sin A}$

= cotA = LHS

31. Let us first construct the table for $d_i \times f_i$,

where $d_i = x_i - A$ (Assumed mean), as shown below:

Class	**Class Marks (x_i)**	**Frequency (f_i)**	**$d_i = x_i - A$**	**$d_i \times f_i$**
0-5	2.5	8	-10	-80
5-10	7.5	7	-5	-35
10-15	12.5 = A	10	0	0
15-20	17.5	13	5	65
20-25	22.5	12	10	120
		$\sum f_i$ = 50		$\sum f_i d_i$ = 70

Let A = 12.5,

Then $\sum f_i d_i$ = 70

Now, the required mean

$(\bar{x}) = A + \frac{\sum f_i d_i}{\sum f_i}$

= 12.5 + $\frac{70}{50}$

= 12.5 + 1.4

= 13.9

Section D

32. Let the two consecutive positive integers be x and (x + 1)

According to given condition,

x(x + 1) = 306

$x^2 + x - 306 = 0$

$x^2 + 18x - 17x - 306 = 0$

x(x + 18) - 17(x + 18) = 0

(x + 18) (x – 17) = 0

(x + 18) = 0 or (x – 17) = 0

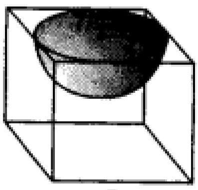
x = - 18 or x = 17

but x = 17 (x is a positive integers)
x + 1 = 17 + 1 = 18
Thus the two consecutive positive integers are 17 and 18.

OR

Let the larger number be x. Then,
Square of the smaller number = 4x
Also, Square of the larger number = x^2
It is given that the difference of the squares of the numbers is 45.
$\therefore$ x^2 - 4x = 45
$\Rightarrow x^2$ - 4x - 45 = 0
$\Rightarrow$ x^2 - 9x + 5x - 45 = 0
$\Rightarrow$ x (x - 9) + 5 (x - 9) = 0
$\Rightarrow$ x - 9 = 0 or, x + 5 = 0 $\Rightarrow$ x = 9, -5
Case I When x = 9: In this case, we have
Square of the smaller number = 4x = 36
$\therefore$ Smaller number = ± 6.
Thus, the numbers are 9, 6 or 9, - 6
CASE II When x = -5: In this case, we have
Square of the smaller number = 4x = - 20. But, square of a number is always positive. Therefore, x = - 5 is not possible.
Hence, the numbers are 9, 6 or 9, - 6.

33. Given A $\triangle ABC$ in which $\angle BAC = 90^\circ$ and DEFG is a square.
Proof
   i. In $\triangle AGF$ and $\triangle DBG$, we have
   $\angle GAF = \angle BDG = 90^\circ$
   $\angle AGF = \angle DBG$ [corresponidng angles]
   [$\because GF \| BC$ and AB is the transversal]
   $\therefore \quad \triangle AGF \sim \triangle DBG$
   ii. In $\triangle AGF$ and $\triangle EFC$, we have
   $\angle FAG = \angle CEF = 90^\circ$
   $\angle GFA = \angle FCE$ [corresponding angles]
   [$\because GF \| BC$ and AC is the transversal]
   $\therefore \quad \triangle AGF \sim \triangle EFC$
   iii. $\triangle DBG \sim \triangle AGF$ and $\triangle AGF \sim \triangle EFC$
   $\Rightarrow \triangle DBG \sim \triangle EFC$
   iv. $\triangle DBG \sim \triangle EFC$
   $\Rightarrow \frac{BD}{FE} = \frac{DG}{EC}$
   $\Rightarrow \frac{BD}{DE} = \frac{DE}{EC}$ [$\because$ DG = DE and FE = DE).
   Hence, $DE^2 = BD \times EC$.

34. 

Edge of the cube, a = 7 cm.
Radius of the hemisphere, $r = \frac{7}{2}$cm.
Surface area of remaining solid
= total surface area of the cube - area of the top of hemispherical part + curved surface area of the hemisphere
$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$
$= \left(6 \times 7 \times 7 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^2$
= (294 + 38.5)cm^2 = 332.5 cm^2.

OR

Given, radius of cone = radius of hemisphere

= r

= 7 cm

Height of cone (h) = 2 × radius

= 2 × 7

= 14 cm

Volume of solid = Volume of cone + Volume of hemisphere

Volume of solid (V) = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(2r) + \frac{2}{3}\pi r^3$ ...($\because$ h = 2r)

$= \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{4312}{3}$

= 1437.33 cm^3

35.

Class Interval	Frequency f_i	Cumulative frequency
0.5 - 5.5	7	7
5.5 - 10.5	10	17
10.5 - 15.5	16	33
15.5 - 20.5	32	65
20.5 - 25.5	24	89
25.5 - 30.5	16	105
30.5 - 35.5	11	116
35.5 - 40.5	5	121
40.5 - 45.5	2	123

Here, N = 123⇒ $\frac{N}{2} = 61.5$

Median class is 15.5 - 20.5

$\therefore l = 15.5, h = 5, f = 32, c.f. = 33$

Now, Median = $l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$= 15.5 + \left[5 \times \frac{(61.5-33)}{32}\right]$

$= 15.5 + 4.45 = 19.95$

Section E

36. i. Child's Day wise are,

$\underbrace{5}_{1\ coin}, \underbrace{10}_{2\ coins}, \underbrace{15}_{3\ coins}, \underbrace{20}_{4\ coins}, \underbrace{25}_{5\ coins}$ .... to $\underbrace{n\ days}_{n\ coins}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

⇒ $\frac{n}{2}[2 \times 1 + (n-1)1]$ = 190

⇒ n(n + 1) = 380 ⇒ n^2 + n - 380 = 0

⇒ (n + 20)(n − 19) = 0 ⇒ (n + 20)(n - 19) = 0

⇒ n = −20 or n = 19 ⇒ n = -20 or n = 19

But number of coins cannot be negative

∴ n = 19 (rejecting n = -20)

So, number of days = 19

ii. Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

= $\frac{19}{2}$[2 × 5 + (19 - 1)5]

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

and total money she shaved = ₹ 950

iii. Money saved in 10 days

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10-1) \times 5]$

$\Rightarrow S_{10}$ = 5[10 + 45}]

$\Rightarrow$ S_{10} = 275

Money saved in 10 days = ₹ 275

OR

Number of coins in piggy bank on 15th day

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 5]$

$\Rightarrow S_{15} = \frac{15}{2}$ [2 + 14]

$\Rightarrow$ S_{15} = 120

So, there are 120 coins on 15th day.

37. i. A(1, 9) and B(5, 13)

ii. C(9, 13) and D(13, 9)

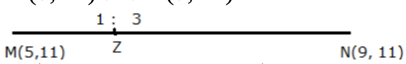
Mid-point of CD is (11, 11)

iii. a. M(5, 11) and Q(9, 3)

MQ = $\sqrt{(9-5)^2 + (3-11)^2} = \sqrt{80}$ or $4\sqrt{5}$

OR

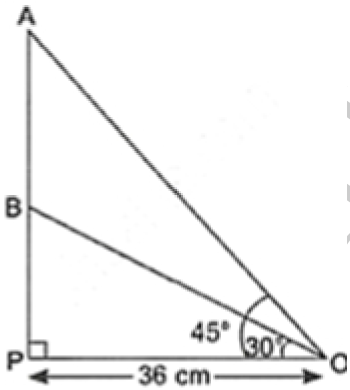
b. M(5, 11) and N(9, 11)



$\text{Z}\left(\frac{1\times 9+3\times 5}{1+3}, \frac{1\times 11+3\times 11}{1+3}\right)$

$\text{Z}(6, 11)$

38. i. Let the length of wire BO = x cm



$\therefore$ cos 30^o = $\frac{PO}{BO}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{36}{x}$

$\Rightarrow$ x = $\frac{36\times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

= 12 $\times$ $2\sqrt{3}$

= $24\sqrt{3}$ cm

ii. In $\triangle$APO, tan 45^o = $\frac{AP}{PO}$

$\Rightarrow$ 1 = $\frac{AP}{36}$

$\Rightarrow$ AP = 36 cm ...(i)

Now, In $\triangle$PBO,

tan 30^o = $\frac{BP}{PO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BP}{36}$

$\Rightarrow$ BP = $\frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{36}{3}\sqrt{3}$

= $12\sqrt{3}$ cm

$\therefore$ AB = AP - BP

= 36 - $12\sqrt{3}$ cm

iii. In $\triangle$OPB

tan 30^o = $\frac{BP}{PO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BP}{36}$

BP = $\frac{36}{\sqrt{3}}$

$= \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

= $12\sqrt{3}$ cm

Now, Area of $\triangle$OPB = $\frac{1}{2}$ $\times$ height $\times$ base

= $\frac{1}{2}$ $\times$ BP $\times$ OP

= $\frac{1}{2}$ $\times$ $12\sqrt{3}$ $\times$ 36

= $216\sqrt{3}$ cm^2

OR

In $\triangle$APO, tan 45^o = $\frac{AP}{36}$

1 = $\frac{AP}{36}$

A = 36 cm

Height of section A from the base of the tower = AP = 36 cm.