SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

MHT - CET - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 100**

1. If x = cos 10° cos 20° cos 40°, then the value of x is **[2]**

a) $\frac{1}{4}$ tan 10^o b) $\frac{1}{8}$ cot 10^o

c) $\frac{1}{8}$ sec 10^o d) $\frac{1}{8}$ cosec 10^o

2. Equations of diagonals of square formed by lines x = 0, y = 0, x = 1 and y = 1 are **[2]**

a) y = x, x + y = 2 b) 2y = x, y + x = $\frac{1}{3}$

c) y = x, y + x = 1 d) y = 2x, y + 2x = 1

3. A circle has radius 3 units and its centre lies on the line y = x - 1. Then the equation of this circle if it passes through point (7, 3), is **[2]**

a) $x^2 + y^2 - 8x - 6y + 16 = 0$ b) $x^2 + y^2 + 8x + 6y + 16 = 0$

c) $x^2 + y^2 + 8x - 6y - 16 = 0$ d) $x^2 + y^2 - 8x - 6y - 16 = 0$

4. If a circle passes through the point (0, 0), (a, 0), (0, b), then its centre is **[2]**

a) $\left(\frac{a}{2}, \frac{b}{2}\right)$ b) $\left(\frac{b}{2}, -\frac{a}{2}\right)$

c) (a, b) d) (b, a)

5. A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is p, 0 < p < 1. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is **[2]**

a) $\frac{5p}{4p+1}$ b) $\frac{4p}{3p+1}$

c) $\frac{5p}{3p+2}$ d) $\frac{3p}{4p+3}$

6. If z = 1 - cos α + i sin α, then amp z = **[2]**

a) $\frac{\alpha}{2}$ b) $\frac{\pi}{2} + \frac{\alpha}{2}$

c) $\frac{\pi}{2} - \frac{\alpha}{2}$ d) $-\frac{\alpha}{2}$

7. If ${}^{12}P_r = 1320$, then r is equal to **[2]**

a) 4 b) 5

c) 3 d) 2

8. Three prizes are to be distributed among six persons. The number of ways in which this can be done, if no person gets all the prizes is **[2]**

a) 120 b) 220

c) 210 d) 216

9. If the function f : $[1, \infty) \rightarrow [1, \infty)$ is defined by f(x) = $2^{x(x - 1)}$, then $f^{-1}(x)$ is **[2]**

a) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$ b) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$

c) $\left(\frac{1}{2}\right)^{x(x-1)}$ d) Not defined

10. $\lim\limits_{x \to 1} \frac{x-1}{2x^2-7x+5} =$ **[2]**

a) $\frac{1}{11}$ b) $\frac{-1}{11}$

c) $\frac{1}{3}$ d) $\frac{-1}{3}$

11. f(x) = x + |x| is continuous for **[2]**

a) x $\in$ (-∞, oo) b) x $\in$ (-∞, ∞) - {0}

c) no value of x d) only x > 0

12. Using quantifier the open sentence ‘x^2 - 4 = 32’ defined on W is converted into true statement as **[2]**

a) $\forall$ x $\in$ W, x^2 - 4 = 32 b) $\exists$ x $\in$ W, such that x^2 - 4 $\leq$ 32

c) $\forall$x $\in$ W, x^2 - 4 > 32 d) $\exists$x $\in$ W, such that x^2 - 4 = 32

13. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ and abc $\neq$ 0, then A^{-1} **[2]**

a) $\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & c \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{0} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & c \end{bmatrix}$

c) $\begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$ d) $\begin{bmatrix} a & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$

14. If A is a singular matrix of order n, then A·(adj A) is **[2]**

a) zero matrix b) unit matrix

c) column matrix d) row matrix

15. If 0 $\leq$ A $\leq$ $\frac{\pi}{4}$, then $\tan^{-1}\left(\frac{1}{2}\tan 2A\right)$ + $\tan^{-1}$ (cot A) + $\tan^{-1}$ ($\cot^3$A) is equal to **[2]**

a) π b) 0

c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

16. The smallest positive value of x and y, satisfying x - y = $\frac{\pi}{4}$ and cot x + cot y = 2, are **[2]**

a) x = $\frac{5\pi}{12}$, y = $\frac{\pi}{6}$ b) x = $\frac{\pi}{3}$, y = $\frac{7\pi}{12}$

c) x = $\frac{\pi}{6}$, y = $\frac{5\pi}{2}$ d) x = $\frac{\pi}{4}$, y = $\frac{5\pi}{12}$

17. A solution of the equation $\tan^{-1}$ (1 + x) + $\tan^{-1}$ (1 - x) = $\frac{\pi}{2}$ is **[2]**

a) x = 1 b) x = π

c) x = 0 d) x = -1

18. The value of $\tan^{-1}\left\{\sin\left(\cos^{-1}\sqrt{\frac{2}{3}}\right)\right\}$ is **[2]**

a) $\frac{\pi}{6}$
b) $\frac{\pi}{2}$
c) $\frac{\pi}{3}$
d) $\frac{\pi}{4}$

19. $\int_0^{\frac{\pi}{2}} \frac{dx}{2+\cos x} =$ **[2]**

a) $\sqrt{3}\tan^{-1}(\sqrt{3})$
b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
c) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
d) $2\sqrt{3}\tan^{-1}(\sqrt{3})$

20. $\int_{-1}^{1} \frac{1+x^3}{9-x^2} =$ **[2]**

a) $\frac{1}{3}$ log 2
b) log 9
c) $\frac{1}{3}$ log 9
d) log 2

21. If f(x) is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non zero x, then $\int_{\cos\theta}^{\sec\theta} f(x)dx =$ **[2]**

a) $\cos^2\theta$
b) 0
c) $\cos\theta + \sec\theta$
d) $\sec^2\theta$

22. $\int_1^3 \left(\frac{x^2+1}{4x}\right)^{-1} dx =$ **[2]**

a) log 100
b) log 5
c) log 25
d) $\frac{1}{2}$ log 5

23. If $\vec{a}\cdot\hat{i} = \vec{a}\cdot(\hat{i}+\hat{j}) = \vec{a}\cdot(\hat{i}+\hat{j}+\hat{k})$, then $\vec{a}$ is equal to **[2]**

a) $\hat{i}+\hat{j}+\hat{k}$
b) $\hat{i}$
c) $\hat{j}$
d) $\hat{k}$

24. If $\vec{a} = \hat{i}+\hat{j}-\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{c} = -\hat{i}+2\hat{j}-\hat{k}$, then a unit vector perpendicular to the vectors $\vec{a}+\vec{b}$ and $\vec{b}-\vec{c}$ is **[2]**

a) $\hat{j}$
b) $\hat{i}$
c) $\hat{k}$
d) All of these

25. If the vectors $5\hat{i} - x\hat{j} + 3\hat{k}$ and $-3\mathrm{i} + 2\hat{j} - y\hat{k}$ are parallel, the values of x and y respectively are **[2]**

a) $\frac{-10}{3}, \frac{-9}{5}$
b) $\frac{10}{3}, \frac{9}{5}$
c) $\frac{9}{5}, \frac{10}{3}$
d) $\frac{-9}{5}, \frac{-10}{3}$

26. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $|\vec{a}+\vec{b}+\vec{c}| = 1$ and $\vec{a} \perp \vec{b}$. If $\vec{c}$ makes angles δ, β with $\vec{a}, \vec{b}$ respectively, then $\cos\delta + \cos\beta$ is equal to **[2]**

a) $\frac{3}{2}$
b) 1
c) 0
d) -1

27. $a(x^2 - y^2) + xy = 0$ represents a pair of straight lines for **[2]**

a) a = 1 only
b) a = 1 or -1 only

c) a = 0 only | d) all real values of 'a'

28. The perpendicular distance of the point P(6, 7, 8) from XY- plane is **[2]**

a) 8
b) 7
c) 6
d) 5

29. If x co-ordinate of a point on the line joining points (2, 2, 1) and (5, 1, -2) is 4, then its z co-ordinate will be **[2]**

a) 1
b) -1
c) -2
d) 2

30. If 4x + 5y $\leq$ 20, x + y $\geq$ 3, x $\geq$ 0, y $\geq$ 0, maximum 2x + 3y is **[2]**

a) 5
b) 20
c) 12
d) 0

31. If y sec x + tan x + x^2y = 0, then $\frac{dy}{dx}$ = **[2]**

a) $\frac{2xy+\sec^2 x+y\sec x\tan x}{x^2+\sec x}$
b) $\frac{2xy+\sec^2 x-\sec^2 x\tan x}{x^2+\sec x}$
c) $-\frac{2xy+\sec^2 x+\sec x\tan x}{x^2+\sec x}$
d) $-\frac{2xy+\sec^2 x+y\sec x\tan x}{x^2+\sec x}$

32. If y = $\frac{2(x-\sin x)^{\frac{3}{2}}}{\sqrt{x}}$, then $\frac{\mathrm{d}y}{\mathrm{d}x}$ = **[2]**

a) $\frac{(x-\sin x^2)^{\frac{3}{2}}}{\sqrt{x}}\left(\frac{3}{2}\cdot\frac{1-\cos x}{x-\sin x}-\frac{3}{2x}\right)$
b) $\frac{2(x-\sin x)^{\frac{1}{2}}}{\sqrt{x}}\left(\frac{3}{2}\cdot\frac{1-\cos x}{x-\sin x}-\frac{1}{2x}\right)$
c) $\frac{2(x-\sin x)^{\frac{3}{2}}}{\sqrt{x}}\left(\frac{3}{2}\cdot\frac{1-\cos x}{1-\sin x}-\frac{1}{2x}\right)$
d) $\frac{2(x-\sin x)^{\frac{3}{2}}}{\sqrt{x}}\left(\frac{3}{2}\cdot\frac{1-\cos x}{x-\sin x}-\frac{1}{2x}\right)$

33. If y = $\left(1+\frac{1}{x}\right)^x$, then $\frac{\mathrm{dy}}{\mathrm{dx}}$ = **[2]**

a) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)-\frac{1}{1+x}\right]$
b) $\left(x+\frac{1}{x}\right)^x\left[\log(x-1)-\frac{x}{x+1}\right]$
c) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)+\frac{1}{1+x}\right]$
d) $\left(1+\frac{1}{x}\right)^x\left[\log\left(1+\frac{1}{x}\right)\right]$

34. If y = $\sin x+\mathrm{e}^x$, then $\frac{\mathrm{d}^2x}{\mathrm{d}y^2}=$ **[2]**

a) $\frac{\sin x-\mathrm{e}^x}{(\cos x+\mathrm{e}^x)^3}$
b) $(-\sin x + e^x)^{-1}$
c) $\frac{\sin x-\mathrm{e}^x}{(\cos x+\mathrm{e}^x)^2}$
d) $\frac{\sin x+\mathrm{e}^x}{(\cos x+\mathrm{e}^x)^3}$

35. A point moves in a straight line during the time t = 0 to t = 3 according to the law s = 15t - $2t^2$. The average velocity is **[2]**

a) 15 units
b) 9 units
c) 27 units
d) 3 units

36. If for a function f(x), f'(a) = 0, f''(a) = 0, f'''(a) > 0, then at x = a, f(x) is **[2]**

a) Minimum
b) Extreme point
c) Maximum
d) Not an extreme point

37. A running track of 440 ft is to be laid out enclosing a football field, the shape of which is a rectangle with a semicircle at each end. If the area of rectangular portion is to be maximum, then lengths of sides are **[2]**

a) 110, 70
b) 140, 40

c) 120, 60

d) 130, 50

38. $\int$ x $\sin^2$ x dx = **[2]**

a) $\frac{x^2}{4}+\frac{x}{4}$ sin 2x + $\frac{1}{8}$ cos 2x + c

b) $\frac{x^2}{4}-\frac{x}{4}$ sin 2x + $\frac{1}{8}$ cos 2x + c

c) $\frac{x^2}{4}+\frac{x}{4}$ sin 2x - $\frac{1}{8}$ cos 2x + c

d) $\frac{x^2}{4}-\frac{x}{4}$ sin 2x - $\frac{1}{8}$ cos 2x + c

39. $\int\left(x+\frac{1}{x}\right)^3 \mathrm{d}x=$ **[2]**

a) $\frac{1}{4}\left(x+\frac{1}{x}\right)^4+c$

b) $\frac{x^4}{4}+\frac{3x^2}{2}+3$ log x $-\frac{1}{2x^2}+c$

c) $\frac{x^4}{4}+\frac{3x^2}{2}+3$ log x $+\frac{1}{x^2}+c$

d) $4\left(x+\frac{1}{x}\right)^4+c$

40. $\int\left(\frac{1}{x-3}-\frac{1}{x^2-3x}\right)$ dx = ________ + c, x > 3 **[2]**

a) $\frac{2}{3}$ log [$\sqrt{x}$ (x - 3)]

b) $\frac{2}{3}$ log [x (x - 3)]

c) $\frac{1}{3}$ log [x (x - 3)]

d) $\frac{1}{3}$ log [$\sqrt{x}$ (x - 3)]

41. If $\int\frac{\tan x}{1+\tan x+\tan^2 x}dx$ = x - $\frac{2}{\sqrt{A}}$ $\tan^{-1}$ $\left(\frac{2\tan x+1}{\sqrt{\mathrm{A}}}\right)$ + c, then A = **[2]**

a) 2

b) 4

c) 5

d) 3

42. Area bounded by the curve y = sin x between x = 0 and x = 2π is **[2]**

a) 16 sq. units

b) 2 sq. units

c) 8 sq. units

d) 4 sq. units

43. The integrating factor (I.F.) of differential equation $\frac{dy}{dx}$(1 + x) - xy = 1 - x is **[2]**

a) $(1 + x)e^{-x}$

b) $(1 - x)e^{-x}$

c) $(x - 1)e^{-x}$

d) $(1 + x)e^{x}$

44. The differential equation representing the family of curves $y^2=2\,\mathrm{d}(x+\sqrt{\mathrm{d}})$, where d is a positive parameter, is of **[2]**

a) degree 2

b) degree 3

c) degree 4

d) order 2

45. The solution of the differential equation (2x - 4y + 3)$\frac{dy}{dx}$ + (x - 2y + 1) = 0 is **[2]**

a) log [2(2x - 4y) + 3] = 2(x - 2y) + c

b) log [(2x - 4y) + 3] = x - 2y + c

c) log [4(x - 2y) + 5] = 4(x + 2y) + c

d) log [2(x - 2y) + 5] = 2(x + y) + c

46. The p.d.f of a r.v. X is $f_X(x) = \begin{cases}\frac{k}{\sqrt{x}}, & 0<x<4\\ 0; & \text{otherwise}\end{cases}$, then the c.d.f. of X is given by **[2]**

a) $\sqrt{x}$

b) $2\sqrt{x}$

c) x

d) $\frac{\sqrt{x}}{2}$

47. Let f(x) = $\begin{cases}\frac{1}{x^2}, & 1<x<\infty\\ 0; & \text{otherwise}\end{cases}$ be the p.d.f. of a r.v. X. If C = {x : 1 < x < 2} and C_2 = {x : 4 < x < 5}, then $P(C_1 \cup C_2)$ **[2]**

a) $\frac{13}{20}$

b) $\frac{11}{20}$

c) $\frac{7}{20}$ d) $\frac{1}{20}$

48. The probability distribution of a r.v. X is **[2]**

X = x	-2	-1	0	1	2
P(X = x)	0.2	0.3	0.15	0.25	0.1

Then F (-1) =

a) 0.5 b) 0.7

c) 1 d) 0.9

49. A coin is tossed 10 times. The probability of getting exactly six heads is **[2]**

a) $\frac{100}{153}$ b) $\frac{105}{512}$

c) $^{10}C_6$ d) $\frac{512}{513}$

50. Let X be the number of successes in 'n' independent Bernoulli trials with probability of success p = $\frac{3}{4}$. The least value of **n** so that P(X $\geq$ 1) $\geq$ 0.9375 is **[2]**

a) 4 b) 2

c) 1 d) 3