SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

MHT - CET - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 100**

1. If $\cos\theta + \cos 7\theta + \cos 3\theta + \cos 5\theta = 0$, then θ is **[2]**

   a) $\frac{n\pi}{2}$ b) $\frac{n\pi}{8}$

   c) $n\pi$ d) $\frac{n\pi}{4}$

2. The triangle formed by the lines x + y - 4 = 0, 3x + y = 4, x + 3y = 4 is **[2]**

   a) equilateral b) isosceles

   c) Obtuse d) right-angled

3. If the lines 2x + 3y + 1 = 0 and 3x - y - 4 = 0 lie along diameters of a circle of circumference 10π, then the equation of the circle is **[2]**

   a) $x^2 + y^2 - 2x + 2y - 23 = 0$ b) $x^2 + y^2 + 2x - 2y + 23 = 0$

   c) $x^2 + y^2 - 2x - 2y - 23 = 0$ d) $x^2 + y^2 + 2x - 2y - 23 = 0$

4. The equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at the point (5, 5), is **[2]**

   a) $x^2 + y^2 - 18x - 16y + 120 = 0$ b) $x^2 + y^2 + 18x - 16y + 120 = 0$

   c) $x^2 + y^2 - 18x - 16y - 120 = 0$ d) $x^2 + y^2 + 18x + 16y - 120 = 0$

5. Let A, B and C be three events such that P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$, then $P(B \cap C)$ satisfies **[2]**

   a) $P(B \cap C) \leq 0.23$ b) $0.23 \leq P(B \cap C) \geq 0.48$

   c) $P(B \cap C) \leq 0.48$ d) $0.23 \leq P(B \cap C) \leq 0.48$

6. If $z = (3\sqrt{7} + 4i)^2 (3\sqrt{7} - 4i)^3$, then Re(z) = **[2]**

   a) $(79)^2 (3\sqrt{7} - 4i)$ b) $79 \times 3\sqrt{7}$

   c) $(79)^2\, 3\sqrt{7}$ d) $-4(79)^2$

7. Three different prizes are to be distributed in a class of 20 boys. In how many ways can this be done, if a boy is eligible to get any number of prizes. **[2]**

   a) 6480 b) 8000

   c) 7220 d) 6840

8. There are four teachers and five classes in a school. One teacher teaches only one class at a time. In how many ways can the teachers choose the classes? **[2]**

a) 16

b) 120

c) 60

d) 14

9. If f(x) = 3x - 5, then $f^{-1}(x)$ is **[2]**

a) Does not exist because f is not onto

b) Does not exist because f is not one-one

c) Is given by $\frac{x+5}{3}$

d) $\frac{1}{3x-5}$

10. If f(x) = $\frac{\sin\left(e^{x-3}-1\right)}{\log(x-2)}$, then $\lim\limits_{x \to 3} f(x) =$ **[2]**

a) -2

b) 0

c) -1

d) 1

11. If $f(x) = \begin{cases} \frac{x}{\frac{1}{e^x}+1}, \text{ when } x \neq 0 \\ 0, \text{ when } x = 0 \end{cases}$, then **[2]**

a) f(x) is not continuous at x = 0

b) $\lim\limits_{x \to 0^-}$ f(x) = 1

c) f(x) is continuous at x = 0

d) $\lim\limits_{x \to 0^+}$ f(x) = 1

12. The negation of the statement **72 is divisible by 2 and 3** is **[2]**

a) 72 is not divisible by 2 and 3.

b) 72 is divisible by 2 and 72 is divisible by 3.

c) 72 is divisible by 2 or 72 is divisible by 3.

d) 72 is not divisible by 2 or 72 is not divisible by 3.

13. If A = $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where A_{11}, A_{12}, A_{13} are co-factors of a_{11}, a_{12}, a_{13} respectively, then the value of $a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$ = **[2]**

a) 0

b) -1

c) 1

d) $\frac{1}{2}$

14. If A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$, then |adj A| is equal to **[2]**

a) 64

b) 12

c) 72

d) 144

15. Which one of the following is true? **[2]**

a) $\tan(\sin^{-1} x) = \sin(\tan^{-1} x)$

b) $\cos(\tan^{-1} x) = \tan(\cos^{-1} x)$

c) $\sec(\tan^{-1} x) = \tan(\sec^{-1} x)$

d) $\sin(\cos^{-1} x) = \cos(\sin^{-1} x)$

16. In a $\triangle$ABC, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and the side a = 2, then area of the triangle is **[2]**

a) $\sqrt{3}$

b) 2

c) 1

d) $\frac{\sqrt{3}}{2}$

17. If the angles of a triangle ABC be in A.P., then **[2]**

a) $c^2 = a^2 + b^2 - ab$

b) $b^2 = a^2 + c^2 - ac$

c) $a^2 = b^2 + c^2 - ac$ d) $b^2 = a^2 + c^2$

18. If $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z = \frac{\pi}{2}$, then x + y + z is equal to **[2]**

a) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ b) $x^2 + y^2 + z^2$

c) xyz d) xy + yz + zx

19. The value of $\int_0^1 x^2 e^x$ dx is equal to **[2]**

a) e - 2 b) e^2

c) $e^2 - 2$ d) e + 2

20. If f(x) = $\begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$. Then $\int_0^{\frac{\pi}{2}} f(x)$ dx = **[2]**

a) 0 b) $\frac{1}{3}$

c) 3 d) $\frac{2}{3}$

21. The value of $\int_3^5 \frac{x^2}{x^2-4}$ dx is **[2]**

a) $2 + 4 \log_e 3 - 4 \log_e 7 + 4 \log_e 5$ b) $2 + \log_e \left(\frac{15}{7}\right)$

c) $2 - \tan^{-1} \left(\frac{15}{7}\right)$ d) $2 - \log_e \left(\frac{15}{7}\right)$

22. $\int_0^{\frac{\alpha}{3}} \frac{f(x)}{f(x)+f\left(\frac{\alpha-3x}{3}\right)}$ dx = **[2]**

a) $\frac{2\alpha}{3}$ b) $\frac{\alpha}{2}$

c) $\frac{\alpha}{6}$ d) $\frac{\alpha}{3}$

23. A unit vector perpendicular to the plane containing the vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-2\hat{i} + \hat{j} + 3\hat{k}$ is **[2]**

a) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$ b) $\frac{-\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

c) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ d) $\frac{-\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$

24. If the position vectors of the points A, B, C are $\vec{a}, \vec{b}$ and $3\vec{a} - 2\vec{b}$ respectively, then the points A, B, C are **[2]**

a) Forming a right angled triangle b) Coplanar

c) Collinear d) Non-collinear

25. If A(a, 2, 2), B(a, b, 1) and C(1, 2, -2) are the vertices of triangle ABC and G(2, 1, c) is its centroid, then values of a, b and c are **[2]**

a) a = −1, b = 1, c = $\frac{3}{2}$ b) a = $\frac{1}{2}$, b = 1, c = 1

c) $a = \frac{1}{2}$, $b = \frac{1}{2}$, c = -1 d) $a = \frac{5}{2}$, $b = -1, c = \frac{1}{3}$

26. If the points (1, 0), (0, 1) and (x, 8) are collinear, then the value of x is equal to **[2]**

a) -7 b) 5

c) -6 d) 6

27. If slope of one of the lines $ax^2 + 2hxy + by^2 = 0$ is 5 times that of the other, then $5h^2$ = **[2]**

a) 9ab
b) 2ab
c) ab
d) 7ab

28. If the planes 3x - 2y + 2z + 17 = 0 and 4x + 3y - kz - 25 are mutually perpendicular, then k = **[2]**

a) -3
b) 3
c) 9
d) -6

29. If the points (1, 1, k) and (-3, 0, 1) are equidistant from the plane 3x + 4y - 12z + 13 = 0, then k = **[2]**

a) 2
b) 3
c) 0
d) 1

30. The coordinates of the corner points of the bounded feasible region are (10, 0), (2, 4), (1, 5) and (0, 8). The maximum of objective function z = 60x + 10y is **[2]**

a) 800
b) 600
c) 110
d) 700

31. If y = $\tan^{-1}\left(\frac{e^{2x}+1}{e^{2x}-1}\right)$, then $\frac{dy}{dx}$ is **[2]**

a) $-\frac{2e^{2x}}{1+e^{2x}}$
b) $\frac{e^{2x}}{1-e^{2x}}$
c) $\frac{e^{2x}}{1+e^{2x}}$
d) $-\frac{2e^{2x}}{1+e^{4x}}$

32. If x = $\frac{1-t^2}{1+t^2}$ and y = $\frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ = **[2]**

a) $\frac{-y}{x}$
b) $\frac{x}{y}$
c) $\frac{-x}{y}$
d) $\frac{y}{x}$

33. If y = $\tan^{-1}\left[\frac{\log ex}{\log \frac{e}{x}}\right] + \tan^{-1}\left[\frac{8-\log x}{1+8\log x}\right]$, then $\frac{d^2y}{dx^2}$ is **[2]**

a) $\frac{1}{2}$
b) 1
c) 0
d) -1

34. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3\left(x^3 - y^3\right)$, then $\frac{dy}{dx}$ = **[2]**

a) $\frac{x^2}{y^2}\sqrt{\frac{1-x^6}{1-y^6}}$
b) $\frac{y^2}{x^2}\sqrt{\frac{1-y^6}{1-x^6}}$
c) $\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$
d) $\frac{x^3}{y^3}\sqrt{\frac{1-x^4}{1-y^4}}$

35. The points on the curve y = 12x - x^3 at which the gradient is zero are **[2]**

a) (2, -16), (-2, 16)
b) (0, 2), (2, 16)
c) (2, 16), (-2, -16)
d) (0, -2), (2, -16)

36. The weight W of a certain stock of fish is given by W = nw, where n is the size of stock and w is the average weight of a fish. If n and w change with time t as n = $2t^2$ + 3 and w = t^2 - 1 + 2, then the rate of change of W with respect to t at t = 1 is **[2]**

a) 13
b) 5
c) 1
d) 8

37. The distance in seconds, described by a particle in t seconds is given by s = $ae^t + \frac{b}{e^t}$. Then acceleration of the **[2]**

particle at time t is

a) Proportional to t
b) Proportional to v
c) s
d) Constant

38. If $\int \frac{dx}{1+\sin x}$ = tan$\left(\frac{x}{2} + \text{a}\right)$ + b, then **[2]**

a) a = $\frac{\pi}{4}$, b = 3
b) a = $-\frac{\pi}{4}$, b = arbitrary constant
c) a = $-\frac{\pi}{4}$, b = 3
d) a = $\frac{\pi}{4}$, b = arbitrary constant

39. $\int \sqrt{\tan x}\ \mathrm{d}x =$ **[2]**

a) $\frac{1}{\sqrt{2}}$ tan-1 $\left(\frac{x^2-1}{\sqrt{2x}}\right) + \frac{1}{\sqrt{2}}$ log $\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|$ + c
b) $\frac{1}{\sqrt{2}}$ tan-1 $\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}$ log $\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|$ + c
c) $\frac{1}{2\sqrt{2}}$ tan-1 $\left(\frac{x^2-1}{\sqrt{2x}}\right) + \frac{1}{\sqrt{2}}$ log $\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|$ + c
d) $\frac{1}{2\sqrt{2}}$ tan-1 $\left(\frac{x^2-1}{\sqrt{2x}}\right) + \frac{1}{2\sqrt{2}}$ log $\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|$ + c

40. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx =$ **[2]**

a) $\frac{2}{\sqrt{\tan x}} + c$
b) $2\sqrt{\tan x} + c$
c) $2\sqrt{\sec x} + \text{c}$
d) $\frac{2}{\sqrt{\sec x}} + c$

41. The value of integral $\int \frac{1}{\left[(x-1)^3(x+2)^5\right]^{\frac{1}{4}}}$ dx is **[2]**

a) $\frac{4}{3}\left(\frac{x+1}{x+2}\right)^{\frac{1}{4}} + c$
b) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + c$
c) $\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + c$
d) $\frac{4}{3}\left(\frac{x-1}{x-2}\right)^{\frac{1}{4}} + c$

42. The area bounded by y = $1 + \frac{8}{x^2}$, X-axis and the x ordinates x = 2, x = 4 is **[2]**

a) 2
b) log 2
c) log 4
d) 4

43. The curve passes through the point (3, 4). The normal to the curve at the point (x, y) passes through the point (3, 0). The equation of the curve is **[2]**

a) $x^2 - y^2 - 6x + 7 = 0$
b) $x^2 + y^2 - 6x + 7 = 0$
c) $x^2 + y^2 - 6x - 7 = 0$
d) $x^2 + y^2 + 6x - 7 = 0$

44. Which of the following differential equations has the same order and degree? **[2]**

a) $\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} + 8\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^6 + 5y = \mathrm{e}^x$
b) $\left[1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3\right]^{\frac{2}{3}} = 4\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}$
c) $5\left(\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}\right)^4 + 8\left(1 + \frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 5y = x^8$
d) $y = x^2\frac{\mathrm{d}y}{\mathrm{d}x} + \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$

45. The solution of the equation $\frac{dy}{dx}$ + 2y tan x = sin x satisfying y = 0 when x = $\frac{\pi}{3}$, is **[2]**

a) $y = 2\sin^2 x + \cos x - 2$
b) $y = 2\cos x - \sin^2 x - 1$
c) $y = 2\cos^2 x - \sin x + 2$
d) $y = 2\sin^2 x - \cos x - 2$

46. A random variable X has the following probability distribution: **[2]**

X	-2	-1	0	1	2	3
P(X)	0.1	K	0.2	2k	0.3	k

Then the value of k is

a) 0.01

b) 0.1

c) 0.05

d) 0.3

47. A random variable X has the following probability distribution: **[2]**

X	1	2	3	4
P(X)	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{1}{7}$

Then, the variance of this distribution is

a) $\frac{40}{49}$

b) $\frac{49}{40}$

c) $\frac{29}{20}$

d) $\frac{20}{29}$

48. The p.m.f. of a r.v. X is P(x) = $\begin{cases} \frac{1}{15}, & \text{for x} = 1, 2, \ldots, 14, 15 \\ 0, & \text{otherwise.} \end{cases}$. Then, E(X) is equal to **[2]**

a) 2

b) 4

c) 6

d) 8

49. A card is drawn at random 4 times, with replacement, from a pack of 52 playing cards. If getting a red card is considered as success, then the mean and variance of the distribution are respectively **[2]**

a) $\frac{3}{4}$, 1

b) 2, 1

c) 1, 2

d) 1, $\frac{3}{4}$

50. In eight throws of a die 1 or 3 is considered a success. Then the standard deviation of the success is **[2]**

a) $\frac{2}{3}$

b) $\frac{4}{3}$

c) $\frac{8}{3}$

d) $\frac{16}{9}$