SATISH SCIENCE ACADEMY

DHANORI PUNE-411015

MATHEMATICS

MHT - CET - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 100**

1. tan A + cot (180° + A) + cot (90° + A) + cot (360°- A) = **[2]**

   a) 0
   b) 2(tan A - cot A)
   c) 2 tan A
   d) 2 cot A

2. The angle between the lines whose intercepts on the axes are (a, -b) and (b, -a) respectively, is **[2]**

   a) $\tan^{-1} \frac{a^2+b^2}{ab}$
   b) $\tan^{-1} \frac{b^2-a^2}{ab}$
   c) $\tan^{-1} \frac{b^2-a^2}{2ab}$
   d) $\tan^{-1} \frac{b^2-a^2}{2}$

3. A circle having centre at the origin passes through the three vertices of an equilateral triangle. The length of its median being 9 units. Then the equation of that circle is **[2]**

   a) $x^2 + y^2 = 36$
   b) $x^2 + y^2 = 9$
   c) $x^2 + y^2 = 18$
   d) $x^2 + y^2 = 81$

4. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to **[2]**

   a) $\frac{32}{5}$
   b) $\frac{64}{5}$
   c) $\frac{56}{5}$
   d) $\frac{52}{5}$

5. Given that $x \in [0, 1]$ and $y \in [0, 1]$. Let A be the event of (x, y) satisfying $y^2 \leq x$ and B be the event of (x, y) satisfying $x^2 \leq y$. Then. **[2]**

   a) A, B are exhaustive
   b) $P(A \cap B) = \frac{1}{3}$
   c) A, B are mutually exclusive
   d) A, B are independent

6. If the imaginary part of $\frac{2+i}{ai-1}$ is zero, where a is a real number, then the value of a is equal to **[2]**

   a) -2
   b) 2
   c) $-\frac{1}{2}$
   d) $\frac{1}{2}$

7. ${}^{10}P_3$ = **[2]**

   a) 760
   b) 560
   c) 720
   d) 680

8. How many 5 digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once? **[2]**

   a) 335
   b) 338

c) 337 | d) 336

9. Consider the function f(x) = cos x^2. Then **[2]**

a) f is not periodic π | b) f is of period $\sqrt{2\pi}$

c) f is not periodic | d) f is of period 2π

10. $\lim\limits_{x \to 0} \frac{12^x - 3^x - 4^x + 1}{x \sin x}$ is equal to **[2]**

a) log $\sqrt{3}$ ·log 4 | b) log $\sqrt{3}$·log 2

c) log 2·log 3 | d) log 4·log 3

11. If f(x) is continuous on [-4, 2], where f(x) = $\begin{cases} 6b - 3ax, & \text{for } -4 \leq x < -2 \\ 4x + 1, & \text{for } -2 \leq x \leq 2 \end{cases}$, then a + b = **[2]**

a) $\frac{1}{6}$ | b) $-\frac{7}{6}$

c) $\frac{7}{6}$ | d) $-\frac{1}{6}$

12. Negation of the proposition $(p \vee q) \wedge (\sim q \wedge r)$ is **[2]**

a) $(\sim \mathrm{p} \wedge \sim \mathrm{q}) \vee (\mathrm{q} \vee \sim \mathrm{r})$ | b) $(p \wedge q) \wedge (q \wedge \sim r)$

c) $(\sim \mathrm{p} \vee \sim q) \wedge (\sim q \wedge r)$ | d) $(\mathrm{p} \wedge \mathrm{q}) \vee (\mathrm{q} \vee \sim \mathrm{r})$

13. If A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then M_{21} = **[2]**

a) 1 | b) 3

c) 2 | d) -1

14. If k is a scalar and I is a unit matrix of order 3, then adj (kI) = **[2]**

a) k^2I | b) $-k^3I$

c) $-k^2I$ | d) k^3I

15. $\cos\left(\sin^{-1} \frac{5}{13}\right)$ = **[2]**

a) $\frac{5}{12}$ | b) $\frac{12}{13}$

c) $-\frac{5}{12}$ | d) $-\frac{12}{13}$

16. $\cos^{-1} \frac{4}{5} + \tan^{-1} \frac{3}{5} =$ **[2]**

a) $\cot^{-1} \frac{27}{11}$ | b) $\tan^{-1} \frac{27}{11}$

c) $\sin^{-1} \frac{11}{27}$ | d) $\cos^{-1} \frac{11}{27}$

17. In $\triangle$ABC, 1 - tan $\frac{A}{2}$ tan $\frac{B}{2}$ = **[2]**

a) $\frac{2c}{a+b+c}$ | b) $\frac{2}{a+b+c}$

c) $\frac{a}{a+b+c}$ | d) $\frac{4a}{a+b+c}$

18. $2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{5\sqrt{2}}{7} + 2 \tan^{-1} \frac{1}{8}$ = **[2]**

a) $\frac{\pi}{4}$ | b) $\frac{\pi}{6}$

c) $\frac{\pi}{8}$ | d) $\frac{\pi}{3}$

19. The value of $\lim\limits_{x \to 0} \frac{1}{x}\left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt\right]$ is equal to **[2]**

a) $e^{2 \sin y}$

b) $e^{\sin^2 y}$

c) $e^{\operatorname{cosec}^2 y}$

d) $e^{|\sin y|}$

20. If f(x) is defined on [-2, 2] by f(x) = $4x^2 - 3x + 1$ and g(x) = $\frac{f(-x)-f(x)}{x^2+3}$, then $\int_{-2}^{2} g(x)$ dx = **[2]**

a) 0

b) 24

c) -48

d) 64

21. If $\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$, then k = **[2]**

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{8}$

d) 1

22. If $\int_1^k (2x-3)$ dx = 12, then k = **[2]**

a) -5

b) 5

c) 2

d) -2 and 5

23. The vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are of the same length and taken pairwise, they form equal angles. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$, then the co-ordinates of $\vec{c}$ are **[2]**

a) (1, 0, 1)

b) (1, 2, 3)

c) (2, 1, 3)

d) (-1, 1, 2)

24. If $\vec{a} + \vec{b} + \vec{c} = \lambda\vec{d}$ and $\vec{b} + \vec{c} + \vec{d} = \mu\vec{a}$ and $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, then $\vec{a} + \vec{b} + \vec{c} + \vec{d}$ is equal to **[2]**

a) $\mu\vec{b}$

b) $\vec{0}$

c) $(\lambda + \mu)\vec{a}$

d) $\lambda\vec{a}$

25. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number then $\left[\lambda(\vec{a}+\vec{b})\lambda^2\vec{b} \quad \lambda\vec{c}\right] = \left[\vec{a}\vec{b} + \vec{c}\vec{b}\right]$ for **[2]**

a) no value of λ

b) exactly one value of λ

c) exactly two values of λ

d) exactly three values of λ

26. If $\vec{\alpha} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{\beta} = -\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{\gamma} = \hat{i} + \hat{j} + \hat{k}$, then $(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\gamma})$ is equal to **[2]**

a) -74

b) 64

c) 60

d) 74

27. Equation of pair of straight lines drawn through (1, 1) and perpendicular to the pair of lines $3x^2 - 7xy + 2y^2 = 0$ is **[2]**

a) $2(x - 1)^2 + 7(x - 1)(y - 1) - 3y^2 = 0$

b) $2(x - 1)^2 + 7(x - 1)(y - 1) - 3(y - 1)^2 = 0$

c) $2x^2 + 7xy - 11x + 6 = 0$

d) $2(x - 1)^2 + 7(x - 1)(y - 1) + 3(y - 1)^2 = 0$

28. The equation of line is $\vec{r} = \vec{p} + t(\vec{q} - \vec{p})$, where P$(\vec{p}) \equiv (3, 4, 1)$ and Q$(\vec{q}) \equiv (5, 1, 6)$. The value of t for which the line crosses XY-plane is **[2]**

a) $t = \frac{1}{6}$

b) $t = \frac{-1}{\sqrt{5}}$

c) $t = \frac{1}{4}$

d) $t = \frac{-1}{5}$

29. The point of intersection of the lines $\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}$, $\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$ is **[2]**

a) $\left(21, \frac{5}{3}, \frac{10}{3}\right)$
b) (2, 10, 4)
c) (5, 7, -2)
d) (-3, 3, 6)

30. A factory owner wants to purchase 2 types of machines, A and B for his factory. The machine A requires an area of 1000 m^2 and 12 skilled men for running it and its daily output is 50 units, whereas the machine B requires 1200 m^2 area and 8 skilled men, and its daily output is 40 units. If an area of 7600 m^2 and 72 skilled men are available to operate the machines. The linear constraints are **[2]**

a) 1000x + 1200y $\leq$ 7600, 12x + 8y $\leq$ 72, x $\geq$ 0, y $\geq$ 0
b) 1000x + 1200y $\geq$ 7600, 12x + 8y $\leq$ 72, x $\geq$ 0, y $\geq$ 0
c) 1000x + 1200y $\leq$ 7600, 12x + 8y $\geq$ 72, x $\geq$ 0, y $\geq$ 0
d) 1000x + 1200y $\geq$ 7600, 12x + 8y $\geq$ 72, x $\geq$ 0,y $\geq$ 0

31. The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is **[2]**

a) $-(\log_{10} x)^2$
b) $(\log_x 10)^2$
c) $\frac{x^2}{100}$
d) 1

32. If y = $\sqrt{\frac{1+x}{1-x}}$, then $\frac{dy}{dx}$ = **[2]**

a) $\frac{1}{2(1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}}$
b) $\frac{1}{(1+x)^2(1-x)^{\frac{3}{2}}}$
c) $\frac{2}{(1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}}$
d) $\frac{1}{(1+x)^{\frac{3}{2}}(1-x)^{\frac{1}{2}}}$

33. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is **[2]**

a) 4
b) 2
c) 1
d) -1

34. If f(x) = $\cot^{-1}(\cos 2x)^{\frac{1}{2}}$, then $f'\left(\frac{\pi}{6}\right)$ is **[2]**

a) $\frac{2}{\sqrt{3}}$
b) $-\frac{2}{\sqrt{3}}$
c) $\sqrt{\frac{2}{3}}$
d) $\frac{1}{\sqrt{3}}$

35. The function f(x) = 2 - 3x is **[2]**

a) decreasing
b) increasing
c) remain constant
d) neither decreasing nor increasing

36. Slope of normal to the curve y = x^3 - x at x = 2 is **[2]**

a) $-\frac{1}{11}$
b) $-\frac{1}{7}$
c) $-\frac{1}{9}$
d) $-\frac{1}{3}$

37. The equation of the tangent to the curve x = 2 $\cos^3\theta$ and y = 3 $\sin^3\theta$ at the point $\theta=\frac{\pi}{4}$ is **[2]**

a) 2x - 3y = $3\sqrt{2}$
b) 3x - 2y = $3\sqrt{2}$
c) 2x + 3y = $3\sqrt{2}$
d) 3x + 2y = $3\sqrt{2}$

38. To find the value of $\int \frac{1+\log x}{x}$ dx, the proper substitution is **[2]**

a) 1 + x = t
b) $\frac{1}{x}$ = t
c) 1 + log x = t
d) log x = t

39. $\int$ x $\cos^2$x dx = **[2]**

a) $\frac{x^4}{4}+\frac{1}{4}$x sin 2x + $\frac{1}{8}$ cos 2x + c
b) $\frac{x^4}{4}-\frac{1}{4}$x sin 2x + $\frac{1}{8}$ cos 2x + c
c) $\frac{x^4}{4}-\frac{1}{4}$x sin 2x - $\frac{1}{8}$ cos 2x + c
d) $\frac{x^4}{4}+\frac{1}{4}$x sin 2x - $\frac{1}{8}$ cos 2x + c

40. $\int \frac{x}{x^4-1}$ dx = **[2]**

a) $\frac{1}{4}\log\left|\frac{x^2+1}{x^2-1}\right|$ + c
b) $\frac{1}{2}\log\left|\frac{x^2+1}{x^2-1}\right|$ + c
c) $\frac{1}{4}\log\left|\frac{x^2-1}{x^2+1}\right|$ + c
d) $\frac{1}{2}\log\left|\frac{x^2-1}{x^2+1}\right|$ + c

41. $\int\left(\frac{x+2}{x+4}\right)^2$ e^x dx is equal to **[2]**

a) $e^x\left(\frac{x+2}{x+4}\right)+c$
b) $e^x\left(\frac{x-2}{x+4}\right)+c$
c) $\left(\frac{2xe^x}{x+4}\right)+c$
d) $e^x\left(\frac{x}{x+4}\right)+c$

42. Area enclosed between the curve $y^2(2a - x) = x^3$ and line x = 2a above X-axis is **[2]**

a) $2\pi a^2$
b) $\frac{3\pi a^2}{2}$
c) πa^2
d) $3\pi a^2$

43. Degree of the given differential equation $\left(\frac{d^2y}{dx^2}\right)^3=\left(1+\frac{dy}{dx}\right)^{\frac{1}{2}}$ is **[2]**

a) 2
b) $\frac{1}{2}$
c) 3
d) 6

44. The solution of the differential equation $\frac{dy}{dx}$ = sin(x + y) tan (x + y) - 1 is **[2]**

a) cosec (x + y) + tan(x + y) = x + c
b) x + cosec (x + y) = c
c) x + sec(x + y) = c
d) x + tan(x + y) = c

45. If a curve y = f(x) passes through the point (1, -1) and satisfies the differential equation, y (1 + xy) dx = x dy, then $f\left(-\frac{1}{2}\right)$ is equal to **[2]**

a) $\frac{4}{5}$
b) $\frac{2}{5}$
c) $-\frac{2}{5}$
d) $-\frac{4}{5}$

46. Let X denote the number of fruits that are grown in a farm house in a particular day. The probability that X can take the value of x has the following form, where k is some unknown constant. **[2]**

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Then the value of k is

a) $\frac{1}{2}$
b) $\frac{1}{6}$
c) $\frac{1}{3}$
d) $\frac{1}{8}$

47. The p.m.f. of a random variable X is **[2]**

X = x	1	2	3	4	5
P(X = x)	$\frac{1}{20}$	$\frac{3}{20}$	a	b	$\frac{1}{20}$

If b = 2a, then

a) a = $\frac{1}{3}$, b = $\frac{1}{2}$

b) a = $\frac{1}{4}$, b = $\frac{1}{2}$

c) a = $\frac{1}{2}$, b = $\frac{1}{3}$

d) a = $\frac{1}{2}$, b = $\frac{1}{4}$

48. Let X = time (in minutes) that lapses between the ringing of the bell at the end of a lecture and the actual time when the professor ends the lecture. Suppose X has p.d.f. f(x) = $\begin{cases} kx^2, & 0 \le x \le 2 \\ 0; & \text{otherwise} \end{cases}$. Then, the probability that lecture ends within 1 minute of the bell ringing is **[2]**

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{8}$

d) $\frac{1}{16}$

49. The probability that an event A occurs in a single trial of an experiment is 0.3. Six independent trials of the experiment are performed. What is the variance of probability distribution of occurrence of event A? **[2]**

a) 12.6

b) 1.26

c) 0.18

d) 1.8

50. In a trial the probability of success is twice the probability of failure. In six trials the probability of at least four successes will be **[2]**

a) $\frac{400}{729}$

b) $\frac{500}{729}$

c) $\frac{496}{729}$

d) $\frac{600}{729}$