SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

MHT - CET - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 100**

1. If $\sin\theta = \frac{12}{13}, \left(0 < \theta < \frac{\pi}{2}\right)$ and $\cos\phi = -\frac{3}{5}, \left(\pi < \phi < \frac{3\pi}{2}\right)$, then $\sin(\theta + \phi)$ will be **[2]**

   a) $\frac{-56}{61}$ b) $\frac{-56}{65}$

   c) $\frac{1}{65}$ d) -56

2. The points (1, 3) and (5, 1) are the opposite vertices of a rectangle. The other two vertices lie on the line y = 2x + c, then the value of c will be **[2]**

   a) 4 b) -2

   c) -4 d) 2

3. The area of triangle formed by the tangent, normal drawn at $(1, \sqrt{3})$ to the circle $x^2 + y^2 = 4$ and positive X-axis, is **[2]**

   a) $4\sqrt{3}$ b) $2\sqrt{3}$

   c) $\sqrt{3}$ d) $3\sqrt{3}$

4. The equation of a circle whose diameter is the line joining the points (-4, 3) and (12, -1) is **[2]**

   a) $x^2 + y^2 + 8x + 2y + 51 = 0$ b) $x^2 + y^2 - 8x - 2y - 51 = 0$

   c) $x^2 + y^2 + 8x + 2y - 51 = 0$ d) $x^2 + y^2 + 8x - 2y - 51 = 0$

5. A bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from a randomly chosen bag and is found to be red. The probability that it was drawn from bag B was **[2]**

   a) $\frac{5}{18}$ b) $\frac{25}{52}$

   c) $\frac{5}{14}$ d) $\frac{5}{16}$

6. If 1, ω, ω^2, ________ ω^{n-1} are the n^{th} roots of unity, then $(2 - \omega)(2 - \omega^2)$ ________ $(2 - \omega^{n-1})$ is equal to **[2]**

   a) 1 b) $2^n - 1$

   c) $2^n + 1$ d) 2^n

7. Everybody in a room shakes hand with everybody else. The total number of hand shakes is 66. The total number of persons in the room is **[2]**

   a) 14 b) 12

   c) 11 d) 13

8. There are 10 lamps in a hall each one of them can be switched on independently. The number of ways in which the hall can be illuminated. **[2]**

a) 1023 b) 2^{10}

c) 10! d) 10^2

9. The composite map fog of the functions f : R $\to$ R, f(x) = sin x and g : R $\to$ R, g(x) = x^2 is **[2]**

a) x^2 b) x^2(sin x)

c) sin x^2 d) $(\sin x)^2$

10. $\lim\limits_{x\to 1}[x] =$ **[2]**

a) 1 b) 0

c) -1 d) does not exist

11. If $f(x) = \begin{cases} \frac{x^2-9}{x-3} + \mathrm{a}, x > 3 \\ 5 \quad , x = 3 \\ 2x^2 + 3x + \mathrm{b}, x < 3 \end{cases}$ is continuous at x = 3, then **[2]**

a) a = 1, b = -22 b) a = -1, b = 22

c) a = -1, b = -22 d) a = 1, b = 22

12. The negation of the statement **If Saral Mart does not reduce the prices, I will not shop there any more** is **[2]**

a) Saral Mart does not reduce the prices and still I will shop there.

b) Saral Mart reduces the prices and I will not shop there.

c) Saral Mart does not reduce the prices or I will shop there.

d) Saral Mart reduces the prices and still I will shop there.

13. If A = $\begin{bmatrix} \frac{\mathrm{k}}{2} & 0 & 0 \\ 0 & \frac{l}{3} & 0 \\ 0 & 0 & \frac{\mathrm{m}}{4} \end{bmatrix}$ = $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ then k + l + m = **[2]**

a) 29 b) 14

c) 9 d) 1

14. If $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ A $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$ = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then matrix A equals **[2]**

a) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ b) $\begin{bmatrix} 7 & 5 \\ -11 & -8 \end{bmatrix}$

c) $\begin{bmatrix} 5 & 3 \\ 13 & 8 \end{bmatrix}$ d) $\begin{bmatrix} 7 & 1 \\ 34 & 5 \end{bmatrix}$

15. If $\sin^{-1}$ x + $\sin^{-1}$ y + $\sin^{-1}$ z = $\frac{\pi}{2}$, then the value of $x^2 + y^2 + z^2 + 2xyz$ is equal to **[2]**

a) 3 b) 2

c) 0 d) 1

16. If in a triangle ABC, 2 cos A = sin B cosec C, then **[2]**

a) a = b b) 2a = bc

c) b = c d) c = a

17. In $\triangle$ABC, $\frac{\cos\frac{1}{2}(B-C)}{\sin\frac{1}{2}A}$ = **[2]**

a) $\frac{a}{b+c}$ b) $\frac{\mathrm{b+c}}{\mathrm{a}}$

c) $\frac{\mathrm{a}}{\mathrm{b-c}}$ d) $\frac{b-c}{a}$

18. If 4 sin A = 4 sin B = 3 sin C in a $\triangle$ABC, then cos C = **[2]**

a) $\frac{1}{9}$ b) $\frac{1}{27}$

c) $\frac{1}{18}$ d) $\frac{1}{3}$

19. $\int\limits_{\frac{-1}{2}}^{\frac{1}{2}}(\cos x)\left[\log\left(\frac{1-x}{1+x}\right)\right]$ dx = **[2]**

a) 0 b) $\mathrm{e}^{\frac{1}{2}}$

c) $2\mathrm{e}^{\frac{1}{2}}$ d) 1

20. $\int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}}$ dx = **[2]**

a) $\frac{1}{2}$ b) $\frac{5}{2}$

c) $\frac{2}{5}$ d) $\frac{3}{2}$

21. $\int\limits_{0}^{\frac{\pi}{2}}\frac{\cos 2x}{\cos x+\sin x}$ dx = **[2]**

a) 1 b) 0

c) 2 d) -1

22. The value of I = $\int\limits_{0}^{\frac{\pi}{2}}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}$ dx is **[2]**

a) 1 b) 3

c) 0 d) 2

23. If l, m, n are direction cosines of the line, then -l, -m, -n can be **[2]**

a) only direction ratios of the line b) neither direction cosines nor direction ratios of the line

c) only direction cosines of the line d) direction cosines and direction ratios of the line

24. A and B are two points. The position vector of A is $6\vec{b}-2\vec{a}$. A point P divides the line AB in the ratio 1 : 2. If $\vec{a}-\vec{b}$ is the position vector of P, then the position vector of B is given by **[2]**

a) $7\vec{a}+15\vec{b}$ b) $15\vec{\mathrm{a}}-7\vec{\mathrm{b}}$

c) $7\vec{\mathrm{a}}-15\vec{\mathrm{b}}$ d) $15\vec{\mathrm{a}}+7\vec{\mathrm{b}}$

25. If $\overrightarrow{\mathrm{OA}}=2\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}}, \overrightarrow{\mathrm{OB}}=2\hat{\mathrm{i}}+4\hat{\mathrm{j}}+4\hat{\mathrm{k}}$ and the length of the internal bisector of $\angle\mathrm{BOA}$ of triangle AOB is k, then 9k^2 = **[2]**

a) $\sqrt{225}$ b) 20

c) 712 d) 136

26. Let $\hat{\alpha},\hat{\beta},\hat{\gamma}$ be three unit vectors such that $\hat{\alpha}\times(\hat{\beta}\times\hat{\gamma})=\frac{1}{2}(\hat{\beta}\times\hat{\gamma})$ where $\hat{\alpha}\times(\hat{\beta}\times\hat{\gamma})$ **[2]**

$= (\hat{\alpha} \cdot \hat{\gamma})\hat{\beta} - (\hat{\alpha} \cdot \hat{\beta})\hat{\gamma}$. If $\hat{\beta}$ is not parallel to $\hat{\gamma}$, then the angle between $\hat{\alpha}$ and $\hat{\beta}$ is

a) $\frac{5\pi}{6}$ | b) $\frac{2\pi}{3}$

c) $\frac{\pi}{3}$ | d) $\frac{\pi}{6}$

27. The angle between the lines represented by the equation $(x^2 + y^2) \sin \theta + 2xy = 0$ is **[2]**

a) $\frac{\theta}{2}$ | b) θ

c) $\frac{\pi}{2} - \frac{\theta}{2}$ | d) $\frac{\pi}{2} - \theta$

28. The d.r.s of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\frac{\pi}{4}$ with 4 plane x + y = 3, are **[2]**

a) 1, 1, 2 | b) 1, 1, $\sqrt{2}$

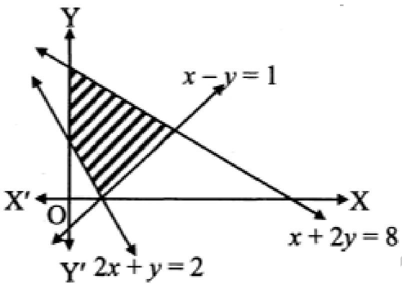
c) $\sqrt{2}$, 1, 1 | d) 1, $\sqrt{2}$, 1

29. The equation of line passing through the midpoint of the line joining the points (-1, 3, -2) and (-5, 3, -6) and equally inclined to the axes is **[2]**

a) x + 3 = y - 3 = z + 4 | b) x + 1 = y - 3 = z + 2

c) x - 3 = y + 3 = z - 4 | d) x + 5 = y + 3 = z + 6

30. Find the linear inequations for which the shaded area in the following figure is the solution set: **[2]**



a) x - y $\geq$ 1, 2x + y $\geq$ 2, x + 2y $\geq$ 8, x $\geq$ 0, y $\geq$ 0 | b) x - y $\leq$ 1, 2x + y $\geq$ 2, x + 2y $\leq$ 8, x $\geq$ 0, y $\geq$ 0

c) x + y $\geq$ 1, 2x + y $\leq$ 2, x + 2y $\geq$ 8, x $\geq$ 0, y $\geq$ 0 | d) x + y $\leq$ 1, 2x + y $\geq$ 2, x - 2y $\geq$ 8, x $\leq$ 0, y $\geq$ 0

31. If y = $\sqrt{\frac{1+\tan x}{1-\tan x}}$, then $\frac{dy}{dx}$ = **[2]**

a) $\frac{1}{2}\sqrt{\frac{1-\tan x}{1+\tan x}} \cdot \sec\left(\frac{\pi}{4} + x\right)$ | b) $\frac{1}{2}\sqrt{\frac{1-\tan x}{1+\tan x}} \cdot \sec^2\left(\frac{\pi}{4} + x\right)$

c) $\sqrt{\frac{1-\tan x}{1+\tan x}} \cdot \sec\left(\frac{\pi}{4} + x\right)$ | d) $\sqrt{\frac{1-\tan x}{1+\tan x}} \cdot \sec^2\left(\frac{\pi}{4} + x\right)$

32. If x = t log t, y = t^t, then $\frac{dy}{dx}$ = **[2]**

a) 1 + log t | b) e^x

c) e^t | d) $\frac{e^t}{1+\log t}$

33. Differential coefficient of $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1} x$ is **[2]**

a) 1 | b) $\frac{1}{2}$

c) $\frac{3}{2}$ | d) 2

34. If y = $\log\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, then $\frac{dy}{dx}$ is equal to **[2]**

a) sec 2x | b) tan x

c) tan 2x		d) sec x

35. The point on the curve $x^2 = 3 - 2y$, where the tangent is parallel to x + y = 2, is **[2]**

a) (-1, 3)
b) $(\sqrt{3}, 0)$
c) (3, -3)
d) (1, 1)

36. The approximate value of $\cot^{-1}$ (1.001) is **[2]**

a) 0.7890
b) 0.7865
c) 0.7845
d) 0.7895

37. If log 3 = 1.0986, then the greatest value of the function f(x) = $\tan^{-1} x - \frac{1}{2}$ log x in $\left(\frac{1}{\sqrt{3}}, \sqrt{3}\right)$ is **[2]**

a) $\frac{\pi}{6} - \frac{1}{4}$ log 3
b) $\frac{\pi}{3} - \frac{1}{4}$ log 3
c) $\frac{\pi}{3} + \frac{1}{2}$ log 3
d) $\frac{\pi}{6} + \frac{1}{4}$ log 3

38. $\int x^{51}$ $(\tan^{-1}x + \cot^{-1}x)$ dx = **[2]**

a) $\frac{x^{52}}{52}$ $(\tan^{-1}x + \cot^{-1}x)$ + c
b) $\frac{x^{52}}{52}$ $(\tan^{-1}x - \cot^{-1}x)$ + c
c) $\frac{\pi x^{52}}{104} + \frac{\pi}{2} + c$
d) $\frac{\pi x^{52}}{52} + \frac{\pi}{2} + c$

39. If $\int \frac{\sin 2x}{\sin 5x \sin 3x}$ dx = $\frac{1}{3}$ log |sin 3x| - $\frac{1}{5}$ log |f(x)| + c, then f(x) = ________. **[2]**

a) sin 2x
b) sin 5x
c) sin 4x
d) sin 6x

40. $\int \frac{1}{x^2}$ $(2x + 1)^3$ dx = **[2]**

a) $4x^2 + 12x + 6 \log x - \frac{1}{x} + c$
b) $2x^2 + 8x + 3 \log x - \frac{2}{x} + c$
c) $4x^2 + 12x - 6 \log x - \frac{2}{x} + c$
d) $8x^2 + 6x + 6 \log x + \frac{2}{x} + c$

41. If $\int \frac{1}{f(x)}$ dx = log $\{f(x)\}^2$ + c, then f(x) is equal to **[2]**

a) $x^2 + \alpha$
b) $\frac{x}{2} + \alpha$
c) $x + \alpha$
d) $2x + \alpha$

42. The area of the region bounded by parabola $y^2 = 16x$ and its latus rectum is ________ square units. **[2]**

a) $\frac{128}{3}$
b) $\frac{16}{3}$
c) $\frac{64}{3}$
d) $\frac{256}{3}$

43. Let y = y (x) be the solution of the differential equation, $(y^2 - x) \frac{dy}{dx} = 1$, satisfying y(0) = 1. This curve intersects the X-axis at a point whose abscissa is **[2]**

a) 2
b) 2 + e
c) 2 - e
d) -e

44. The solution of the equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is **[2]**

a) $x\sqrt{1-y^2} + y\sqrt{1-x^2}$ = c
b) None of these
c) $x\sqrt{1+y^2} + y\sqrt{1+x^2}$ = c
d) $x\sqrt{1-y^2} - y\sqrt{1-x^2}$ = c

45. The differential equation of all straight lines passing through the origin is **[2]**

a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x}$

b) x + y $\frac{dy}{dx}$ = 0

c) y = $\sqrt{x\frac{\mathrm{d}y}{\mathrm{d}x}}$

d) $\frac{dy}{dx}$ = y + x

46. For the following distribution function F(x) of a r.v. X **[2]**

X	1	2	3	4	5	6
F(X)	0.2	0.37	0.48	0.62	0.85	1

P(3 < X $\leq$ 5) =

a) 0.48

b) 1.47

c) 0.37

d) 0.27

47. If the probability function of a random variable X is defined by P(X = k) = a$\left(\frac{k+1}{2^k}\right)$ for k = 0, 1, 2, 3, 4, 5, then the probability that X takes a prime value is **[2]**

a) $\frac{13}{20}$

b) $\frac{23}{60}$

c) $\frac{19}{60}$

d) $\frac{11}{20}$

48. A random variable X takes values -1, 0, 1, 2 with probabilities $\frac{1+3\text{p}}{4}, \frac{1-\text{p}}{4}, \frac{1+2p}{4}, \frac{1-4\text{p}}{4}$ respectively, where p varies over R. Then the minimum and maximum values of the mean of X are respectively **[2]**

a) $-\frac{1}{16}$ and $\frac{5}{16}$

b) $-\frac{1}{16}$ and $\frac{5}{4}$

c) $-\frac{7}{4}$ and $\frac{5}{16}$

d) $-\frac{7}{4}$ and $\frac{1}{2}$

49. A r.v. X ~ B (n, p). If values of mean and variance of X are 18 and 12 respectively then total number of possible values of X are **[2]**

a) 55

b) 54

c) 12

d) 18

50. Probability that a person will develop immunity after vaccination is 0.8. If 8 people are given the vaccine then probability that all develop immunity is **[2]**

a) 1

b) ${}^8C_6\ (0.2)^6\ (0.8)^2$

c) $(0.8)^8$

d) $(0.2)^8$